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**All questions must be answered on this test form!**

*For each question you must show your work and (or) provide a clear argument.*

**Question 1** Suppose that a person's utility function is  $u(x_1, x_2) = x_1^2 x_2^3$  and that prices are  $p_1 = 12$ ,  $p_2 = 9$ . Then the equation of the income offer curve is

*8 points*

$x_2 =$

Suppose you know that the person consumes 30 units of good 1.

*6 points*

**Then the person's income is  $I =$**

**Question 2** Suppose that person's utility function is  $u(x_1, x_2) = x_1 + 2 \ln(x_2)$ . Prices are  $p_1, p_2$  and income is  $I$ . Then the demand function for goods 1 and 2 are given by *6 points*

$x_1(p_1, p_2, I) =$	$, x_2(p_1, p_2, I) =$
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Thus, the indirect utility function is

$v(p_1, p_2, I) =$	
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*6 points*

**Question 3** A utility function is given by  $u(x_1, x_2) = x_1 + 2\sqrt{x_2}$ . Then Hicksian demand is

6 points

$$h_1(p_1, p_2, u) =$$

$$, h_2(p_1, p_2, u) =$$

.

The expenditure function is

$$e(p_1, p_2, u) =$$

6 points

**Question 4** A person's utility function is  $u(x_1, x_2) = (1/4)x_1^2x_2$ . Recall that the demand functions are  $x_1 = 2I/(3p_1)$ ,  $x_2 = I/(3p_2)$ . Suppose that originally prices are  $p_1 = 1$ ,  $p_2 = 1$ , and income is  $I = 45$ . Suppose that the government introduces a tax of 7 Dollars on good 2 that raises the prices to  $p_2 = 8$ . At the same time, the government wants to provide a lump-sum subsidy,  $s$ , to the person such that his/her utility is the same at the original prices and income.

*8 points*

**The lump sum subsidy is  $s =$**

The government's tax revenue from the tax on good 2 after  $s$  is added to the person's income is .

*3 points*

Why is the lump-sum subsidy not equal to the amount of money raised by the tax? (Your answer must fit into the box below).

*3 points*

**Question 5** Suppose that all income offer curves are straight lines starting at (0, 0). At prices  $p_1 = 2$ ,  $p_2 = 2$  the person's optimal consumption is (20, 20). When the price of good 1 increases to  $p_1 = 5$  (income stays the same) demand of good 1 decreases to 4. Then Slutsky substitution effects for goods 1 and 2 are

*10 points*

$$\Delta_1^s = \quad , \Delta_2^s = \quad .$$

The Slutsky income effects are

$$\Delta_1^I = \quad , \Delta_2^I = \quad .$$

**Question 6** Suppose that there is a riskless asset that has a return of 10%, and a risky asset that has a return of 100% in state  $g$  and  $-40\%$  in state  $b$ . State  $g$  occurs with probability 0.4, and state  $b$  with the remaining probability. The person has 3,000 Dollars to invest, and preferences are described by a Bernoulli utility function  $u(x) = \ln(x)$ . Suppose the person puts  $m$  Dollars into the risky asset and the remainder into the riskless asset. Determine the amount of money the person will have in states  $g$  or  $b$ , respectively (you answer is of course a function of  $m$ ).

*6 points*

**State  $g$ : \$ \_\_\_\_\_, state  $b$ : \$ \_\_\_\_\_ .**

Using calculus, determine the optimal  $m$ :

*8 points*

**The person invests \$ \_\_\_\_\_ into the risky asset.**

**Question 7** A person's Bernoulli utility function is given by  $u(x) = \sqrt{x}$ . Consider the following lottery: With probability 0.5 the payoff is 1, with probability 0.2 the payoff is 4, with probability 0.2 the payoff is 25 and with probability 0.1 the payoff is 100. The person's current wealth is zero. Then

**The expected payoff of the lottery is**

*4 points*

**The person's expected utility from the lottery is**

*4 points*

**The lottery's certainty equivalent is**

*4 points*

*Note:* Recall that the certainty equivalent is a payment  $y$  that the person receives with certainty such that expected utility of  $y$  is the same as that of the lottery.

**Question 8** Suppose there are two risky assets,  $A$ , and  $B$  and riskless asset. The riskless asset has a return of 2%. The returns of assets  $A$  and  $B$  are 10% and 30% respectively. Denote by  $x_r$ ,  $x_A$ , and  $x_B$  the fraction of your money invested in the three assets. Then  $x_r + x_A + x_B = 1$ . The return of the portfolio is  $0.02x_r + 0.1x_A + 0.3x_B$  (Note that this is the return rather than the gross return). Suppose that the variance of the portfolio is  $0.01x_A^2 + 0.01x_Ax_B + 0.04x_B^2$ . The person has mean variance preferences of the form  $u(\mu, \sigma) = \mu - 20\sigma^2$ , where  $\mu$  is the expected return and  $\sigma$  the standard deviation of the portfolio.

The optimal portfolio is

*12 points*

$x_r =$ , $x_A =$ , $x_B =$ .
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