**Question 1** A utility function is given by \( u(x_1, x_2) = -(1/x_1) - (1/x_2) \). Suppose that prices are \( p_1 = 1, p_2 = 9 \).

The partial derivatives are \( 1/x_1^2 \) and \( 1/x_2^2 \). Thus, \( MRS = x_2^2/x_1^2 = 1/9 \)

Then the equation of the income offer curve is

\[
x_2 = (1/3)x_1
\]

**Question 2** The indirect utility function is given by

\[
v(p_1, p_2, I) = \frac{I}{p_1^{0.7} p_2^{0.3}}
\]

The expenditure function is

\[
e(p_1, p_2, u) = u p_1^{0.7} p_2^{0.3}
\]

and the Hicksean demand functions are

\[
h_1(p_1, p_2, u) = 0.7u \left( \frac{p_2}{p_1} \right)^{0.3}, h_2(p_1, p_2, u) = 0.3u \left( \frac{p_1}{p_2} \right)^{0.7}
\]

**Question 3** A utility function is given by \( u(x_1, x_2) = \ln(x_1) + x_2 \). Suppose that prices are \( p_1 = 1, p_2 = 4 \) and income is \( I \). Specify the utility maximization problem in the box below

\[
\max_{x_1, x_2} \ln(x_1) + x_2, \text{ subject to (i) } x_1 + 4x_2 \leq I, \text{ (ii) } x_1 \geq 0, \text{ (iii) } x_2 \geq 0.
\]

Thus, the Lagrangean is given by (specify all constraints for the Lagrangean).

\[
\mathcal{L} = \ln(x_1) + x_2 - \lambda_1(x_1 + 4x_2 - I) + \lambda_2 x_1 + \lambda_3 x_2
\]

The first order conditions are

\[
\frac{\partial \mathcal{L}}{\partial x_1}: \quad \frac{1}{x_1} - \lambda_1 + \lambda_2 = 0
\]

\[
\frac{\partial \mathcal{L}}{\partial x_1}: \quad x_2 - 4\lambda_1 + \lambda_3 = 0
\]

**Question 4** Income offer curves and indifference curves are depicted below. Originally price are \( p_1 = 2, p_2 = 8 \) and income is \( I = 80 \). Then the price of good 1 increases to \( p_1 = 8 \). Determine graphically the Hicks and the Slutsky substitution and income effect.

The Slutsky substitution effect for goods 1 and 2 is \( \Delta^s x_1 = -7.5, \Delta^s x_2 = 7.5 \)
The Slutzky income effect for goods 1 and 2 is $\Delta_I x_1 = -7.5, \Delta_I x_2 = -7.5$.

The Hicks substitution effect for goods 1 and 2 is $\Delta_s x_1 = -10, \Delta_s x_2 = 5$.

The Hicks income effect for goods 1 and 2 is $\Delta_I x_1 = -5, \Delta_I x_2 = -5$.

Question 5 A person’s utility function is given by $u(L, c) = L^2 c$, where $L$ is leisure and $c$ is consumption. The person’s income is derived from work.

(a) Suppose the hourly wage is $w = 8$. Thus, $MRS = 2c/L = 8$. Hence $c = 4L$.

$c = 8\ell$, where $\ell = 24 - L$. Thus, $c = 192 - 8L$, i.e., $c + 8L = 192$. Thus, $12L = 192$, which implies $L = 16$ and $c = 64$. Thus, utility is

Then, the person’s maximum utility is $16,384$. 


(b) At the new prices we have \(2c/L = 6\), i.e., \(c = 3L\). In addition, \(L^2c = 16,384\), i.e., \(3L^3 = 16,384\). Hence, \(L = 17.61\), and \(c = 52.83\). \(\ell = 24 - L\) implies \(\ell = 6.39\). Since the after-tax wage is 6, the total wage income is 38.34. However, to afford 52.83, the person needs a lump sum subsidy of 14.49. The tax revenue is \(4\ell = 12.78\). Thus, the deadweight loss is 1.71.

Tax revenue is: 12.78

The lump sum subsidy is: 14.49

The deadweight loss is: 1.71

**Question 6** Suppose preferences are given by \(u(x_1, x_2) = 2\sqrt{x_1x_2}\), which yields the indirect utility function \(v(p_1, p_2, I) = I/\sqrt{p_1p_2}\), and the expenditure function \(e(p_1, p_2, u) = u\sqrt{p_1p_2}\). Suppose prices are \(p_1 = 9\), \(p_2 = 1\) and income is \(I = 180\). Then the price increases of good 2 increases to \(p_2 = 9\). Determine the compensating and the equivalent variation associated with the price change.

The compensating variation is \(180 - e(9, 9, v(9, 1, 180))\). Note that \(v(9, 1, 180) = 60\), and \(e(9, 9, 60) = 540\).

**The compensating variation is: -360.**

The equivalent variation is \(e(9, 1, v(9, 9, 180)) - 180\). Note that \(v(9, 9, 180) = 20\), and \(e(9, 1, 20) = 60\).

**The equivalent variation is: -120**

**Question 7** Suppose a person’s Bernoulli utility function is given by \(u(x) = \ln(x)\). The person has 1,000 Dollars to invest. There are two investments available: (a) A riskless asset that pays an interest rate of 0%. A risky asset that either pays 100% with probability 0.4 or -40% with probability 0.6.

Suppose the person invests \(\alpha\) Dollars into the risky asset 1,000 – \(\alpha\) Dollars into the riskless asset. The person’s expected utility is

\[
0.4 \ln((1,000 - \alpha) + 2\alpha) + 0.6 \ln((1,000 - \alpha) + 0.4\alpha)
\]

The first order conditions is

\[
\frac{0.4(-1 + 2)}{(100 - \alpha) + 2\alpha} + \frac{0.6(-1 + 0.6)}{(1,000 - \alpha) + 0.6\alpha} = 0.
\]

Thus, \(24((1,000 - \alpha) + 2\alpha) = 40((1,000 - \alpha) + 0.6\alpha)\), which implies

\[
\alpha = 400
\]

**Question 8** A person has mean variance preferences of the form \(30E[X] - \text{Var}[X]\), where \(X\) is the random variable that describes the portfolio return.
Suppose the person has 100 Dollars. He invests $\alpha$ Dollars in a risky asset with mean return 1.4 and a variance of 0.1. The remainder ($100 - \alpha$) is invested in a riskless asset with return 1.1.

Thus, the person solves

$$\max_{\alpha} 30(1.4\alpha + 1.1(100 - \alpha)) - 0.1\alpha^2.$$

The first order condition is $42 - 33 - 0.2\alpha = 0$.

The optimal $\alpha = 45$

**Question 9** A person has 1,000 Dollars of wealth. He/she has the opportunity to make an investment which has an initial cost of 400 Dollars, but has a payoff of 3,000 Dollars with probability 0.2, 1,000 with probability 0.4, and zero otherwise.

(a) The expected utility is

$$-0.2/3,600 - 0.4/1,600 - 0.4/600.$$

The expected utility of making the investment is $-0.000972$
The expected utility of not making the investment is $-0.001$

Therefore the person should (circle the correct answer) make the investment

(b) Now suppose that the highest payoff is $m$ instead of 20,000 Dollars. Determine the value of $m$ such that the person is just indifferent between making and not making the investment. The expected utility is

$$-0.2/(m + 4000) - 0.4/14,000 - 0.2/4,000.$$

$m = 1,800$