Question 1 The optimization problem is

$$\min_{x_1,x_2} 4x_1 + x_2 \text{ s.t. } (1)\ x_1x_2 \geq 100;$$
$$(2)\ x_1 \geq 0;$$
$$(3)\ x_2 \geq 0.$$

The Lagrangean is

$$\mathfrak{L} = 4x_1 + x_2 - \lambda(x_1x_2 - 100).$$

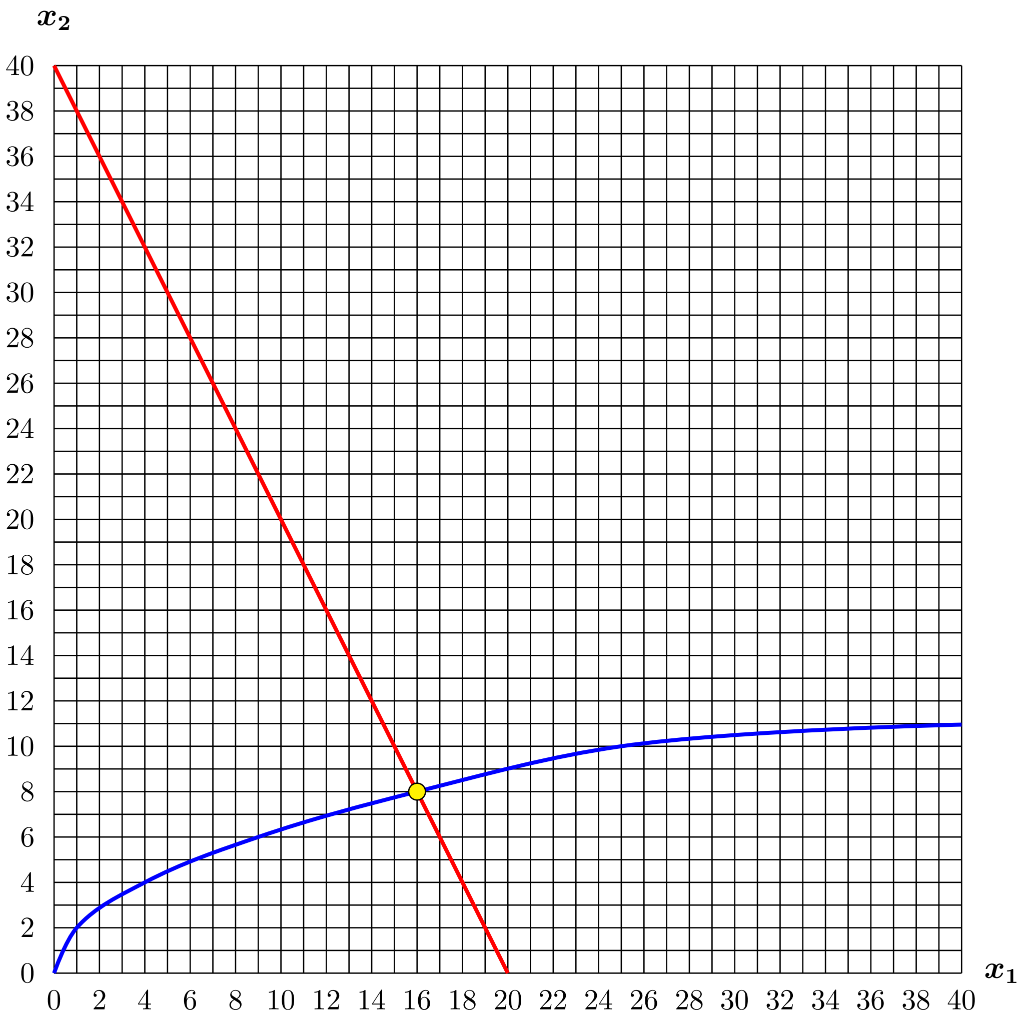
The first order conditions are

$$4 - \lambda x_2 = 0;$$
$$1 - \lambda x_1 = 0.$$

Thus, $x_2/4 = x_1$, i.e, $4x_1 = x_2$. In addition $x_1x_2 = 100$. Thus, $4x_1^2 = 100$, which implies $x_1 = 5$. Thus,

The optimal values are $x_1 = 5$, $x_2 = 20$ *10 points*

Question 2 **The optimal consumption is $x_1 = 16$ $x_2 = 8$**



Question 3 The person solves

$$\max_{\alpha,\beta} 1.1\alpha + 1.3\beta - 0.4\beta^2, \text{ s.t. } \alpha + \beta = 1.$$

The Lagrangean is

$$1.1\alpha + 1.3\beta - 0.4\beta^2 - \lambda(\alpha + \beta - 1).$$

Thus, $1.1 = \lambda$, $1.3 - 0.8\beta = \lambda$, which implies $0.8\beta = 0.2$, i.e., $\beta = 1/4$.

The optimal consumption is $\boldsymbol{\alpha = 0.75}$, $\boldsymbol{\beta = 0.25}$ *15 points*

Question 4

(a) The MRS is $x_2/x_1 = 1.44$. Thus, $1.44x_1 = x_2$. Inserting this into the budget line equation $1.44x_1 + x_2 = 9,000$ yields $2.88x_1 = 9,000$, i.e., $x_1 = 3,125$ and $x_2 = 4,500$.

The consumption after the tax is *10 points*

$x_1 = 3,125$, $x_2 = 4,500$. Utility is 14,062,500

The government's tax revenue from the person is 1,375.

(b) We must solve $\min_{x_1,x_2} x_1+x_2$ s.t. $x_1x_2 \geq 14,062,500$. The Lagrangean is $x_1+x_2-\lambda(x_1x_2-14,062,500)$. The first order conditions are $1 = \lambda x_2$, $1 = \lambda x_1$. Thus, $x_1 = x_2$. Hence $x_1^2 = 14,062,500$ which implies $x_1 = x_2 = 3,750$. The cost of this consumption is 7,500. Thus, the person is willing to pay 1,500.

The person's maximum willingness to pay is \$1,500.

The difference between the tax revenue and the person's willingness to pay (which should be larger than the tax revenue) is the deadweight loss from taxation.

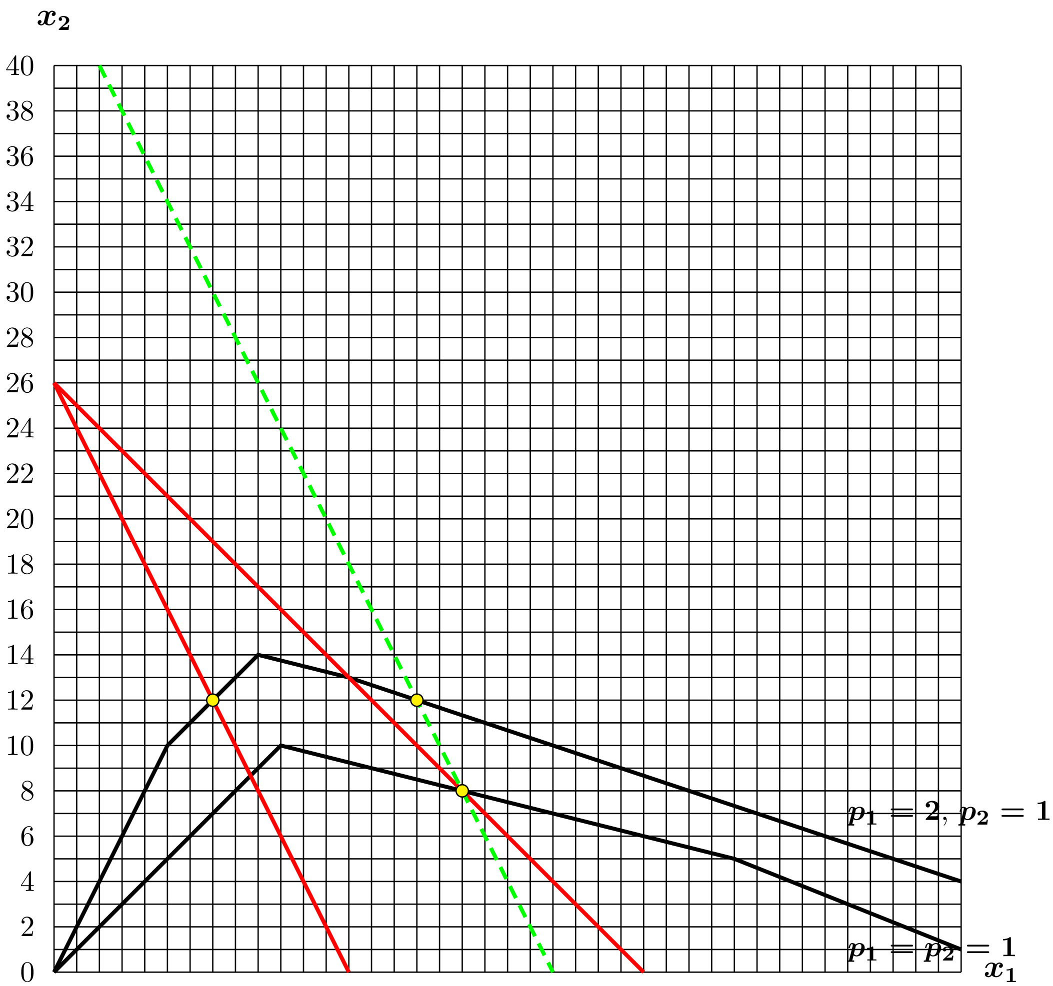
Per Dollar of taxes raised, the deadweight loss is 9 cents.

Question 5 A person's income is $I = 26$. Originally, prices are $p_1 = p_2 = 1$, but then the price of good 1 increase to $p_1 = 2$. Using the income offer curves depicted below, determine the Slutzky substitution an income effect.

The substitution effect is $\Delta^s x_1 = -2$, $\Delta^s x_2 = 4$.

The income effect is $\Delta^I x_1 = -9$, $\Delta^I x_2 = 0$.

(*Denote an increase by positive sign, an a decrease by a negative sign*).



Question 6 A person has an income of $I = 10,000$. With probability p he has in accident which results in a loss of $9,100. At a cost of $396 he can get complete coverage. His Bernoulli utility function is $\sqrt{x}$.

(a) The utility from being insured is $\sqrt{10,000 - 396} = 98$. The utility from being uninsured is $(1-p)\sqrt{10,000} + p\sqrt{900} = 100(1-p) + 30p = 100 - 70p$. Thus, $100 - 70p = 98$, which implies

$$\boldsymbol{p = 2/70 = 0.0285.}$$

(b) The utility from being uninsured is again $100 - 70p$. The utility from being insured is $(1-p)\sqrt{10,000 - 199} + p\sqrt{10,000 - 1,701 - 199} = 99(1-p) + 90p = 99 - 9p$. Thus, $100 - 70p = 99 - 9p$, i.e., $1 = 63p$. Thus,

$$\boldsymbol{p = 1/63 = 0.0159.}$$

Question 7 The expected utility is $0.2(4) + 0.1(10) + 0.01(100) = 2.8$

the expected utility from playing the lottery is 2.8.

the certainty equivalent of the lottery is 7.84.