

**Question 1** The Lagrangean is given by

$$\mathcal{L} = x_1 x_2^2 - \lambda(x_1 - 3x_2 - 180).$$

The first order conditions are  $x_2^2 = \lambda$  and  $2x_1 x_2 = 3\lambda$ . Thus,  $2x_1 x_2 = 3x_2^2$ .

The equation of the income offer curve is  $x_2 = (2/3)x_1$ . The budget line equation is  $x_1 + 3x_2 = 180$ .

$$x_1 = 60, x_2 = 40.$$

**Question 2** The Lagrangean is given by

$$\mathcal{L} = x_1 + 2x_2 - \lambda(x_1^2 x_2 - 2,000).$$

The first order conditions are  $2 = 2\lambda x_1 x_2$ , and  $2 = \lambda x_1^2$ . Thus,  $2x_1 x_2 = x_1^2/2$ , i.e.,  $4x_2 = x_1$ . In addition,  $x_1^2 x_2 = 2,000$ . Thus,  $16x_2^3 = 2,000$ . Thus,

$$\text{The least costly consumption bundle is } x_1 = 20, x_2 = 5$$

**Question 3** 1. Suppose the interest rate is 0%. Then

5 points

**Current consumption is  $c_1 = 10$ . The person saves \$ 10.**

2. Suppose the interest rate increases to 50%.

**Current consumption is  $c_1 = 12$ . The person saves \$ 8.**

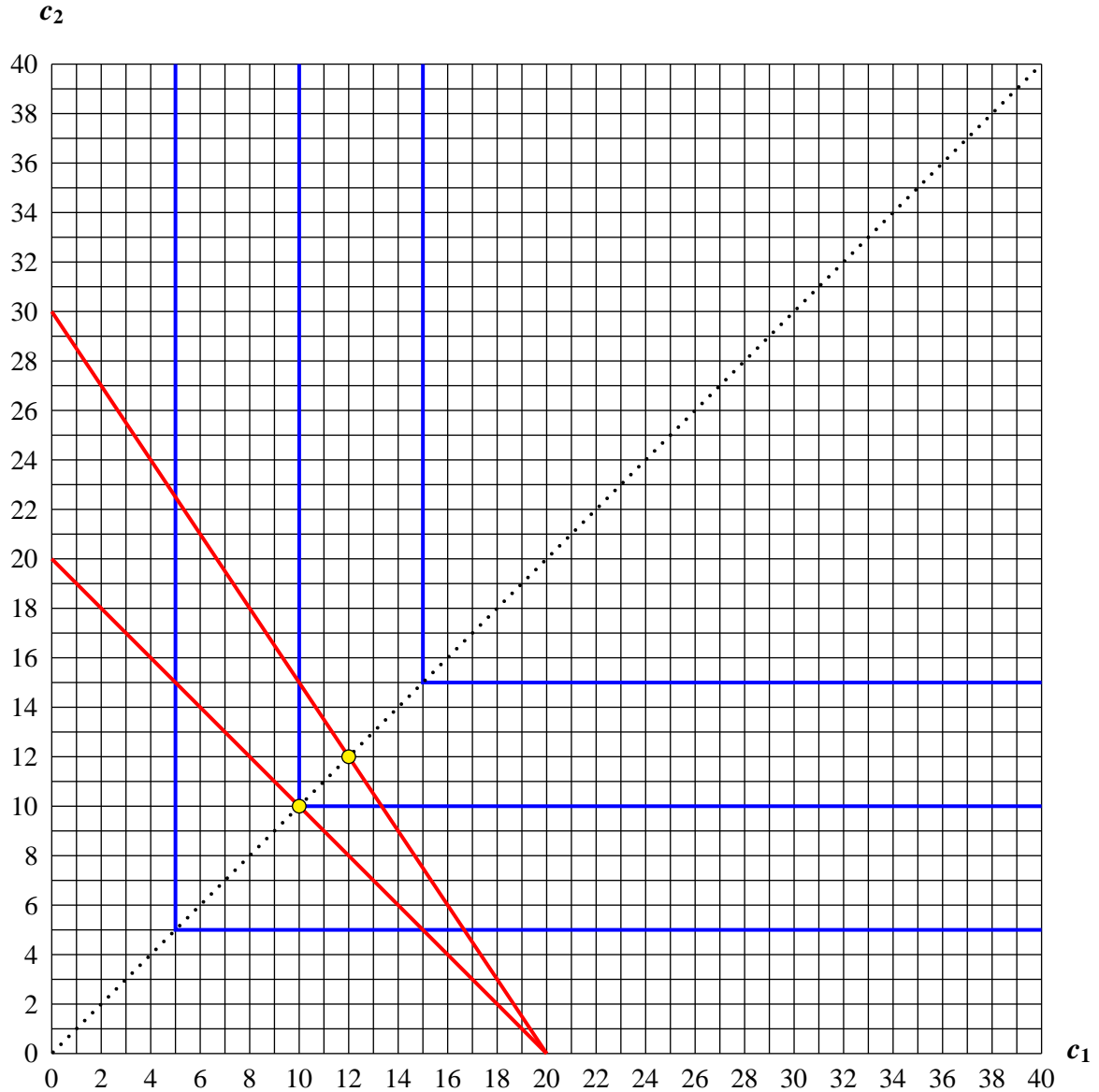
3. The change in saving due to the income effect is **-2.**

the change in saving due to the substitution effect is **0.**

Explain briefly why raising the interest rate lowers savings (*Your answer must be brief, i.e., fit in the box below*).

5 points

The interest rate increase results in a positive income effect, i.e., the person is able to spend more money since the return on his or her savings is increased. Because preferences are perfect complements, there is no substitution effect that can counteract this income effect.



**Question 4** Before the price change  $x_1(1, 9, 84) = 21$ , and  $x_2(1, 9, 84) = 7$

**Demand before the price change is  $x_1 = 21$ ,  $x_2 = 7$ .**

After the price change  $x_1(4, 9, 84) = 8.4$ , and  $x_2(4, 9, 84) = 5.6$

**Demand after the price change is  $x_1 = 8.4$ ,  $x_2 = 5.6$ .**

In order to afford  $(21, 7)$  at the new prices, income must be  $I = 147$ . Then,  $x_1(4, 9, 147) = 14.7$ , and  $x_2(4, 9, 147) = 9.8$

**The income effect is  $\Delta^I x_1 = -6.3$ ,  $\Delta^I x_2 = -4.2$ .**

**The substitution effect is  $\Delta^s x_1 = -6.3$ ,  $\Delta^s x_2 = 2.8$ .**

**Question 5**

**The expected payoff of the lottery is 13.6.**

**The expected utility of the lottery is -0.496.**

**The certainty equivalent of the lottery is 2.016.**

**Question 6** A person has 10,000 Dollars, but faces a loss of 5,000 Dollars with probability 0.2. The person's Bernoulli utility is  $u(x) = -1/x$ . Determine the maximum amount the person is willing to pay to get insurance with full coverage. *10 points*

The expected utility from being uninsured is  $-1/10,000 * 0.8 - 1/5,000 * 0.2 = -0.00012$ . The expected utility from being insured is  $-1/(10,000 - p)$ . Thus,  $-1/10,000 * 0.8 - 1/5,000 * 0.2 = -1/(10,000 - p)$ . This implies  $10,000 - p = 1/0.00012$ . Thus,

**The person is willing to pay at most \$1,666.67.**

**Question 7**

(a) The overall probability that a person becomes ill is  $0.01(0.9) + 0.2(0.08) + 0.9(0.02) = 0.043$ . Thus, the expected costs are  $0.043(20,000) = 860$  per person. Average fixed costs are 6 Dollars. Thus,

**The insurance premium is \$866**

(b) At \$300 everyone to signs up. The expected costs per person are \$860. Thus, profit is  $300,000,000 - 860,000,000 - 6,000,000$ , i.e.,  $-566$  Million. Thus, the firm's total profit is **-566 Million Dollars**.

Alternatively, at \$6,000 only *B* and *C* will sign up. The probability of an illness is  $(0.2(0.08) + 0.9(0.02))/0.1 = 0.34$ . Thus, expected costs per person are 6,800. There are 100,000 type *B* and *C* people. Thus, the firm's total profit is **-86 Million Dollars**.

Finally, at a premium of 18,200 only type *C* sign up. Thus, the firm's total profit is **-2 Million Dollars**.

**0% of the population will be insured by the firm.**

**Question 8** The person solves

$$\max_{x_A, x_B} 0.5 \ln(0.2x_A + 1.8x_B) + 0.5 \ln(2x_A),$$

subject to  $x_A + x_B = 2,000$ .

The Lagrangean is

$$\mathcal{L} = 0.5 \ln(0.2x_A + 1.8x_B) + 0.5 \ln(2x_A) - \lambda(x_A + x_B - 2,000).$$

Thus, the first order conditions are

$$\frac{0.1}{0.2x_A + 1.8x_B} + \frac{1}{2x_A} = \lambda$$

$$\frac{0.9}{0.2x_A + 1.8x_B} = \lambda$$

Thus,

$$\frac{0.8}{0.2x_A + 1.8x_B} = \frac{1}{2x_A}$$

which implies  $1.6x_A = 0.2x_A + 1.8x_B$ , i.e.,  $1.4x_A = 1.8x_B$ . In addition,  $x_A + x_B = 2,000$ . Thus,

**The optimal investment amounts are  $x_1 = 1,125$ ,  $x_2 = 875$ .**