The homework is due on Wednesday, December 2 at 4pm. Each question is worth 0.8 points.

**Question 1** A production function is given by \( f(K, L) = \sqrt{KL} \). Input costs are \( r \) and \( w \).

Determine the firm’s cost function. You can use the fact that you know that the indirect utility function for the utility function \( u(x_1, x_2) \) is given by

\[
v(p, I) = \frac{I}{2 \sqrt{p_1 p_2}}.
\]

Suppose that there are many such firms in a competitive industry. The market price for the output is given by \( P = 40 \). Suppose that the wage \( w = 2 \). Determine the value of \( r \) if equilibrium supply and demand is strictly positive (but of course not infinite).

**Question 2** Suppose all firms in a competitive industry have cost function \( c(Q) = 4Q^2 \).

Suppose that in equilibrium each firm makes a profit of 20 Dollars per unit. Determine the equilibrium price \( P \) for the firm’s output, each firm’s supply \( Q \), and total profit.

**Question 3** A production function that uses only labor as input is given by

\[
f(L) = \begin{cases} 
0 & \text{if } L \leq 10 \\
10 \sqrt{L-10} & \text{if } L > 10.
\end{cases}
\]

As you can see, the firm needs more than 10 units of labor to produce a positive amount of output. Let \( w \) be the cost of a unit of labor.

(a) Determine the (minimum) cost of producing 0, 10, 20, and 30 units of output, (this will depend on \( w \)).

(b) Specify the firm’s cost function for general \( Q \) assuming that \( w = 10 \).

**Question 4** Suppose that all firms in a competitive industry have cost function \( c(Q) = Q^2 + 100 \). The demand function is \( Q_D(P) = 1,000 - 10P \).

(a) Suppose that there are currently 10 firms in that industry. Determine equilibrium prices and profits.

(b) Now suppose there is free entry, and firms will enter until equilibrium profits are zero. How many firms will operate in equilibrium?
(e) Suppose we are again in (a) with 10 firms, but these firms manage to lobby policy makers to introduce a law that prevents new entry. Suppose that there 1,000 consumers with the same utility functions $u(q, m) = 100q - 50q^2 + m$, where $q$ is the firm’s product and $m$ money spent on other goods. Each consumer has an income $I = 200$. You can check (but you don’t have to) that aggregate demand for the firm’s product is then $Q_D(P) = 1,000 - 10P$.

Since $m$ enters linearly, we can interpret utilities as monetary amounts, i.e., we can add utilities of consumers together and compare them to firm profits. Determine the utility loss of each consumer from the new law, i.e., compare the utility from (a) to that from (b). Don’t forget that purchasing $q$ units of the firm’s product reduces consumption of other goods by $qP$, where $P$ is the price charged by the firms. To get the total utility loss just add up the losses of all consumers.

Now determine the profit gains of the 10 firms if the law is introduced and new entry is prevented, i.e., compare profits in (a) to those in (b) for each firm, and then multiply this by the number of firms, i.e., 10.

Is this law a good idea? What are the total gains or losses to society if the law is introduced?