The homework is due on Wednesday, October 28 at 4pm. No partial credits.

0.8point

Question 1 A person’s Bernoulli utility function is given by $u(x) = \ln(x)$. The person has wealth $w$ and invests fraction $\alpha$ into a risky asset and $1 - \alpha$ into a riskless asset, where $\alpha \in \mathbb{R}$, i.e., $\alpha$ can be negative or greater than one. The riskless asset has a return of 20%. The risky asset has a return of 100% if the state is $g$ and −40% if the state is $b$. State $g$ occurs with probability 0.75 and state $b$ with probability 0.25. Determine the optimal $\alpha$ analytically, i.e., do not use Excel.

Now suppose that the Bernoulli utility function is $u(x) = \sqrt{x}$ (everything else remains the same). Determine the new value of $\alpha$ analytically. Is $\alpha$ smaller or larger than for $u(x) = \ln(x)$? Explain?

0.8points

Question 2 A person’s utility function is given by $u(I) = -1/x$. Suppose the person’s income is $10,000. There is a probability $p$ that the person has an accident which results in a loss of $8,000. There are two types of insurances available. For a price of $800 the person can get full coverage. For a price of $200 the person can get insurance with a deductible of $2,000.

There is an interval of probabilities $p$ at which the person prefers to be uninsured, an interval at which the person prefers complete coverage, and an interval for which the person prefers the deductible. Determine these intervals. Explain your result.

0.8points

Question 3 Suppose a person’s Bernoulli utility function is given by $u(x) = 10x - 0.01x^2$. The person has 1 unit of money to invest. There are two investments available: (a) A riskless asset that pays an interest rate of 10%. A risky asset that that either pays 50% with probability 0.6 or -10% with probability 0.4.

(a) Suppose the person invests $\alpha$ Dollars into the risky asset $100 - \alpha$ Dollars into the riskless asset. Specify the person’s expected utility (do not forget to add the principal back in, e.g., if the interest rate is 10% and you have 50 Dollars in the riskless asset, then your payoff is $50 + 5 = 55$.)

(b) Determine the optimal portfolio choice of $\alpha$ analytically (i.e., using calculus rather than Excel).

0.8points

Question 4 A person has mean variance preferences given by $u(\mu, \sigma) = 10\mu - \sigma^2$, where $\sigma$ is the standard deviation (and $\sigma^2$, the variance).

There are two investments $X$ and $Y$. The expected returns are $\mu_X = 1.1$, $\mu_Y = 1.2$, the standard deviations are $\sigma_X = 0.2$, $\sigma_Y = 0.4$ and the covariance $\text{cov}(X,Y)$ is −0.1.

Consider a portfolio consisting of $\alpha$ units of $X$ and $(1 - \alpha)$ units of $Y$. 
Determine the mean and variance of a portfolio with shares $\alpha$ and $1-\alpha$. Then determine the optimal alpha using calculus (rather than Excel). Note that solutions with $\alpha < 0$ or $\alpha > 1$ could occur. If you have such a solution, what is the interpretation? 

**Question 5** Using daily data from October 22, 2014 and October 22, 2015 estimate $\alpha$ and $\beta$ for the following stocks. 1) Microsoft (MSFT), and 2) Goldman and Sachs (GS). Then estimate $\alpha$ and $\beta$ for the same two stocks using daily data between October 22, 2013 and October 22, 2014. In both cases use the 3 month Treasury bill rate to determine the risk-free rate.

Do the estimates differ? Discuss your results.