Question 1 1. Does $A$ sell any cars?  No

2. Does dealer $B$ sell any cars? Yes The sales price is $6,500 \leq p \leq 7,000$.

Question 2 $A$ currently runs a fast food restaurant and wants to open a restaurant at a different location, which will be run by a manager. Let $e$ denote $B$’s effort, then the profit of the restaurant will be $f(e) = 100e$. The utility of the manager is $u(m, e) = m - 2e^2$, where $m$ is the manager’s income and $e$ is effort. The manager is only willing to take the job if his payoff $u(m, e) \geq 1,000$.

(a) $A$ solves $\max_{e,m} 100e - m$, subject to $2e^2 - m \leq 100$. The Lagrangean is $\mathcal{L} = 100e - m - \lambda(2e^2 - m - 100)$. Thus, the first order conditions are $100 = 4\lambda e$, and $1 = \lambda$. Thus, $e = 25$. As a consequence, $m = 1,150$.

$b = 25$

A’s payoff is 1,350

The payment to the manager is $m = 1,150$

(b) The payment $K$ must correspond to $A$’s payoff. Thus, $K = 1,350$

Question 3 Suppose a cost function is given by $C(Q) = 200 + 4Q^2$. Then

Average variable costs are $4Q$.

Average total costs are $200/Q + 4Q$.

Marginal costs are $8Q$.

Fixed costs are 200.

Average Fixed costs are $200/Q$.

Question 4 There are 100 firms with costs $C(Q) = 5Q^2 + 10Q$. Thus, $MC = P$ implies $10Q + 10 = P$. Hence, $Q = P/10 - 1$. Since there are 100 firms, we get $Q = 10P - 100$.

Demand is given by $Q_D(P) = 2,000 - 40P$. Thus, $2,000 - 40P = 10P - 100$.
Now suppose the government introduces a tax of 10 Dollars on firms be unit sold. Thus, \( MC = P \) implies \( 10Q + 10 = P \). Hence, \( Q = P/10 - 2 \). Since there are 100 firms, we get \( Q = 10P - 200 \).

The governments tax revenue is 2,400

**Question 5** A firm’s cost function is given by \( C(Q) = 1,000 + 4Q \). Suppose there are 100 costumers, each of them has a demand function \( Q_D(P) = 20 - 2P \). The firm wants to do two-part pricing, i.e., charge a fixed fee \( F \) and a price per unit \( P \) to maximize profits.

The firm will set \( P = MC \). Thus, \( P = 4 \). At \( P = 4 \), we get \( Q = 12 \). Thus, the net surplus is 36, which is the maximum amount the firm can choose for \( F \).

The profit maximizing \( F = 36, P = 4 \)

Excluding fixed costs, the profit from each consumer is 36. Since there are 100 consumer, we get 3,600. Accounting for fixed costs:

**The firm’s total profit (from all costumers) is 2,600**

**Question 6** For low demand consumers, \( Q_D(P) = 10 - 2P \), the gross benefit of getting 2 units is 9. Thus, \( F_l = 9 \). The benefit of a high demand costumer of consuming 2 units is 18. Thus, the net benefit is 9. The benefit of consuming 10 units for high demand costumers is 50. Thus, \( F_h = 41 \) is the highest price it can charge.

\( F_l = 9, F_h = 41 \)

**Question 7**

(a) A good is produced both by 100 firms that are in country \( A \) and 100 foreign firms that only sell the product in country \( A \). We assume the market is competitive, i.e., firms are price takers. All firms have the same cost function given by \( C(Q) = Q^2 \). The demand for the the product in country \( A \) is given by \( Q_D(P) = 1,200 - 100P \).

Since \( P = MC \) we get \( P = 2Q \), i.e., \( Q = 2/P \). Thus, supply by domestic and foreign firms is \( Q = 100P \). Therefore, \( 1,200 - 100P = 100P \). i.e., \( P = 6 \). Each firm produces 3 units. A firm’s revenue is therefore 18, and costs are 9.
The equilibrium price is $P = 6$
The equilibrium quantity $Q = 600$
The profit of each firm is 9
The net surplus of all domestic consumers is 1,800

(Recall the net surplus is the gross benefit of consuming $Q$ units instead of 0, minus the amount of money paid for the $Q$ units)

(b) Now the government introduces a tariff of 8 Dollars per unit on the foreign form, thus raising the costs of the foreign firms to $C(Q) = Q^2 + 8Q$. Thus, supply of each foreign firm is $Q = P/2 - 4$. Total supply is $Q_S(P) = 100P - 400$. Therefore, $100P - 400 = 1,200 - 100P$ implies $P = 8$. The equilibrium quantity is therefore $Q = 400$. Each domestic firm produces $Q = 4$. Thus, revenue is 32. Costs are 16. Therefore, the profit is 16.

The equilibrium price is $P = 8$
The equilibrium quantity $Q = 400$
The profit of each domestic firm is 16
The net surplus of all domestic consumers is 800

(c) Consumers’ welfare loss is 1,000. The profit gain of all firms is 700. Thus, the welfare loss (circle the correct answer) is 300

Question 8 (a) Cost functions are given by $C_1(Q_1) = 10 + 2Q_1$ and $C_2(Q_2) = 2 + 4Q_2$. Thus, marginal costs are 2 and 4, respectively. The price elasticity of demand is $\epsilon = P/(P - 18)$. Thus, we have

$$2 = P \left(1 + \frac{(P - 18)s_1}{P}\right), \quad 4 = P \left(1 + \frac{(P - 18)s_2}{P}\right),$$

which is equivalent to $2 = P + s_1(P - 18)$, and $4 = P + s_2(P - 18)$. Adding the two equations yields $6 = 2P + (P - 18)$. Thus, $P = 8$. Inserting $P = 8$ into these equations yields $s_1 = 0.6$, $s_2 = 0.4$. Total demand is 10 units. Thus, $Q_1 = 6$, and $Q_2 = 4$. The revenue of the firms is 48 and 32, respectively. Costs are 22 and 18. Thus, profits are 26 and 14.
The equilibrium price \( P = 8 \)

The firms’ market shares are \( s_1 = 0.6, s_2 = 0.4 \)

Firm 1’s profit is 26, Firm 2’s profit is 14

(b) Now suppose that firm 1 buys firm 2 for \( q \) Dollars. After the purchase, they produce only with the technology that has costs \( C_1(Q_1) = 10 + 2Q_1 \). Thus, \( 2 = P(1 + (P - 18)/P) = P + P - 18 = 2P - 18 \). Therefore, \( P = 10 \). The firm sells 8 units. Revenue is 80 and costs are 26. Therefore, the profit is 54. Originally, the profit was 26. Thus, \( q \leq 54 - 26 = 28 \).

The equilibrium price \( P = 10, \) and \( Q = 8 \)

The firm’s profit is 54

Thus, the purchase price \( q \leq 28 \).

(c) We need to compute the change in total surplus. Before the merger, total firm profits were 40. After the merger they are 54, i.e., the benefit to the firms is 14. Next, we need to compute the change in the net-consumer surplus, which will be negative, since the price is higher after the merger. If the sum of the two numbers is positive, then the merger increases surplus and it desirable, otherwise, the merger is not desirable.