Question 1 Suppose that a demand function is given by $Q_D(P) = 20 - P$. The supply is $Q_S(P) = 2 + P$. Solve the following questions graphically (using the grid below).

(a) Then the equilibrium price and quantity are.  

$$P^* = \quad \text{and} \quad Q^* = \quad .$$

(b) Now suppose that the government introduces a tax of 100% on each unit of good 1 that a person consumes. Thus, if the price is $P$, a consumer pays $2P$ for the product and demand is therefore $Q_D(P) = 20 - 2P$. After the tax is introduced the new equilibrium price and quantity are

$$P^* = \quad \text{and} \quad Q^* = \quad .$$
**Question 2** Suppose that if the price is $P = 20$ then demand is $Q = 40$. The price elasticity of demand is $-0.2$. If demand is linear, then the demand function is given by

$$Q_D(P) = \cdot$$

**Question 3** Suppose that both $(4, 12)$ and $(6, 6)$ are on the budget line. If $p_1 = 12$ then

$$p_2 = \cdot$$
**Question 4** Suppose that the demand for a product is \( Q_D(P) = 100 - P \) and the supply is \( Q_S(P) = 10 + P \). Similar to Question 1, the government wants to impose a tax of \( \tau \) per unit on consumers. Thus, demand after the tax is \( Q_D(P) = 100 - (P + \tau) = 100 - P - \tau \).

(a) The equilibrium quantity (which depends on \( \tau \) is)

\[
Q^*(\tau) = \ldots \quad 7 \text{ points}
\]

(b) Now suppose that the government wants to choose \( \tau \) to maximize the revenue from the tax. Since the government receives \( \tau \) for each unit sold, the revenue is given by \( \tau Q^*(\tau) \). Find \( \tau \) that maximizes this (of course, you’ll have to take a derivative).

The revenue maximizing tax is given by \( \tau^* = \ldots \). 5 points
Question 5 A farmer grows soybeans and corns. He can sell soybeans for 2 Dollars per unit and corn for 4 Dollars per unit. Thus, he wants to maximize \(2x_S + 4x_C\). The amount of corn and soybeans he can grow is constraint by \(x_S + x_C \leq 30\). Furthermore, in order to rotate crops, he needs to plant at least twice as much soybeans as corn, i.e., \(x_S \geq 2x_C\). However, he also needs to grow at least 5 units of corn and 16 units of soybeans to feed his own livestock. The optimal quantity of soybeans and corn is

\[
x_S = \quad , \quad x_C = \quad .
\]

Clearly indicate the feasible set by shading it. You must also draw a few of the lines that represent the objective.
Question 6  A firm wants to produce two outputs, $x_1$, $x_2$ from three inputs that are subject to resource constraints. The price of both outputs is the same, i.e., the firm maximizes $x_1 + x_2$ subject to the following resource constraints $4x_1 + x_2 \leq 80$, $x_1 + 2x_2 = 48$, $2x_1 + x_2 \leq 60$, and $6x_1 + x_2 = 132$. Clearly, $x_1 \geq 0$ and $x_2 \geq 0$. Determine the optimal choice of $x_1$ and $x_2$, graphically.

Clearly indicate the feasible set by shading it. You must also graph some lines that represent the objective.

$x_1 = \hspace{1cm}, x_2 = \hspace{1cm}$

15 points
**Question 7** Suppose a person has the following preferences: Independently of the consumption of good 2:

1. If the person consumes at most 5 units of good 1 then the MRS is 3;
2. If if the person consumes at least 5 units of good 1 but not more than 15 units then the MRS is 1;
3. If if the person consumes more than 15 units of good 1 then the MRS is 1/2.

(a) Graph the indifference curves through (0, 40), (0, 35) and (0, 30), using the grid on the following page.  

(b) Suppose that a person’s income is $I = 90$ and that prices are $p_1 = 6$, $p_2 = 3$. Graph the budget line, determine the optimal consumption choice, and graph the indifference curve through the optimal choice point.

(c) Now suppose that there is a quantity discount, i.e., the price per unit is $p_1 = 6$ if the consumer purchases 10 units. Every additional unit costs $p_1 = 1$ (the price of good 2 remains $p_2 = 3$ and $I$ is again 90). Graph the budget line. The optimal consumption is given by

$$x_1 = \quad , \quad x_2 = \quad .$$
**Question 8** The demand for rental apartments is \( Q_D(P) = 1,000 - AP \), where \( A > 0 \). The supply is \( Q_S(P) = 0.2P \).

(a) Show that demand becomes more elastic when \( A \) is increased (keeping \( P \) fixed):
\[
\epsilon_D^P = \text{.}  
\]
Taking the derivative of \( \epsilon_D^P \) with respect to \( A \) we get (recall that \( \left( \frac{u}{v} \right)' = \frac{u'v - uv'}{v^2} \)).
\[
\frac{d\epsilon_D^P}{dA} = \text{.} 
\]
This derivative is **positive** **negative** (mark the correct answer).

(b) Now suppose that the government wants to subsidize rental apartments. That is, every landlord receives a subsidy of \( s \) per apartment. As a consequence, the supply of apartments is \( Q_S(P + s) = 0.2(P + s) \).
The equilibrium price (which depends on \( A \) and \( s \)) is therefore
\[
P^* = \text{.}  
\]
(e) Considering the results in (a) and (b), provide a brief argument below, whether the impact of \( s \) on the equilibrium rental price \( P \) becomes larger or smaller when you compare any situation with a less elastic consumer demand to one with a more elastic consumer demand. Note that the impact of \( s \) on the equilibrium price is given by \( \frac{\partial P^*(A,s)}{\partial s} \). Formally, the question is how this impact changes when \( A \) changes. 5 points