



**Question 2** A person's utility function is  $u(x_1, x_2, L) = \ln(x_1) + \ln(x_2) + \ln(600 - L)$ , where  $x_1$  and  $x_2$  are consumption of two goods and  $L$  is the amount of time the person works. Suppose that prices are  $p_1 = 1$ ,  $p_2 = 4$  and that the wage is  $w = 20$ . The person's income solely comes from labor. Thus, the person solves  $\max_{x_1, x_2, L} \ln(x_1) + \ln(x_2) + \ln(600 - L)$  subject to  $x_1 + 4x_2 \leq 20L$ . *4 points*

**The Lagrangean is given by**

Determine the optimal level of  $L$  (recall that the derivative of  $\ln(x)$  is  $1/x$ ).

**At the optimum  $L =$**

*10 points*

**Question 3** A utility function is given by  $u(x_1, x_2) = 27x_1^2x_2$ . The resulting demand functions are

$$x_1(p_1, p_2, I) = \frac{2I}{3p_1}, \quad x_2(p_1, p_2, I) = \frac{I}{3p_2}.$$

Then the indirect utility function is

$v(p_1, p_2, I) =$

*4 points*

The expenditure function is

$e(p_1, p_2, u) =$

*4 points*

Hicksian demand for good 1 is

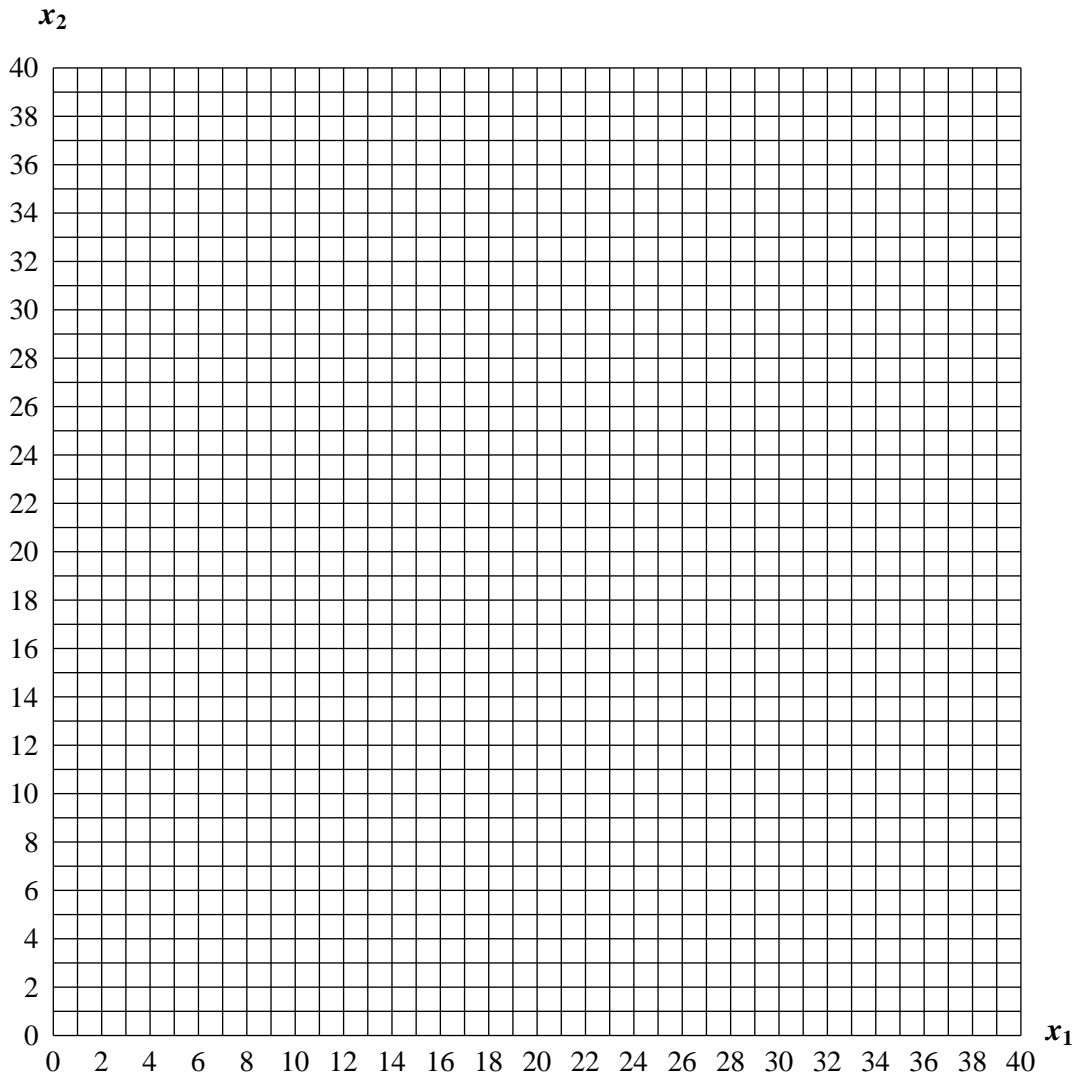
$h_1(p_1, p_2, u) =$

*4 points*

**Question 4** Suppose a person's income offer curves are always straight lines from (0, 0) (for example, this is true for all CES utility functions). Suppose the person's income is  $I = 90$ . Originally, prices are  $p_1 = 3$ ,  $p_2 = 5$  and consumption is (15, 9). Then the price of good 1 increases to  $p_1 = 5$ , and consumption changes to (6, 12). Determine graphically the Slutsky income and substitution effect. *12 points*

The Slutsky substitution effect for goods 1 and 2 is  $\Delta^s x_1 =$   $\Delta^s x_2 =$

The Slutsky income effect for goods 1 and 2 is  $\Delta^I x_1 =$   $\Delta^I x_2 =$



**Question 5** Suppose the demand functions are

$$x_1(p_1, p_2, I) = \frac{I}{2p_1}, \quad x_2(p_1, p_2, I) = \frac{I}{2p_2}.$$

Indirect utility is

$$v(p_1, p_2, I) = \frac{I}{\sqrt{p_1 p_2}},$$

and the expenditure function is

$$e(p_1, p_2, u) = u \sqrt{p_1 p_2}.$$

A person's income is  $I = 900$ , and prices are  $p_1 = 1$ ,  $p_2 = 1$ . Then the government introduces a tax of 8 Dollars on each unit of good 1, raising the price to  $p_1 = 9$ . *12 points*

**The government's tax revenue is**

**After tax utility is**

Suppose prices are  $p_1 = p_2 = 1$ . At what income level  $I'$  would the person get the above after tax utility.

**$I' =$**

**The deadweight loss of the tax is therefore**

**Question 6** A person has mean variance preferences of the form  $40E[X] - \text{Var}[X]$ , where  $X$  is the random variable that describes the portfolio return.

Suppose the person has 100 Dollars. He invests  $\alpha$  Dollars in a risky asset with mean return 1.5 and a variance of 0.2. The remainder  $(100 - \alpha)$  is invested in a riskless asset with return 1.1. Thus, the portfolio's mean return is  $1.5\alpha + 1.1(100 - \alpha)$  and the variance is  $0.2\alpha^2$ .

**The optimal  $\alpha =$**

*12 points*

**Question 7** A person has a net worth of  $I = 120,000$ . The person can either get a job which would pay \$40,000 or start a business. The business requires an initial investment of \$110,000. With probability  $1 - p$  it is a complete failure, i.e., the payoff of the business is 0 (and of course the owner loses his initial investment). With probability  $p$  the business is a success, in which case it pays \$350,000 (so the net payoff is \$240,000).

- (a) Suppose the person's Bernoulli utility function is  $u(x) = \sqrt{x}$ . Determine the value of  $p$  at which the person is indifferent between starting the business and getting the job.

5 points

To start the business  $p \geq$

- (b) Now consider someone with the Bernoulli utility function  $u(x) = -1/x$ . Determine the value of  $p$  at which this person is indifferent between starting the business and getting the job.

5 points

To start the business  $p \geq$

- (c) Who is more risk averse? (circle the correct answer)

3 points

The Person in (a)

The Person in (b)

Explain:

**Question 8** A lottery has the following possible payoffs: With probability  $1/10$  the payoff is 36 Dollars. With probability  $1/5$  the payoff is 9 Dollars and with probability  $1/2$  the payoff is 1 Dollar; with the remaining probability the payoff is zero. Suppose the person's Bernoulli utility is  $\sqrt{x}$ .

(a) Then

*6 points*

The certainty equivalent of the lottery is

(b) Now suppose that the highest payoff is  $m$  instead of 36 Dollars. Determine the value of  $m$  such that certainty equivalent is 3 Dollars.

$m =$

*6 points*