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All questions must be answered on this test form!

For each question you must show your work and (or) provide a clear argument.

All graphs must be accurate to get credit.

Question 1 A utility function is given by $u(x_1, x_2) = 4 \ln(x_1) + \ln(x_2)$, where $\ln(x)$ is the logarithm of x . Recall that the derivative of $\ln(x) = 1/x$, e.g., the partial derivative of $u(x_1, x_2)$ with respect to x_1 is $1/x_1$. Suppose that prices are $p_1 = 2$, $p_2 = 1$. Then the equation of the income offer curve is

12 points

$x_1 =$

Question 2 A consumer's utility function is given by $u(x_1, x_2) = x_1x_2$. The person has an income $I = 90$. The price of each unit of good 2 is 2 Dollars. However, the price of each unit of good 1 is $0.1x_1$. As a consequence, the budget constraint is given by $0.1x_1^2 + 2x_2 = 90$. Further, when solving this problem you can ignore the constraints that $x_1, x_2 \geq 0$. Solve for the optimal consumption by using the Lagrangean (you only need to solve the first order conditions for x_2)

4 points

The Lagrangean is given by

The optimal consumption of good 2 is $x_2 =$

10 points

Question 3 Suppose that a city is considering purchasing some land for a public park. The per capita cost of purchasing the land would be 200 Dollars.

- (a) Suppose that the city wants to pay the cost of 200 Dollars by charging an entrance fee of p Dollars. The estimated per capita demand function is $x(p) = 40 - 2p$, i.e., $x(p)$ is the number of times an individual would visit the park if the entrance fee is p . The cities' revenue per capita is therefore $px(p)$. Determine the entrance fee p that the city must charge such that its revenue per individual is 200 Dollars (*Hint*: The answer is an integer, so you can just solve this numerically).

The entrance fee is $p =$

3 points

- (b) You are an economic consultant of the city who wants to determine which of following policy is better: (i) Charge the entrance fee p determined in part (a); (ii) impose a tax of 200 Dollars on every individual and allow free entry to the park (i.e., choose $p = 0$).

8 points

Let x be the number of times an individual visits the park, and let m be the amount of money the person spends on other goods. Let I be an individual's income. For example, if the person spends 40 Dollars on the park, then m would be $I - 40$. One can easily verify that the utility function $u(x, m) = 20x - 0.25x^2 + m$ generates the demand function $x(p) = 40 - 2p$.

In case (i), i.e., if the city charges the entrance fee p that you determined in (a) then

a person's utility is

(The answer will depend on I . Also, if you were unable to find p in (a) then use $p = 8$ for the argument.)

Now suppose each citizens must pay the tax of 200, but gets unlimited access to the park (i.e., $p = 0$). Then

a person's utility is

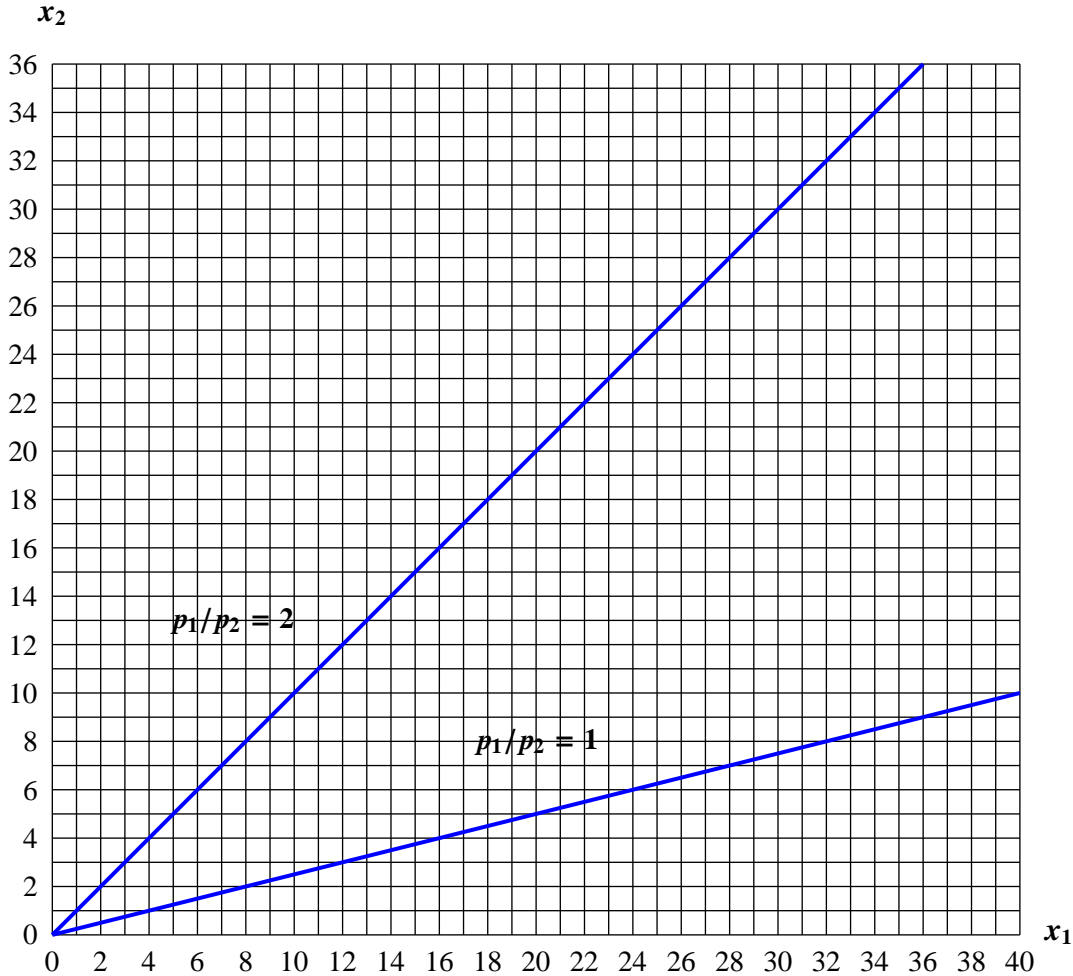
Given the computation, which policy is better (*mark the correct answer*)

An entrance fee p

A tax of 200 Dollars.

Question 4 A person's income offer curves for price ratios $p_1/p_2 = 1$ and $p_1/p_2 = 2$ are depicted below. Suppose the person's income is $I = 30$. Originally, prices are $p_1 = 2, p_2 = 1$. Then the price of good 1 decreases to $p_1 = 1$.

12 points



At prices $p_1 = 2, p_2 = 1$ demand is $x_1 =$ $x_2 =$

At prices $p_1 = 1, p_2 = 1$ demand is $x_1 =$ $x_2 =$

Determine the Slutsky substitution and income effects. Recall, that in the Slutsky substitution effect, the person is compensated with exactly the level of income at which the consumer can afford the original consumption choice at the new prices.

The change in demand due to the substitution effect is $\Delta^s x_1 =$ $\Delta^s x_2 =$

The change in demand due to the income effect is $\Delta^I x_1 =$ $\Delta^I x_2 =$

Question 5 A utility function is given by $u(x_1, x_2) = x_1 x_2^2$. Suppose prices are $p_1 = 1$, $p_2 = 2$. Then the least costly consumption bundle that gives a utility of 125 is

$x_1 =$, $x_2 =$

12 points

Question 6 Suppose there are two types of costumers that use cellular phones. High demand types have a utility function $u_h(t,m) = 10t - t^2 + m$ while low demand types have a utility function $u_l(t,m) = 20t - t^2 + m$. The cellular phone company offers a contracts T_h, F_h for the high demand types, where T_h is the number of hours the person can call and F_h is the total fee for the contract. Similarly, T_l, F_l is the contract for the low demand types. Since 20% of the population are high and 80% low demand costumers, the firms solves the following maximization problem.

$$\max_{F_h, T_h, F_l, T_l} 0.2F_h + 0.8F_l,$$

subject to

1. $10T_l - T_l^2 - F_l \geq 0$
2. $20T_h - T_h^2 - F_h \geq 20T_l - T_l^2 - F_l.$

The objective is the firm's expected revenue per costumer (with probability 0.2 the costumer is a high demand type, and the revenue is F_h and with probability 0.8 the demand is low and the probability is F_l). The first constraint ensures that the low type is better off accepting the contract rather than not accepting it and getting a payoff of zero. The second constraint ensures that the high demand type choose the contract that is designed for them (we will show in a future lecture that the low demand types will never choose the contract designed for the high demand types).

(a)

4 points

The Lagrangean is given by

- (b) Solve the Lagrangean to determine the optimal values for T_h and T_l (you will have to take the first order conditions with respect to all four variables, however, you only need to find T_h and T_l .)

10 points

$T_h =$	$T_l =$
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Question 7 A person has an income if $I = 10,000$ and faces the loss of 9,900 in case of an accident. The Bernoulli utility function is $u(x) = \sqrt{x}$.

(a) Suppose that the person's probability of having an accident is 0.02. Then

5 points

The expected utility when being uninsured is

(b) Now suppose the an insurance company offers full coverage at a price of p . What is the maximum price the insurance company can charge (at this price the person must be indifferent between being insured and being uninsured).

5 points

$p =$

(c) Now suppose that the person's probability of an accident is q . The insurance company offers coverage at a price $p = 1,900$. Determine the least risky person, i.e., the lowest q , who would be just willing to sign up.

5 points

$q =$

Question 8 A lottery has the following possible payoffs: With probability $1/1,000$ the payoff is 900 Dollars. With probability $1/100$ the payoff is 9 Dollars and with probability $1/10$ the payoff is 1 Dollar. Suppose the person's Bernoulli utility is \sqrt{x} .

(a) Then

5 points

The certainty equivalent of the lottery is

$m =$

5 points