The homework is due on Wednesday, September 14. Each questions is worth 0.8 points. No partial credits.

For the graphic arguments, use the graphing paper that is attached. *Clearly indicate the feasible set by shading it. Always graph at least three lines representing the objective, and draw an arrow in the direction in which the objective increases.*

For the computer exercises, you need to attach a printout of each Excel worksheet. In addition, you need to write down all the formulae that you use.

Question 1 In the optimization class we considered the utility function $\min\{x_1+4x_2, 40+x_2\}$ and realized that at prices $p_1 = 8$, $p_2 = 100$, and income $I = 500$ good 1 was a Giffen good. Now consider the utility function

$$u(x_1, x_2) = \left((x_1 + 4x_2)^\sigma + (40 + x_2)^\sigma\right)^{1/\sigma}.$$

where $\sigma = -10$. The utility maximization problem is given by

$$\max_{x_1,x_2} \left((x_1 + 4x_2)^{-10} + (40 + x_2)^{-10}\right)^{-0.1}$$

subject to

1. $p_1x_1 + p_2x_2 = 500$
2. $x_1 \geq 0$,
3. $x_2 \geq 0$.

Suppose that $p_2 = 100$, $I = 500$. Using the Solver from Excel, determine the solution for sufficiently many values of p_1 between 1 and 20 such that you can graph the demand function. Graph x_1 on the horizontal axis and p_1 on the vertical axis using the attached grid.

Question 2 Consider the following function:

$$f(x_1, x_2) = \left(x_1^2 + x_2^2\right)\Big(\cos(x_1\pi - \pi) + 2\Big)\Big(\cos(x_2\pi - \pi) + 2\Big).$$

We want to solve

$$\min_{x_1,x_2} f(x_1, x_2) \text{ s.t. } x_1 \geq -10;\ x_2 \geq -10\,;x_1 \leq 10;\ x_2 \leq 10$$

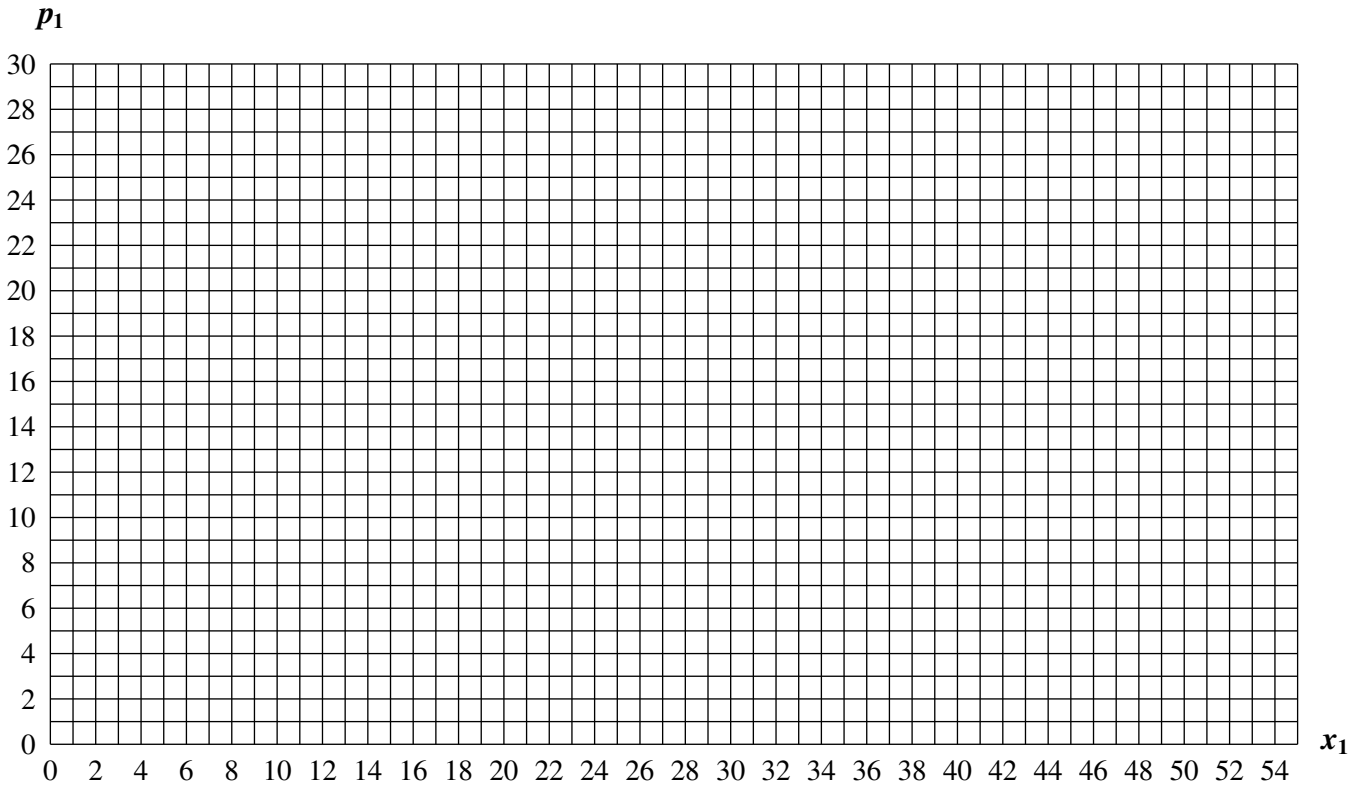
Using the function rand() in Excel, use 10 different start values (x_1, x_2) that satisfy the constraints, i.e., using `20*(rand()-0.5)` you can generate values for x_1 and x_2 that are between -10 and 10. Then use "GRG nonlinear" from Excel to find solutions. Be careful to uncheck the box "make unconstrained variables non-negative."

Do you always get the same answer? Do believe you found the global minimum? Now use the "evolutionary" solver instead. In "options" check require bounds on variables. Again, start from the same 10 values that you used for GRG nonlinear. Do you get different answers? Did you find the global minimum?

Note: To get the number π in Excel use function `PI()`.

Question 3 Suppose there are 100 consumers who have willingness to pay of \$4 for the first hamburger, \$1.50 for the second, and \$0.50 for the third. Nobody wants to eat more than three. Suppose each hamburger costs \$1 to produce, and there are fixed costs of \$200 from operating the restaurant. What pricing schedule maximizes the restaurant's profit? What is the profit?

Now suppose there are two types of consumers (which you cannot distinguish). 50 Consumers have a willingness to pay of \$4 for the first hamburger, \$1.50 for the second, and \$0.50 for the third (as in the previous case), and the remaining 50 have willingness to pay of \$4 for the first hamburger, \$2.00 for the second, and \$1.30 for the third. What pricing schedule maximizes the restaurant's profit (you are only allowed to use one pricing schedule)? What is the profit?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