Question 1 A person’s utility function is given by $u(x_1, x_2) = x_1 x_2$. Prices are $p_1 = 1$, $p_2 = 9$. The person wants to find the least costly consumption bundle that gives him the same utility as (30, 30).

(a) Specify the optimization problem in the box below: 

(b) Solve the optimization problem by using the Lagrangean (assuming that the non-negative constraints on $x_1$ and $x_2$ are slack.)

The optimal values are $x_1 = \quad x_2 = \quad 0$ points
Question 2 The equation of a person’s income offer curve is

\[ x_2 = \frac{p_1}{p_2} x_1^{0.5}, \]

where \( p_1 \) is the price of good 1 and \( p_2 \) is the price of good 2. Suppose that prices are \( p_1 = 4, p_2 = 1 \). Suppose the person’s income is \( I = 24 \). Solve graphically for the optimal consumption choice:

The optimal consumption is \( x_1 = \quad x_2 = \quad 12 \text{ points} \)
Question 3  A person’s utility depends on the mean and the variance of his investments. Suppose that the person has 1 unit of money, that he can invest either at the riskless rate of 10% or in a risky investment with a return of 40% but a variance of 30%. Thus, if the person puts \( \alpha \)% of the money in the riskless investment and \( \beta \)% in the risky investment, then the return is \( 1.1\alpha + 1.4\beta \) and the variance is \( 0.3\beta^2 \). The person’s utility is given by \( \mu - 4\text{Var} \), where \( \mu \) is the return and Var is the variance of the portfolio. The person maximizes this utility, subject to the constraint that \( \alpha + \beta = 1 \). Determine the optimal investment by using the Lagrangean

The optimal consumption is \( \alpha = \) \( \beta = \) 15 points
Question 4 A person utility function is given by $u(x_1, x_2) = x_1 x_2$. Prices are $p_1 = 1$, $p_2 = 1$. Income is $I = 9,000$. Now suppose that the government introduces a tax of 1.25 Dollars per unit of good 1, thus raising the price of good 1 to 2.25 Dollars.

(a) The consumption after the tax is $x_1 = \quad \text{, } x_2 = \quad$. Utility is

The government’s tax revenue from the person is $\quad$. 
(b) We want to determine the maximum amount of money the person would be willing to pay to the government to avoid the tax. To do this, determine the minimum amount of money the person needs in order to obtain the after tax utility from part (a) when prices are at the pretax level \( p_1 = p_2 = 1 \). The difference between this amount and his actual income of \( I = 9,000 \) is the willingness to pay.

The person’s maximum willingness to pay is $10 points

The difference between the tax revenue and the person’s willingness to pay (which should be larger than the tax revenue) is the deadweight loss from taxation.

Per Dollar of taxes raised, the deadweight loss is cents.
Question 5 A person’s income is $I = 26$. Originally, prices are $p_1 = p_2 = 1$, but then the price of good 2 increase to $p_2 = 2$. Using the income offer curves depicted below, determine the Slutsky substitution and income effect. \[15\text{ points}\]

The substitution effect is $\Delta^s x_1 = \Delta^s x_2 = \ldots$.

The income effect is $\Delta^I x_1 = \Delta^I x_2 = \ldots$.

(Denote an increase by positive sign, and a decrease by a negative sign).
Question 6  A person has an income of $I = 10,000$. With probability $p$ he has an accident which results in a loss of $7,500$. At a cost of $396$ he can get complete coverage. The person’s Bernoulli utility function is $\sqrt{x}$

(a) Determine the probability $p$ at which the person is indifferent between being insured and being uninsured.  

\[ p = \ldots \]

7 points

(b) Now suppose there also exists a policy with a deductible. In particular, at a cost of $199$ the person would get insurance with a deductible of $1,701$. Determine the probability $p$ at which the person is indifferent between having the insurance with the deductible, and being uninsured.  

\[ p = \ldots \]

7 points
Question 7 Suppose a person’s Bernoulli utility function is $\sqrt{x}$. The person considers playing a lottery with the following outcomes: (a) with probability 0.3 win 9 Dollars; (b) with probability 0.1 win 1,600 Dollars; (c) with probability 0.05 win 10,000 Dollars. Then

the expected utility from playing the lottery is .

the certainty equivalent of the lottery is .
Scratch Paper: Not graded!
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