The homework is due on Wednesday, September 30. Each question is worth 0.8 points. No partial credits.

For the graphic arguments, use the graphing paper that is attached.

**Question 1** Suppose utility is given by \( u(x_1, x_2) = x_1^2x_3 \). Prices are \( p_1 = p_2 = 2 \). For the following do not use the formula from class, but instead show each step of the derivation for this particular case.

(a) Determine the equation of the income offer curve.

(b) Suppose your income is \( I = 120 \). Determine the optimal consumption.

(c) Determine the least costly consumption bundle that gives a utility of 262,144.

**Question 2** Consider the utility function given by

\[ u(x_1, x_2) = -\frac{1}{x_1} - \frac{1}{x_2}. \]

(a) Determine the Walrasian demand functions for goods 1 and 2 by using the equation of the income offer curve and the equation of the budget line.

(b) Determine the Hicksean demand functions for goods 1 and 2 by using the equation of the income offer curve and the equation of the indifference curves.

(c) Determine the expenditure function using (b).

(d) Determine the indirect utility function using (a) and the utility function.

(e) Determine the elasticity of substitution.

(f) Determine the deadweight loss of taxation using the example from the class on Wednesday, and compare the result to those for Cobb Douglas and the CES with \( \alpha = \rho = 0.5 \) that we did in class. Why is the deadweight smaller?

**Question 3** Let \( u(x_1, x_2) = \min\{x_1, x_2\} \). Determine the Walrasian demand functions, the indirect utility function, the expenditure function, and Hicksean demand.

**Question 4** Suppose utility is given by \( u(x_1, x_2) = \min\{2x_1 + x_2, x_1 + 2x_2\} \). Prices are currently \( p_1 = p_2 = 1 \) and \( I = 40 \). The government imposes a tax of 2 Dollars on good 1, raising the price to \( p_1 = 3 \).

Determine the following graphically:

1. The consumption after the tax.

2. The amount of income the person would need to get the after-tax utility at the pre-tax prices \( p_1 = p_2 = 1 \).
Using these results you determine the government’s tax revenue, the loss to the consumers from the tax and the deadweight loss of taxation.

**Question 5** Suppose that utility is given by \( u(x_1, x_2) = \log(x_1 + x_2) + x_2 \). Prices are \( p_1 = 1, p_2 = 21 \). Determine the equation of the income offer curve for interior solutions, i.e., where \( x_1, x_2 > 0 \).

Then determine for what values of income, \( I \), consumption is on the boundary.

Graph the entire income offer curve in the grid below. Is one of the goods inferior for some levels of income?