The homework is due on Wednesday, October 14 at 4pm. Each question is worth 0.8 points.

**Question 1** A person’s utility function is \( u(x_1, x_2) = x_1 + \ln(x_2) \). Prices are \( p_1 = p_2 = 1 \), and income is \( I \).

1. Specify the Lagrangean using all three constraints.
2. We know that constraint \( x_2 \geq 0 \) must be slack, else if \( x_2 = 0 \) utility would be negative infinity. Specify the first order conditions taking this into account.
3. Using the Lagrangean determine value of \( I \) and \( x_1 \) where constant \( x_2 \geq 0 \) just becomes slack. Note that it this point that Lagrange multiplier of the constraint must be zero and of course \( x_2 = 0 \). Using this information you can solve the first order conditions for \( x_2 \) and \( I \).

**Question 2** A person has an income of \( m_0 \) this year, and \( m_1 \) next year, and can save or borrow money at a fixed interest rate \( r \). The person’s consumption in the two years is denoted by \( x_0 \) and \( x_1 \). Utility of consumption in each period is \( \ln(x) \), and next period’s utility is discounted by a factor of \( \delta \). Let \( y \) be the amount of money the person saves or borrows in the first period. Then, the person solves

\[
\max_{x_0, x_1, y} \ln(x_0) + \delta \ln(x_1)
\]

subject to

1. \( x_0 + y \leq m_0 \);
2. \( x_1 \leq m_1 + (1 + r)y \).

(a) Determine the Lagrangean (assume that both constraints bind), and denote the Lagrange multipliers by \( \lambda_1 \) and \( \lambda_2 \).

(b) Determine the solution, i.e., determine how \( x_0 \) and \( x_1 \) depend on \( \delta \) and \( r \).

(c) What condition must \( \delta \) and \( r \) satisfy such that consumption in both periods is the same?

(d) Suppose the government wants to stimulate the economy by providing a tax rebate that increases the person’s income to \( m_0(1 + s) \). This rebate costs the government \( m_0s \) Dollars. In period 1 the government’s debt is therefore \( m_0s(1 + r) \) which must be recovered by taxing the consumer at period 1, thereby reducing income from \( m_1 \) to \( m_1 - m_0s(1 + r) \).

What is the effect of this policy? Does the policy increase consumption at \( t = 0 \)?
(e) Finally suppose that the consumer can only save but not borrow money, i.e., $y \geq 0$. Find a numerical example in which the government’s policy raises consumption at $t = 0$.

**Question 3** Using the CES utility function for $\alpha = \rho = 0.5$ compute the deadweight loss of taxation using the compensating variation for the example on slide 66 of lecture 7. (Note: in the lecture we computed the deadweight loss using the equivalent variation.)

**Question 4** Using the same numbers as in question 3, i.e., income $I = 64$, prices $p_1 = 4$, $p_2 = 1$, and a tax of 3 Dollars on good 2 raising the price to $p_2 = 3$, determine the deadweight loss using the equivalent compensation, and the deadweight loss using the compensating variation for the CES utility with $\alpha = 0.5$, $\rho = -1$. Is the deadweight loss larger or smaller than in the case where $\rho = 0.5$? Explain your answer.

**Question 5** We want to check whether the following are legitimate demand functions, in the sense that they are derived from utility maximization.

$$x_1(p_1, p_2, I) = I - 2p_1 + p_2, \quad x_2(p_1, p_2, I) = \frac{I(1 - p_1) + p_1(2p_1 - p_2)}{p_2}, \quad (1)$$

around $p_1 = p_2 = 0.5$ and $I = 10$.

First determine the substitution matrix by using the Slutzky equation. Then check whether or not all properties for the substitution matrix are satisfied.

*Note:* To determine the actual derivatives, it is ok to use the Mathematica, Wolfram alpha or a similar program.