Question 1  **The marginal rate of substitution at (2, 4) is 4**

The partial derivatives are

\[
\frac{\partial u(x_1, x_2)}{\partial x_1} = 3x_1^2x_2, \quad \frac{\partial u(x_1, x_2)}{\partial x_2} = x_1^3.
\]

The MRS is therefore

\[
MRS(x_1, x_2) = \frac{3x_1^2x_2}{x_1^3} = \frac{3x_2}{x_1}.
\]

Therefore, \(MRS(2, 4) = 6\).

**Question 2**  The equation of the income offer curve is given by

\[
x_2 = 5x_1
\]

**10 points**

The partial derivatives are

\[
\frac{\partial u(x_1, x_2)}{\partial x_1} = \frac{2}{x_1}, \quad \frac{\partial u(x_1, x_2)}{\partial x_1} = \frac{5}{x_2}.
\]

The MRS is therefore

\[
MRS(x_1, x_2) = \frac{2/x_1}{5/x_2} = \frac{x_2}{2.5x_1}.
\]

Therefore, \(x_2/(2.5x_1) = 2/1\), i.e., \(x_2 = 5x_1\).

**Question 3**

(a) The person’s expenditure minimization problem is

\[
\min_{x_1, x_2} 2x_1 + 4x_2
\]

subject to

1. \(\sqrt{x_1} + x_2 \geq 4\);
2. \(x_1 \geq 0\);
3. \(x_2 \geq 0\).

(b) The Lagrangean is

\[
\mathcal{L} = 2x_1 + 4x_2 - \lambda(\sqrt{x_1} + x_2 - 4).
\]

The first order conditions are

\[
\frac{\partial \mathcal{L}}{\partial x_1} = 2 - \frac{\lambda}{2\sqrt{x_1}} = 0, \quad \frac{\partial \mathcal{L}}{\partial x_2} = 4 - \lambda = 0.
\]
Therefore, $2 - \frac{4}{\sqrt{x_1}} = 0$, which implies $1 = \sqrt{x_1}$. Thus, $x_1 = 1$. Inserting $x_1 = 1$ in the constraint implies $x_2 = 3$.

The optimal consumption is $x_1 = 1 \quad x_2 = 3$

**Question 4**

The change in demand is $\Delta x_1 = -10 \quad \Delta x_2 = 0$

Determine the Slutsky substitution and income effects, where the person is compensated with just enough income to afford the original consumption choice at the new prices.

The change due to the substitution effect is $\Delta^s x_1 = -5 \quad \Delta^s x_2 = 15$
The change due to the income effect is $\Delta^I x_1 = -5 \quad \Delta^I x_2 = -15$

**Question 5**  The expected payoff is $0.6 + 0.25(4) + 0.1(25) + 0.05(400) = 24.1$. The expected utility is $0.6 + 0.25\sqrt{4} + 0.1\sqrt{25} + 0.05\sqrt{400} = 2.6$.

The expected payoff of the lottery is 24.1

The person’s expected utility from the lottery is 2.6

The lottery’s certainty equivalent is 6.76

**Question 6**

(a) The expected payoff must be 1, i.e., $0.1 + 0.01(10) + 0.001x = 1$. Thus, $x = 800$.

In order for a risk neutral person to be indifferent between buying and not buying a lottery ticket,

The jackpot must be $x = 800$

(b) The expected utility of playing the lottery is $0.889 \ln(9,999) + 0.1 \ln(10,000) + 0.01 \ln(10,009) + 0.001 \ln(9,999 + x) = \ln(10,000)$. Thus, $9.201048951 + 0.001 \ln(9,999 + x) = \ln(10,000)$, which implies $\ln(9,999 + x) = 7.608982$. Therefore, $9,999 + x = 10,826.27586$, which implies $x = 832.88$.

The jackpot must be $x = 832.88$

**Question 7**

(a) The utility maximization problem is

$$\max_\alpha 0.7 \ln(2\alpha + (1 - \alpha)) + 0.34 \ln(1 - \alpha)$$

subject to $0 \leq \alpha \leq 1$.

This is equivalent to

$$\max_\alpha 0.7 \ln(1 + \alpha) + 0.3 \ln(1 - \alpha)$$

subject to $0 \leq \alpha \leq 1$. 

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(b) If the constraint does not bind then the first order condition is

$$\frac{0.7}{1 + \alpha} - \frac{0.3}{1 - \alpha} = 0.$$ 

Thus, $7(1 - \alpha) = 3(1 + \alpha)$, which implies $10\alpha = 4$ and $\alpha = 0.4$.

**the person should invest 40 % of his/her wealth in the risky asset**

**Question 8 (a)** $\beta = 1.2/0.4 = 3$. Thus, $R_i = 4 + 3(6) = 22$.

**The stock’s expected return is 22 %**

(b) The returns predicted by the CAPM are 7% and 13% percent, respectively. The expected returns are 6% and 14%, respectively. Thus, you should buy stock $B$ because it is above the market line.