Question 1 A utility function is given by $u(x_1, x_2) = -(1/x_1) - (1/x_2)$. Suppose that prices are $p_1 = 4$, $p_2 = 1$.

The partial derivatives are $1/x_1^2$ and $1/x_2^2$. Thus, MRS $= x_2^2/x_1^2 = 4$

Then the equation of the income offer curve is

$$\boxed{x_2 = 2x_1}$$

Question 2 The indirect utility function is given by

$$\boxed{v(p_1, p_2, I) = \frac{I}{p_1^{0.4} p_2^{0.6}}}$$

The expenditure function is

$$\boxed{e(p_1, p_2, u) = u p_1^{0.4} p_2^{0.6}}$$

and the Hicksean demand functions are

$$\boxed{h_1(p_1, p_2, u) = 0.4u\left(\frac{p_2}{p_1}\right)^{0.6}, h_2(p_1, p_2, u) = 0.6u\left(\frac{p_1}{p_2}\right)^{0.4}}$$

Question 3 A utility function is given by $u(x_1, x_2) = x_1 + \ln x_2$. Suppose that prices are $p_1 = 2$, $p_2 = 1$ and income is I. Specify the utility maximization problem in the box below

$\max_{x_1,x_2} x_1 + \ln x_2$, subject to (i) $2x_1 + x_2 \leq I$, (ii) $x_1 \geq 0$, (iii) $x_2 \geq 0$.

Thus, the Lagrangean is given by (specify *all* constraints for the Lagrangean).

$\mathfrak{L} = x_1 + \ln x_2 - \lambda_1(2x_1 + x_2 - I) + \lambda_2 x_1 + \lambda_3 x_2$

The first order conditions are

$$\boxed{\frac{\partial \mathfrak{L}}{\partial x_1} : \quad 1 - 2\lambda_1 + \lambda_2 = 0}$$

$$\boxed{\frac{\partial \mathfrak{L}}{\partial x_1} : \quad (1/x_2) - \lambda_1 + \lambda_3 = 0}$$

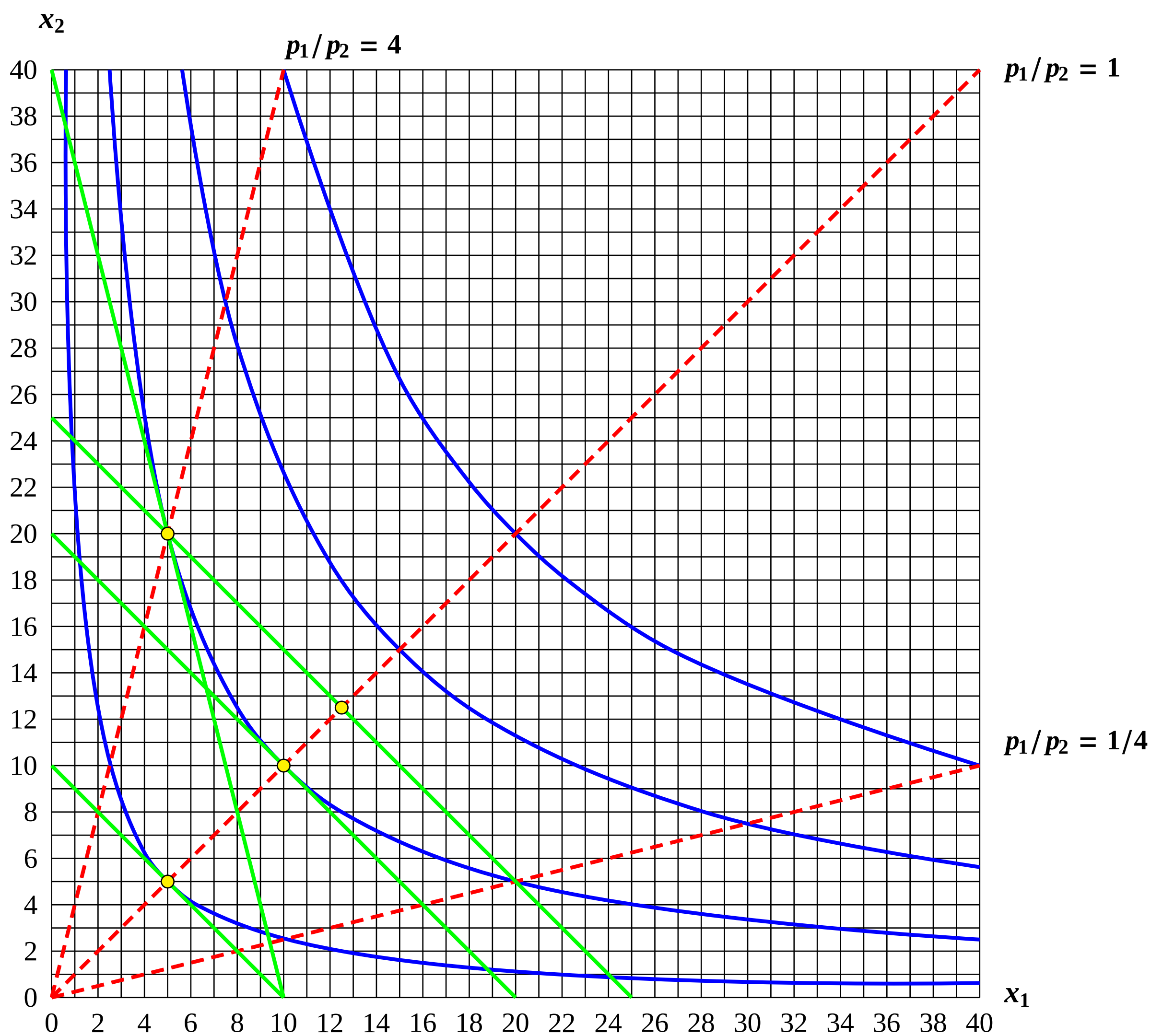
Question 4 Income offer curves and indifference curves are depicted below. Originally price are $p_1 = 4$, $p_2 = 1$ and income is $I = 40$. Then the price of good 2 increases to $p_2 = 4$. Determine graphically the Hicks and the Slutzky substituion and income effect.

The Slutzky substitution effect for goods 1 and 2 is $\boxed{\Delta^s x_1 = 7.5, \Delta^s x_2 = -7.5}$

The Slutzky income effect for goods 1 and 2 is **$\Delta^I x_1 = -7.5, \Delta^I x_2 = -7.5$**

The Hicks substitution effect for goods 1 and 2 is **$\Delta^s x_1 = 5, \Delta^s x_2 = -10$**

The Hicks income effect for goods 1 and 2 is **$\Delta^I x_1 = -5, \Delta^I x_2 = -5$**



Question 5 A person's utility function is given by $u(L, c) = L^2c$, where L is leisure and c is consumption. The person's income is derived from work.

(a) Suppose the hourly wage is $w = 10$. Thus, MRS $= 2c/L = 10$. Hence $c = 5L$. $c = 10\ell$, where $\ell = 24 - L$. Thus, $c = 240 - 10L$, i.e., $c + 10L = 240$. Thus, $15L = 240$, which implies $L = 16$ and $c = 80$. Thus, utility is

Then, the person's maximum utility is **20,480**.

(b) At the new prices we have $2c/L = 6$, i.e., $c = 3L$. In addition, $L^2c = 20,480$, i.e., $3L^3 = 20,480$. Hence, $L = 18.97$, and $c = 56.91$. $\ell = 24 - L$ implies $\ell = 5.03$. Since the after-tax wage is 6, the total wage income is 30.18. However, to afford 56.91 the person needs a lump sum subsidy of 26.73. The tax revenue is $4\ell = 20.12$. Thus, the deadweight loss is 6.61.

Tax revenue is: 20.12

The lump sum subsidy is: 26.73

The deadweight loss is: 6.61

Question 6 Suppose preferences are given by $u(x_1, x_2) = 2\sqrt{x_1 x_2}$, which yields the indirect utility function $v(p_1, p_2, I) = I/\sqrt{p_1 p_2}$, and the expenditure function $e(p_1, p_2, u) = u\sqrt{p_1 p_2}$. Suppose prices are $p_1 = 1$, $p_2 = 4$ and income is $I = 60$. Then the price increases of good 1 increases to $p_1 = 9$.

The compensating variation is $60 - e(9, 4, v(1, 4, 60))$. Note that $v(1, 4, 60) = 30$, and $e(9, 4, 30) = 180$.

The compensating variation is: $-$120.

The equivalent variation is $e(1, 4, v(9, 4, 60)) - 60$. Note that $v(9, 4, 60) = 10$, and $e(1, 4, 10) = 20$.

The equivalent variation is: -40

Question 7 Suppose a person's Bernoulli utility function is given by $u(x) = ln(x)$. The person has 100 Dollars to invest. There are two investments available: (a) A riskless asset that pays an interest rate of 0%. A risky assets that that either pays 40% with probability 0.6 or -40% with probability 0.4.

Suppose the person invests α Dollars into the risky asset $100 - \alpha$ Dollars into the riskless asset. The person's expected utility is

$$\mathbf{0.6\ln((100-\alpha)+1.4\alpha)+0.4\ln((100-\alpha)+0.6\alpha)}$$

The first order conditions is

$$\frac{0.6(-1+1.4)}{(100-\alpha)+1.4\alpha}+\frac{0.4(-1+0.6)}{(100-\alpha)+0.6\alpha}=0.$$

Thus, $4((100-\alpha)+1.4\alpha) = 6((100-\alpha)+0.6\alpha)$, which implies

$$\boldsymbol{\alpha = 50}$$

Question 8 A person has mean variance preferences of the form $10E[X] - \mathrm{Var}[X]$, where X is the random variable that describes the portfolio return.

Suppose the person has 100 Dollars. He invests α Dollars in a risky asset with mean return 1.6 and a variance of 0.1. The remainder $(100 - \alpha)$ is invested in a riskless asset with return 1.1.

Thus, the person solves

$$\max_{\alpha} 10(1.6\alpha + 1.1(100 - \alpha)) - 0.1\alpha^2.$$

The first order condition is $16 - 11 - 0.2\alpha = 0$.

The optimal $\alpha = 25$

Question 9 A person has 10,000 Dollars of wealth. He/she has to opportunity to make an investment which has an initial cost of 6,000 Dollars, but has a payoff of 20,000 Dollars with probability 0.2, 10,000 with probability 0.6, and zero otherwise.

(a) The expected utility is

$$-0.2/24,000 - 0.6/14,000 - 0.2/4,000.$$

The expected utility of making the investment is −0.0001012

The expected utility of *not* making the investment is −0.0001

Therefore the person should (*circle the correct answer*) **not make the investment**.

(b) Now suppose that highest payoff is m instead of 20,000 Dollars. Determine the value of m such that the person is just indifferent between making and not making the investment. The expected utility is

$$-0.2/(m + 4000) - 0.6/14,000 - 0.2/4,000.$$

$m = 24,000$