The homework is due on Wednesday, September 16. Each question is worth 0.8 points. No partial credits.

For the graphic arguments, use the graphing paper that is attached. For the computer exercises, you need to attach a printout of each Excel worksheet.

**Question 1** Solve the following optimization problem graphically.

\[
\begin{align*}
\text{max } & \quad x_1 + x_2 \quad \text{subject to} \\
& \quad \begin{align*}
(i) & \quad 2x_1 + x_2 \leq 150 \\
(ii) & \quad x_1 - 2x_2 \leq 0 \\
(iii) & \quad x_1 + 2x_2 \leq 150 \\
(iv) & \quad x_1 \geq 30.
\end{align*}
\end{align*}
\]

**Question 2** Suppose an oil company has supplies of four crude products. In the refinery the crude products can be used to make two refined products \( x_1 \) and \( x_2 \), which the company can sell at prices 3 and 2, respectively. In order to produce \( x_1 \) units of the first refined product one needs 1 unit of crude product 1, 2 of product 2, 2 of product 3, and 1 of product 4. In order to produce \( x_2 \) units of the second refined product one needs 2 units of crude product 1, 1 unit of product 2, 2 of product 3, and 4 of product 4. The company has a fixed supply of the crude products. In particular supplies of the crude products are 160, 200, 220, and 280 units, respectively. The company wants to maximize the total revenue from selling the product. As a consequence, the company solves the following optimization problem.

\[
\begin{align*}
\text{max } & \quad 3x_1 + 2x_2 \quad \text{subject to} \\
& \quad \begin{align*}
(i) & \quad x_1 + 2x_2 \leq 160 \\
(ii) & \quad 2x_1 + x_2 \leq 200 \\
(iii) & \quad 2x_1 + 2x_2 \leq 220 \\
(iv) & \quad x_1 + 4x_2 \leq 280 \\
(v) & \quad x_1 \geq 0 \\
(vi) & \quad x_2 \geq 0.
\end{align*}
\end{align*}
\]

Determine the optimum graphically. Note: When you graph the lines representing (i)–(iv) then one of the lines will be strictly to the right of the feasible set, i.e., the boundaries of the feasible set are determined by only three of the lines in addition to the conditions that \( x_1, x_2 \geq 0 \).
Question 3 A firm wishes to produce 10 units of a product at the lowest possible cost. Two inputs are needed. The costs of the inputs are 4 and 6 Dollars, respectively. In order to produce 10 units of output, the inputs must fulfill $3x_1 + 4x_2 \geq 300$. Thus, the firm solves:

$$\min_{x_1, x_2} 4x_1 + 6x_2 \text{ subject to}$$

(i) $3x_1 + 4x_2 \geq 300$

(ii) $x_1 \geq 0$

(iii) $x_2 \geq 0$.

Determine the optimal choice of $x_1$ and $x_2$ graphically. Note: This is a minimization problem, i.e., moving down and to the left and your graph decreases costs.