Question 1 (a) The equilibrium price and quantity are
\[ P^* = 9 \text{ and } Q^* = 11 \] .

(b) The new equilibrium price and quantity are
\[ P^* = 6 \text{ and } Q^* = 8 \] .

Question 2 If demand is \( Q_D = a - bP \), then \( \epsilon^D_P = -bP/Q \). Thus, \(-0.2 = -bP/Q = -20b/40 = 0.5b\), i.e., \( b = -0.4 \). Thus, \( Q_D = a - 0.4P \). Since \( Q_D(20) = 40 \), we get \( 40 = a - 8 \) and hence \( a = 48 \). Thus, the demand function is given by
\[ Q_D(P) = 48 - 0.4P \] .

Question 3 Changing consumption from \((4, 12)\) to \((6, 6)\) means that \( \Delta x_1 = 2 \) and \( \Delta x_2 = -6 \). The slope of the budget line is therefore \( \Delta x_2/\Delta x_1 = -3 \). We know that the slope of the budget line is also given by \( p_1/p_2 \). Thus,
\[ p_2 = 4 \].

Question 4  (a) In equilibrium, \( Q_D(P) = 100 - P - \tau = 10 + P = Q_S(P) \). Thus, 
\[ P = 45 - 0.5\tau. \]
The quantity is 
\[ Q^*(\tau) = 55 - 0.5\tau. \]
(b) The tax revenue is \( R(\tau) = 55\tau - 0.5\tau^2 \). This is maximized if \( R'(\tau) = 0 \). 
Thus, 
\[ 55 - \tau = 0. \]
The revenue maximizing tax is given by \( \tau^* = 55 \).

Question 5
\[ x_S = 20, \ x_C = 10. \]
Question 6

\[ x_1 = 16, x_2 = 16. \]
Question 7 (b) \( x_1 = 5, x_2 = 20. \)

(c) \( x_1 = 40, x_2 = 0. \)

Question 8 (a) \( \epsilon_P^D = \frac{-AP \cdot 1000}{1-AP}. \)

Taking the derivative of \( \epsilon_P^D \) with respect to \( A \) we get (recall that \( \left( \frac{u}{v} \right)' = \frac{u'v - uv'}{v^2} \)).

\[ \frac{\partial \epsilon_P^D}{\partial A} = -\frac{P}{1.00 - AP}. \]

This derivative is negative.
(b) In equilibrium \( Q_S(P + s) = 0.2(P + s) = 1,000 - AP \). Thus,

\[
P^* = \frac{5,000 - S}{5A + 1}.
\]

(c) Note that \( \frac{\partial P^*(A,s)}{\partial s} = -\frac{25,000 - 5S}{(5A + 1)^2} < 0 \) since the subsidy decreases the price. Note that \( A \) appears only in the denominator. In particular, if \( A \) is increased then the denominator becomes larger, i.e., \( \left| \frac{\partial P^*(A,s)}{\partial s} \right| \) is decreased. Thus, a more elastic demand result in a smaller price response when the subsidy is introduced.