The homework is due on Wednesday, November 18. Each question is worth 0.8 points.

**Question 1** Assume you consider purchasing a used car. If the car is of good quality then your benefit of getting the car is $4,000. If the car is of low quality, your benefit is $2,000. You believe that the seller is willing to sell the car to you if you pay at least $3,000 if the car is good, and $1,500 if the car is bad.

(a) Assume that you believe that the seller's car is good with probability 0.8. Then in order to maximize your expected net-benefit, you should offer how much money for the car? Determine you expected net-benefit?

(b) How do your answers to (a) change if you believe that the seller’s car is good with probability 0.4.

**Question 2** Assume there are three types of drivers. Excellent drivers have an accident probability of 1/1000. Good drivers have an accident probability of 10/1000 and bad drivers have an accident probability of 49/1000. There is the same number of drivers of each type in the population. Therefore, the overall accident probability is (1/3)(1/1000) + (1/3)(1/100) + (1/3)(49/1000) = 1/50. The expected loss from an accident is $10,000. Insurance company A offers full coverage at a premium of $250 (i.e., $250 is the price of the insurance). Each person would get insurance under the same conditions, because the insurance company cannot tell the type of a driver.

By law every driver must sign up for insurance. Therefore, if A is the only insurance company then they expect to pay (1/50)10,000 = 200 per customer. The expected profit per customer is then 250 − 200 = 50 (because the insurance premium is $250).

Insurance company B enters the market and decides to offer insurance only to drivers who have a clean driving record. It turns out that all excellent and good drivers have a clean record. Bad drivers do not have a clean record.

(a) Suppose that insurance company B wishes to make an expected profit of $50 per driver, then it should charge how much for an insurance contract that offers complete coverage?

(b) Assume insurance company A does not adjust the premium and the contract after B has entered the market. Determine A’s expected profit per customer.

(c) Suppose that insurance company A is unwilling to offer insurance only to drivers with a clean record, and wishes to have an expected profit of $50 per customer. How much should company A charge?
**Question 3** The profit of a firm is given by \( f(e) = 200e - e^2 \), where \( e \) is the manager’s effort. The manager’s cost of effort is given by \( c(e) = 2e \). The manager receives a share \( s \) of the firm’s profit as compensation, i.e., the compensation is \( sf(e) \). Including the cost of effort, the manager’s net-payoff is \( sf(e) - c(e) \). The owner of the firm receives \( f(e) \) minus the payment to manager, i.e., \( f(e) - sf(e) \).

(a) Currently the manager receives a share \( s = 0.1 \) of the firm’s profit. However, the owner is not satisfied with the manager’s effort and the firm’s profit and considers increasing the manager’s compensation to \( s = 0.2 \). Determine the owner’s payoff for \( s = 0.1 \) and \( s = 0.2 \).

(b) Determine the share \( s \) that maximizes the owner’s profit.

(c) Find a contract (that possibly allows negative payoffs to the managers) that implements the efficient effort choice.

**Question 4** Let \( q \) be the probability that a person has an accident. Assume that the loss in case of an accident is 10,000. Hence, the cost of providing insurance that person is 10,000q. Suppose the person’s willingness to pay for complete insurance coverage is given by \( 12,000q - 2,000q^2 \). You can check that for \( q \) with \( 0 < q < 1 \) the willingness to pay exceeds the costs.

1. Suppose the type of the person is known. Is it possible to find an insurance premium \( p \) such that the person is better off getting insured, and the insurance company makes money.

2. Now suppose that the types are private information, i.e., the insurance company does not know \( q \). All that is known is that the \( q \) is uniformly distributed between 0 and 1. Thus, the average type is 0.5. Suppose that insurance company charges the price \( p = 5,000 \), which would correspond to the expected cost of insurance if all types sign up. Determine what types sign up at this price, and the profit of the expected profit of the insurance company.

**Question 5** Consider the following principal-agent problem. The principal (think of it as a firm) hires a worker, and can monitor the worker at a cost of \( m = 4 \). The worker can choose either to work at the high effort level, in which case, output is \( y > 0 \), or the low effort level (he shirks), in which case output \( y \) is zero. The agent’s cost of choosing the high effort level is \( c = 1 \), and the cost of choosing the low effort level is \( c = 0 \). The principal promises to pay a wage \( w \) to the agent, unless the agent is caught shirking. In this latter case, the agent is fired and receives a payoff of \( w_0 \). Thus, if \( p \) is the probability of monitoring (this probability is chosen by the principal), then the payoff to a worker from shirking is \( pw_0 + (1 - p)w \), while the payoff from not shirking is \( w - c \).

(a) The principal wants to choose \( p \) and \( w \) such that the worker does not shirk and such that the principal’s payoff, \( y - pm - w \), is maximized. Specify and solve the principal’s optimization problem and determine the optimal value of \( p \) and \( w \) (the value of \( w \) will depend on \( w_0 \)).
We now model how \( w_0 \) is determined endogenously.

Suppose that the supply curve for labor is \( S(w) = w \). There are ten identical firms corresponding to the principals described above, each requiring one unit of labor to produce. Thus, if \( S(w) = 10 \) then the supply for labor equals demand for labor. In contrast, if \( S(w) > 10 \) then the supply of labor exceeds demand, and there is unemployment in the economy. The unemployment rate is \( u = (S(w) - 10)/S(w) \), since \( S(w) \) is the total number of individuals who wish to work, while \( S(w) - 10 \) is the number of individuals who are unemployed.

Suppose that if an individual is caught shirking and is fired, then he will be reemployed at a different firm with probability \( 1 - u \), in which case he will receive a wage of \( w \). Otherwise, he will receive an unemployment benefit payment \( b \). Thus, \( w_0 = (1 - u)w + ub \).

Suppose that \( b = 0 \). Using the relationship between \( w \) and \( w_0 \) from (a) to determine the unemployment rate.

What happens to the unemployment rate and to wages if the unemployment benefit is increased? Plot \( w \) and \( u \) for \( b \) between 0 and 10. You can do this last part numerically. Be careful, since you are solving a quadratic equation, there are two solutions, but only one solution makes sense.

What does the model imply about the relationship between unemployment benefits, wages, and the unemployment rate?

**Question 6** A production function is given by \( f(K, L) = \sqrt{K} + \sqrt{L} \).

(a) Suppose that \( K = 1 \) and \( L = 4 \). Determine the average and marginal products of capital and labor, respectively. Determine what happens to the average and marginal product of labor and capital if \( K \) is increased to \( K = 9 \).

(b) Determine the MRTS.

(c) Suppose you know that the Walrasian demand functions for the utility function \( u(x_1, x_2) = \sqrt{x_1} + \sqrt{x_2} \) are given by

\[
x_1(p_1, p_2, I) = \frac{Ip_2}{p_1(p_1 + p_2)}, \quad x_2(p_1, p_2, I) = \frac{Ip_1}{p_2(p_1 + p_2)}.
\]

Use this information to determine the firm’s cost function \( C(Q, r, w) \). Further, determine the firm’s optimal demand for inputs \( K \) and \( L \) as a function of \( Q, r, \) and \( w \).