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All questions must be answered on this test form!
For each question you must show your work and (or) provide a clear argument.
All graphs must be accurate to get credit.

**Question 1** A utility function is given by $u(x_1, x_2) = -(1/x_1) - (1/x_2)$. Suppose that prices are $p_1 = 1$, $p_2 = 9$. Then the equation of the income offer curve is

$$x_2 =$$
Question 2 The utility function

\[ u(x_1, x_2) = \frac{1}{0.70.70.30.3} x_1^{0.7} x_2^{0.3} \]

generates demand functions

\[ x_1(p_1, p_2, I) = \frac{0.7I}{p_1}, \quad x_2(p_1, p_2, I) = \frac{0.3I}{p_2}. \]

Then the indirect utility function is given by

\[ v(p_1, p_2, I) = \]

The expenditure function is

\[ e(p_1, p_2, u) = \]

and the Hicksean demand functions are

\[ h_1(p_1, p_2, u) = , \quad h_2(p_1, p_2, u) = \]
**Question 3** A utility function is given by \( u(x_1, x_2) = \ln(x_1) + x_2 \). Suppose that prices are \( p_1 = 1, \ p_2 = 4 \) and income is \( I \). Specify the utility maximization problem in the box below

Thus, the Lagrangean is given by (specify all constraints for the Lagrangean. Also, the Lagrangean must be written in such a way that the multipliers are all greater or equal to zero).

The first order conditions are

\[ \frac{\partial u}{\partial x_1} : \]

\[ \frac{\partial u}{\partial x_2} : \]
**Question 4** Income offer curves and indifference curves are depicted below. Originally price are $p_1 = 2$, $p_2 = 8$ and income is $I = 80$. Then the price of good 1 increases to $p_1 = 8$. Determine graphically the Hicks and the Slutsky substitution and income effect.

The Slutsky substitution effect for goods 1 and 2 is

\[ \Delta^s x_1 = \quad \Delta^s x_2 = \]

The Slutsky income effect for goods 1 and 2 is

\[ \Delta^' x_1 = \quad \Delta^' x_2 = \]

The Hicks substitution effect for goods 1 and 2 is

\[ \Delta^s x_1 = \quad \Delta^s x_2 = \]

The Hicks income effect for goods 1 and 2 is

\[ \Delta^' x_1 = \quad \Delta^' x_2 = \]
Question 5 A person’s utility function is given by \( u(L, c) = L^2 c \), where \( L \) is leisure and \( c \) is consumption. The person’s income is derived from work. I.e., if \( \ell \) is the number of house the person works per day, then \( L = 24 - \ell \) corresponds to leisure.

(a) Suppose the hourly wage is \( w = 8 \). Then, the person’s maximum utility is

(b) Now suppose that the government introduces an income tax of 25%. Thus, the person’s after tax income is \( w = 6 \) and the government receives 2 Dollars in tax revenue for each hour worked. However, at the same time the government pays a lump sum subsidy to the the consumer such that he/she is at the pre tax utility level that you determined in (a).

Tax revenue is: 

The lump sum subsidy is: 

The deadweight loss is: 

Note: The solutions will not be integers.
Question 6 Suppose preferences are given by $u(x_1, x_2) = 2 \sqrt{x_1 x_2}$, which yields the indirect utility function $v(p_1, p_2, I) = I / \sqrt{p_1 p_2}$, and the expenditure function $e(p_1, p_2, u) = u \sqrt{p_1 p_2}$. Suppose prices are $p_1 = 9$, $p_2 = 1$ and income is $I = 180$. Then the price of good 2 increases to $p_2 = 9$. Determine the compensating and the equivalent variation associated with the price change.

The compensating variation is: 

The equivalent variation is:
Question 7 Suppose a person’s Bernoulli utility function is given by \( u(x) = \ln(x) \). The person has 1,000 Dollars to invest. There are two investments available: (a) A riskless asset that pays an interest rate of 0%. A risky asset that the either pays 100% with probability 0.4 or -40% with probability 0.6.

(a) Suppose the person invests \( \alpha \) Dollars into the risky asset, \( 1,000 - \alpha \) Dollars into the riskless asset. Specify the person’s expected utility: 5 points

(b) Determine the optimal portfolio choice of \( \alpha \) analytically. Note that \( \alpha \) is not restricted to be between 0 and 1,000. 7 points

\[ \alpha = \]
Question 8 A person has mean variance preferences of the form $30E[X] - \text{Var}[X]$, where $X$ is the random variable that describes the portfolio return.

Suppose the person has 100 Dollars. He invests $\alpha$ Dollars in a risky asset with mean return 1.4 and a variance of 0.1. The remainder $(100 - \alpha)$ is invested in a riskless asset with return 1.1.

The optimal $\alpha =$ \hspace{1cm} 10 points
**Question 9** A person has 1,000 Dollars of wealth. He/she has the opportunity to make an investment which has an initial cost of 400 Dollars, but has a payoff of 3,000 Dollars with probability 0.2, 1,000 with probability 0.4, and zero otherwise.

(a) Then

The expected utility of making the investment is

The expected utility of not making the investment is

Therefore the person should (circle the correct answer) **make the investment** or **not make the investment**.

(b) Now suppose that highest payoff is \( m \) instead of 3,000 Dollars. Determine the value of \( m \) such that the person is just indifferent between making and not making the investment.

\[ m = \]