Question 1 Suppose that a person's utility function is $u(x_1, x_2) = x_1^3 x_2$ and that prices are $p_1 = 9$, $p_2 = 12$. Then the equation of the income offer curve is $3x_2/x_1 = 9/12$, i.e.,

$$\boxed{\mathbf{x_2 = 0.25x_1}}$$

Suppose you know that the person consumes 36 units of good 1. Then $x_2 = 9$.

$$\boxed{\textbf{Then the person's income is } \boldsymbol{I = 432}}$$

Question 2 Suppose that person's utility function is $u(x_1, x_2) = \ln(x_1) + 4x_2$. Prices are p_1, p_2 and income is I. Then MRS $= 1/(4x_1) = p_1/p_2$. Hence, $x_1 = p_2/(4p_1)$. Then the demand function for goods 1 and are given by

$$\boxed{\boldsymbol{x_1(p_1, p_2, I) = \frac{p_2}{4p_1},\ x_2(p_1, p_2, I) = \frac{4I - p_2}{4p_2}.}}$$

Thus, the indirect utility function is

$$\boxed{\boldsymbol{v(p_1, p_2, I) = \ln(p_2) - \ln(4p_1) + \frac{4I - p_2}{p_2}.}}$$

Question 3 A utility function is given by $u(x_1, x_2) = 4\sqrt{x_1} + x_2$. The equation of the income offer curve is $\frac{2}{\sqrt{x_1}} = p_1/p_2$. Thus, $x_1 = 4p_2^2/p_1^2$. The equation of the indifference curve is $4\sqrt{x_1} + x_2 = u$. Thus,

Then Hicksean demand is

$$\boxed{\boldsymbol{h_1(p_1, p_2, u) = 4p_2^2/p_1^2,\ h_2(p_1, p_2, u) = u - 8\frac{p_2}{p_1}.} \qquad .}$$

The expenditure function is

$$\boxed{\boldsymbol{e(p_1, p_2, u) = up_2 - 4\frac{p_2^2}{p_1}.}}$$

Question 4 A person's utility function is $u(x_1, x_2) = (1/4)x_1^2 x_2$. $x_1 = 2I/(3p_1)$, $x_2 = I/(3p_2)$. If $p_1 = 1$, $p_2 = 1$, $I = 30$, then demand is $x_1 = 20$, $x_2 = 10$. Thus, utility is 1,000. We now have to determine how much money the person needs at prices $p_1 = 8$, $p_2 = 1$ to obtain utility 1,000.

The indirect utility function is $v(p_1, p_2, I) = \frac{I^3}{27p_1^2 p_2}$. Thus, $I^3 = 27p_1^2 p_2 u$, i.e., $e(p_1, p_2, u) = 3p_1^{2/3} p_2^{1/3} u^{1/3}$. Thus, the person needs an income of $e(8, 1, 1000) = 120$

$$\boxed{\textbf{The lump sum subsidy is } \boldsymbol{s = 90}}$$

Demand for good 1 after the subsidy is 10.

The government's tax revenue (after the lump sum subsidy is introduced is) $\boxed{\mathbf{70}}$.

Why is the lump-sum subsidy not equal to the amount of money raised by the tax? (Your answer must fit into the box below).

$120 - 90 = 30$ `is the deadweight loss of the tax measured by using the compensating variation.  The tax generates a distortion (consumers substitute away from good 1 to avoid some of the tax).`

Question 5 Suppose that all income offer curves are straight lines starting at $(0, 0)$. At prices $p_1 = 2$, $p_2 = 4$ the person's optimal consumption is $(20, 10)$. Thus, income is $I = 80$. When the price of good 1 increases to $p_1 = 4$ demand of good 1 decreases to 5. Thus, $20 + 4x_2 = 80$. Thus, $x_2 = 15$. The optimal consumption is $(5, 15)$. Thus, the equation of the income offer curve is $x_2 = 3x_1$.

In order to be able to afford $(20, 10)$ at the new prices, the person's income must be $I' = 120$. Thus, $4x_1 + 4x_2 = 120$ and $x_2 = 3x_1$. Compensated demand is therefore $(7.5, 22.5)$.

Then Slutzky substitution effects for goods 1 and 2 are

$\boxed{\boldsymbol{\Delta_1^s = -12.5, \Delta_2^s = 12.5.}}$

The Slutzky income effects are

$\boxed{\boldsymbol{\Delta_1^I = -2.5, \Delta_2^I = -7.5.}}$

Question 6 Suppose that there is a riskless asset that has a return of 10%, and a risky asset that has a return of 60% in state g and -20% in state b. The two states occur with probability 0.5 each. The person has 1,000 Dollars to invest, and preferences are described by a Bernoulli utility function $u(x) = \ln(x)$.

$\boxed{\textbf{State } \boldsymbol{g}\textbf{: \$ 1.1(1000} - \boldsymbol{m}\textbf{) + 1.6}\boldsymbol{m}\textbf{, state } \boldsymbol{b}\textbf{: \$ 1.1(1000} - \boldsymbol{m}\textbf{) + 0.8}\boldsymbol{m}\textbf{.}}$

The person solves

$$\max_m 0.5 \ln(1.1(1000 - m) + 1.6m) + 0.5 \ln(1.1(1000 - m) + 0.8m),$$

The first order condition is $0.25/(1100 + .5m) = 0.15/(1100.0 - 0.3m)$.

$\boxed{\textbf{The person invests \$733.33 into the risky asset.}}$

Question 7 A person's Bernoulli utility function is given by $u(x) = \sqrt{x}$. Consider the following lottery: With probability 0.6 the payoff is 1, with probability 0.25 the payoff is 4, with probability 0.1 the payoff is 25 and with probability 0.05 the payoff is 400. The person's current wealth is zero. Then

$\boxed{\textbf{The expected payoff of the lottery is 24.10}}$

$\boxed{\textbf{The person's expected utility from the lottery is 2.6}}$

$\boxed{\textbf{The lottery's certainty equivalent is 6.76}}$

Note: Recall that the certainty equivalent is a payment y that the person receives with certainty such that expected utility of y is the same as that of the lottery.

Question 8 Suppose there are two risky assets, A, and B and riskless asset. The riskless asset has a return of 4%. The returns of assets A and B are 20% and 30% respectively. Denote by x_r, x_A, and x_B the fraction of your money invested in the three assets. Then $x_r + x_A + x_B = 1$. The return of the portfolio is $0.04x_r + 0.2x_A + 0.3x_B$ (Note that this is the return rather than the gross return). Suppose that the variance of the portfolio is $0.01x_A^2 + 0.01x_A x_B + 0.04x_B^2$. The person has mean variance preferences of the form $u(\mu, \sigma) = \mu - 10\sigma^2$, where μ is the expected return and σ the standard deviation of the portfolio.

The person solves

$$\max_{x_r, x_A, x_B} 0.04x_r + 0.2x_A + 0.3x_B - 10(0.01x_A^2 + 0.01x_A x_B + 0.04x_B^2),$$

subject to $x_r + x_A + x_B = 1$.

The optimal portfolio is $\boxed{\boldsymbol{x_r = 0.08, x_A = 0.68, x_B = 0.24.}}$