All questions must be answered on this test form!
For each question you must show your work and (or) provide a clear argument.
All graphs must be accurate to get credit.

Question 1  Suppose the marginal rate of substitution is $MRS(x_1, x_2) = \frac{2x_2}{3x_1}$. Suppose that prices are $p_1 = 2$, $p_2 = 1$, and the person’s income is $I = 30$. Then the person’s demand is

$$x_1 = \quad , \quad x_2 =$$
**Question 2** A person’s utility function is \( u(x_1, x_2, L) = \ln(x_1) + \ln(x_2) + \ln(300 - L) \), where \( x_1 \) and \( x_2 \) are consumption of two goods and \( L \) is the amount of time the person works. Suppose that prices are \( p_1 = 1 \), \( p_2 = 2 \) and that the wage is \( w = 10 \). The person’s income solely comes from labor. Thus, the person solves 
\[
\max_{x_1, x_2, L} \ln(x_1) + \ln(x_2) + \ln(300 - L) \text{ subject to } x_1 + 2x_2 \leq 10L.
\]

4 points

The Lagrangean is given by

Determine the optimal level of \( L \) (recall that the derivative of \( \ln(x) \) is \( 1/x \)).

At the optimum \( L = \)

10 points
Question 3  A utility function is given by $u(x_1, x_2) = (1/4)x_1x_2^2$. The resulting demand functions are

$$x_1(p_1, p_2, I) = \frac{I}{3p_1}, \quad x_2(p_1, p_2, I) = \frac{2I}{3p_2}.$$ 

Then the indirect utility function is

$$v(p_1, p_2, I) =$$

4 points

The expenditure function is

$$e(p_1, p_2, u) =$$

4 points

Hicksean demand for good 1 is

$$h_1(p_1, p_2, u) =$$

4 points
**Question 4** Suppose a person’s income offer curves are always straight lines from (0, 0) (for example, this is true for all CES utility functions). Suppose the person’s income is \( I = 20 \). Originally, prices are \( p_1 = 1, \ p_2 = 1 \) and consumption is \((12, 8)\). Then the price of good 1 increases to \( p_1 = 2 \), and consumption changes to \((5, 10)\). Determine graphically the Slutsky income and substitution effect.

The Slutsky substitution effect for goods 1 and 2 is 
\[
\Delta s_{x_1} = \ \ \ \ \ \ \ \Delta s_{x_2} =
\]

The Slutsky income effect for goods 1 and 2 is 
\[
\Delta I_{x_1} = \ \ \ \ \ \ \ \Delta I_{x_2} =
\]
**Question 5** Suppose the demand functions are

\[ x_1(p_1, p_2, I) = \frac{I}{2p_1}, \quad x_2(p_1, p_2, I) = \frac{I}{2p_2}. \]

Indirect utility is

\[ v(p_1, p_2, I) = \frac{I}{\sqrt{p_1p_2}}, \]

and the expenditure function is

\[ e(p_1, p_2, u) = u\sqrt{p_1p_2}. \]

A person’s income is \( I = 200 \), and prices are \( p_1 = 1, p_2 = 1 \). Then the government introduces a tax of 3 Dollars on each unit of good 1, raising the price to \( p_1 = 4 \). \[ 12 \text{ points} \]

**The government’s tax revenue is**

**After tax utility is**

Suppose prices are \( p_1 = p_2 = 1 \). At what income level \( I' \) would the person get the above after tax utility.

\[ I' = \]

**The deadweight loss of the tax is therefore**
Question 6  A person has mean variance preferences of the form $20E[X] - \text{Var}[X]$, where $X$ is the random variable that describes the portfolio return.

Suppose the person has 100 Dollars. He invests $\alpha$ Dollars in a risky asset with mean return 1.4 and a variance of 0.2. The remainder $(100 - \alpha)$ is invested in a riskless asset with return 1.1. Thus, the portfolio’s mean return is $1.4\alpha + 1.1(100 - \alpha)$ and the variance is $0.2\alpha^2$.

The optimal $\alpha =$  

12 points
Question 7 A person has a net worth of $I = 120,000$. The person can either get a job which would pay $40,000 or start a business. The business requires an initial investment of $80,000. With probability $1 - p$ it is a complete failure, i.e., the payoff of the business is 0 (and of course the owner loses his initial investment). With probability $p$ the business is a success, in which case it pays $320,000 (so the net payoff is $240,000$).

(a) Suppose the person’s Bernoulli utility function is $u(x) = \sqrt{x}$. Determine the value of $p$ at which the person is indifferent between starting the business and getting the job.

To start the business $p \geq$

(b) Now consider someone with the Bernoulli utility function $u(x) = -1/x$. Determine the value of $p$ at which this person is indifferent between starting the business and getting the job.

To start the business $p \geq$

(c) Who is more risk averse? (circle the correct answer)

The Person in (a) The Person in (b)

Explain:
Question 8  A lottery has the following possible payoffs: With probability 1/10 the payoff is 25 Dollars. With probability 1/5 the payoff is 4 Dollars and with probability 1/2 the payoff is 1 Dollar; with the remaining probability the payoff is zero. Suppose the person’s Bernoulli utility is $\sqrt{x}$.

(a) Then

The certainty equivalent of the lottery is

(b) Now suppose that the highest payoff is $m$ instead of 25 Dollars. Determine the value of $m$ such that certainty equivalent is 3 Dollars.

$m =$