All questions must be answered on this test form!
For each question you must show your work and (or) provide a clear argument.
All graphs must be accurate to get credit.

Question 1 Suppose that a person’s utility function is \( u(x_1, x_2) = x_1^2 x_2 \). Prices are \( p_1 = 3 \), \( p_2 = 1 \) and income is \( I = 360 \).

The Lagrangean is given by

\[
\text{The equation of the income offer curve is } x_2 = \ldots. \\
\text{The optimal consumption choice is } x_1 = \ldots, x_2 = \ldots.
\]
Question 2 A utility function is given by $u(x_1, x_2) = x_1 x_2^2$. Suppose prices are $p_1 = 2$, $p_2 = 1$. Determine the least costly consumption bundle that gives the person a utility of 1,024. 

The Lagrangean is given by

The least costly consumption bundle is $x_1 = \quad , x_2 = \quad$
Question 3 Suppose a consumer’s preference between current consumption, \( c_1 \), and future consumption \( c_2 \) are given below. The person has an income of 30 Dollars in the first period. In the second period, the person retires, i.e., his income is 0 and he has to finance consumption from saving. If the interest rate is \( r \) then the equation of the budget line is

\[
c_1 + \frac{c_2}{1 + r} = 30,
\]

since the present value of consumption is given by \( c_1 + \frac{c_2}{1+r} \), which must equal the person’s income.

Consumer preferences are depicted on the next page.

1. Suppose the interest rate is 0%. Then

\[
\text{Current consumption is } c_1 = . \text{ The person saves } $ .
\]

2. Suppose the interest rate increases to 50%. Then

\[
\text{Current consumption is } c_1 = . \text{ The person saves } $ .
\]

3. The change in saving due to the income effect is .

The change in saving due to the substitution effect is .

Explain briefly why raising the interest rate lowers savings (Your answer must be brief, i.e., fit in the box below).
Question 4 Suppose there are two goods. The demand functions are given by

\[ x_1(p_1, p_2, I) = \frac{I}{p_1 + \sqrt{p_1p_2}}, \quad x_2(p_1, p_2, I) = \frac{I}{p_2 + \sqrt{p_1p_2}} \]

Suppose that originally income is \( I = 168 \) and \( p_1 = 1, \ p_2 = 9 \). Then the price of good 1 increases to \( p_1 = 4 \).

\[
\begin{align*}
\text{Demand before the price change is } x_1 &= \quad , x_2 = \quad . \\
\text{Demand after the price change is } x_1 &= \quad , x_2 = \quad . \\
\text{The income effect is } \Delta^I x_1 &= \quad , \Delta^I x_2 = \quad . \\
\text{The substitution effect is } \Delta^s x_1 &= \quad , \Delta^s x_2 = \quad . 
\end{align*}
\]

(We use the Slutsky income and substitution effect. To determine the substitution effect, we give the person just enough money such that the person can afford the original consumption.)
Question 5  Suppose a lottery has the following payoffs. With probability 0.3 you win 2 Dollars, with probability 0.2 you win 5 Dollars, and with probability 0.1 you win 20 Dollars. With the remaining probability you win 1 Dollar. The Bernoulli utility function is \( u(x) = -1/x \).

- The expected payoff of the lottery is .
- The expected utility of the lottery is .
- The certainty equivalent of the lottery is .
Question 6  A person has 1,000 Dollars, but faces a loss of 800 Dollars with probability 0.1. The person’s Bernoulli utility is \( u(x) = -1/x \). Determine the maximum amount the person is willing to pay to get insurance with full coverage.  

**The person is willing to pay at most $ \ldots \).**

(You must find the amount \( p \) at which the person is indifferent between being insured and being uninsured.)
Question 7  Suppose there are three types of people A, B, and C in a population of 1 Million. Type A people have a probability of 0.01 of becoming seriously ill. For type B the probability is 0.2, and for type C it is 0.9. The medical costs from being treated for the illness is $30,000. The maximum willingness to pay for insurance is $400 for type A, $8,000 for type B, and $27,500 for type C. Also, assume that 90% of the population are type A, 8% are type B and 2% are type C. The type is private information, i.e., only each person knows their true probability of becoming ill, but not the insurance company. The insurance firm has an additional cost of 20 Million Dollars if is provides health insurance — these costs are not present if no insurance is provided.

(a) Suppose that insurance is mandatory (everyone must be insured). Because of government regulation the firm must provide insurance at a premium such that profits are exactly zero. Then

\[
\text{The insurance premium is}
\]

(b) Now suppose that insurance is voluntary. If the insurance premium is $400, then the firm’s total profit is

\[
\text{Million Dollars}
\]

If the insurance premium is $8,000, then the firm’s total profit is

\[
\text{Million Dollars}
\]

If the insurance premium is $27,500, then the firm’s total profit is

\[
\text{Million Dollars}
\]

If the insurance firm maximizes profits (and an option is always to not offer insurance in which case profits are zero). Then

\[
\% \text{ of the population will be insured by the firm.}
\]
Question 8 (Difficult) A person has 4,000 Dollars to invest. There are two investments, A and B, whose payoffs depend on the realized state of the economy. Suppose there are two states, denoted by $\omega_1$ and $\omega_2$, that each occur with probability 0.5. If the person invests $x_A$ Dollars in A then he will receive $0.2x_A$ Dollars in state $\omega_1$ and $2x_A$ Dollars in state $\omega_2$. If the person invests $x_B$ Dollars in B then he will receive $1.8x_B$ Dollars in state $\omega_1$ and zero Dollars in state $\omega_2$.

Suppose that all money must be invested, i.e., $x_A + x_B = 4,000$. The person’s Bernoulli utility function is $\ln(x)$. Thus, the person solves the following optimization problem.

10 points

The optimal investment amounts are $c_1 = \phantom{1}$, $c_2 = \phantom{1}$.