The homework is due on Wednesday, September 9. Each questions is worth 0.8points. No partial credits.

For the computer exercises, you need to attach a printout of each Excel worksheet.

Question 1 A firm produces three outputs using 5 inputs which are in fixed supply. The prices of the outputs are 2, 1, and 3, respectively. The input amounts are 150, 165, 100, 100, and 300. Thus, the firms solves

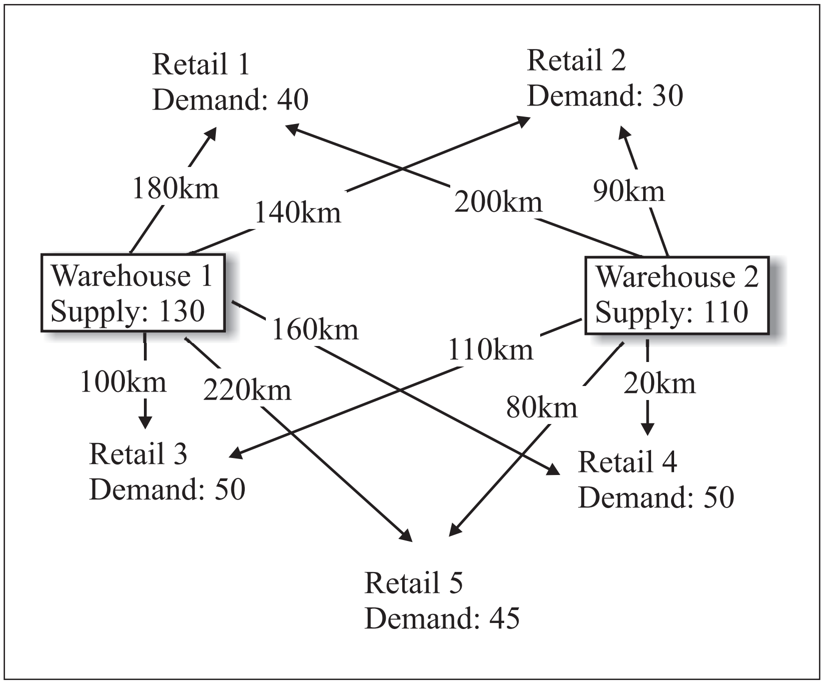
$$\begin{aligned}
&\max_{x_1,x_2} 2x_1 + x_2 + 3x_3 \text{ subject to} \\
&\text{(i) } 2x_1 + x_2 + 4x_3 \leq 150 \\
&\text{(ii) } x_1 + 2x_2 + 2x_3 \leq 165 \\
&\text{(iii) } 5x_1 + 4x_2 + x_3 \leq 100 \\
&\text{(iv) } x_1 + 3x_2 + x_3 \leq 100 \\
&\text{(v) } 5x_1 + x_2 + 3x_3 \leq 300 \\
&\text{(vi) } x_1 \geq 0 \\
&\text{(vii) } x_2 \geq 0 \\
&\text{(viii) } x_3 \geq 0
\end{aligned}$$

1. Solve the optimization problem using Excel.
2. You can also do integer programming using Excel, by using the "int" option in the constraint. On the right-hand side you can pluck in any number.
   (a) Suppose that the quantities produced must be integers. Determine the solution.
   (b) Now suppose that the smallest production unit is 10. Modify the problem such that you can solve it (*Hint:* the integer constraint must be imposed on variables x_1, x_2, and x_3. Thus, have to renormalize the variables appropriately.

Question 2 This question considers a transportation problem that is similar to that in problem 2 of our lab class. All we changed are some of the numbers and we added a fifth retail store. The transportation problem is graphed above. Again, suppose that the transportation costs are 10cents per kilometer and let $x_{i,j}$ denote the quantity of the product shipped from store i to store j. Thus, the company solves

$$\min 18x_{1,1} + 14x_{1,2} + 10x_{1,3} + 16x_{1,4} + 22x_{1,5} + 20x_{2,1} + 9x_{2,2} + 11x_{2,3} + 2x_{2,4} + 8x_{2,5}$$

subject to constraints. Determine the constraints and solve for the optimal values of $x_{i,j}$ by using Excel. Also determine which constraints bind and which are slack.



Question 3 In Question 2 we used the integer constraint to restrict variables to integer values. Another option in Excel is to constraint variables to be binary (the option "binary" in the constraint dialog box — as in the case of "int" you must put some number, e.g., 0, on the right-hand side of the constraint). We use this to solve the following traveling salesman problem.

Suppose there 6 cities, numbered from 1 to 6. The distances between the cities in km are as follows:

	City 1	**City 2**	**City 3**	**City 4**	**City 5**	**City 6**
City 1	—	50	40	32	100	60
City 2	—	—	22	59	72	57
City 3	—	—	—	29	104	11
City 4	—	—	—	—	63	25
City 5	—	—	—	—	—	33

Obviously, the distances are symmetric, e.g., the distance between cities 1 and 2 is the same as that between 2 and 1. For $i < j$, let $x_{i,j}$ be a variable that is 1 if the person travels directly between cities i and j, and 0 otherwise (i.e., if the person travels between i and j by going through some other city). Then the objective is to minimize $50x_{1,2} + 40x_{1,3} + 32x_{1,4} + 100x_{1,5} + 60x_{1,6} + 22x_{2,3} + 59x_{2,4} + \cdots + 33x_{5,6}$. The constraints are that all $x_{i,j}$ are binary, and that each city is on the trip, i.e., for each city i there must exist two other cities j that are directly connected, i.e., for which $x_{i,j} = 1$. Thus, we must add 6 additional constraints. The constraints for the first two cities are $x_{1,2} + x_{1,3} + x_{1,4} + x_{1,5} + x_{1,6} \geq 2$, $x_{12} + x_{2,3} + x_{2,4} + x_{2,5} + x_{2,6} \geq 2$.

The constraint for city 6 is $x_{1,6} + x_{2,6} + x_{3,6} + x_{4,6} + x_{5,6} \geq 2$. Obviously, you must also add the constraints for cities 4, 5, and 6.

Determine the optimal travel path. Also indicate the path in a graph, by drawing the cities and the travel route.

Question 4 A consumer has a utility function $u(x_1, x_2) = x_1^3 x_2^2$. The prices are $p_1 = 2$, $p_2 = 4$ and the person's wealth is 90. Thus, the consumer solves

$$\max_{x_1, x_2} x_1^3 x_2^2 \text{ subject to}$$
$$\text{(i) } 2x_1 + 4x_2 \leq 90$$
$$\text{(ii) } x_1 \geq 0$$
$$\text{(iii) } x_2 \geq 0.$$

(a) Determine the optimal consumption, i.e., the optimal values of x_1 and x_2 by using Excel. *Note:* This is a nonlinear optimization problem!

(b) Determine the percentage of wealth that the person spends on each good. In particular, to determine this percentage you compute

$$100 \frac{2x_1}{90}, \text{ and } 100 \frac{4x_2}{90}.$$

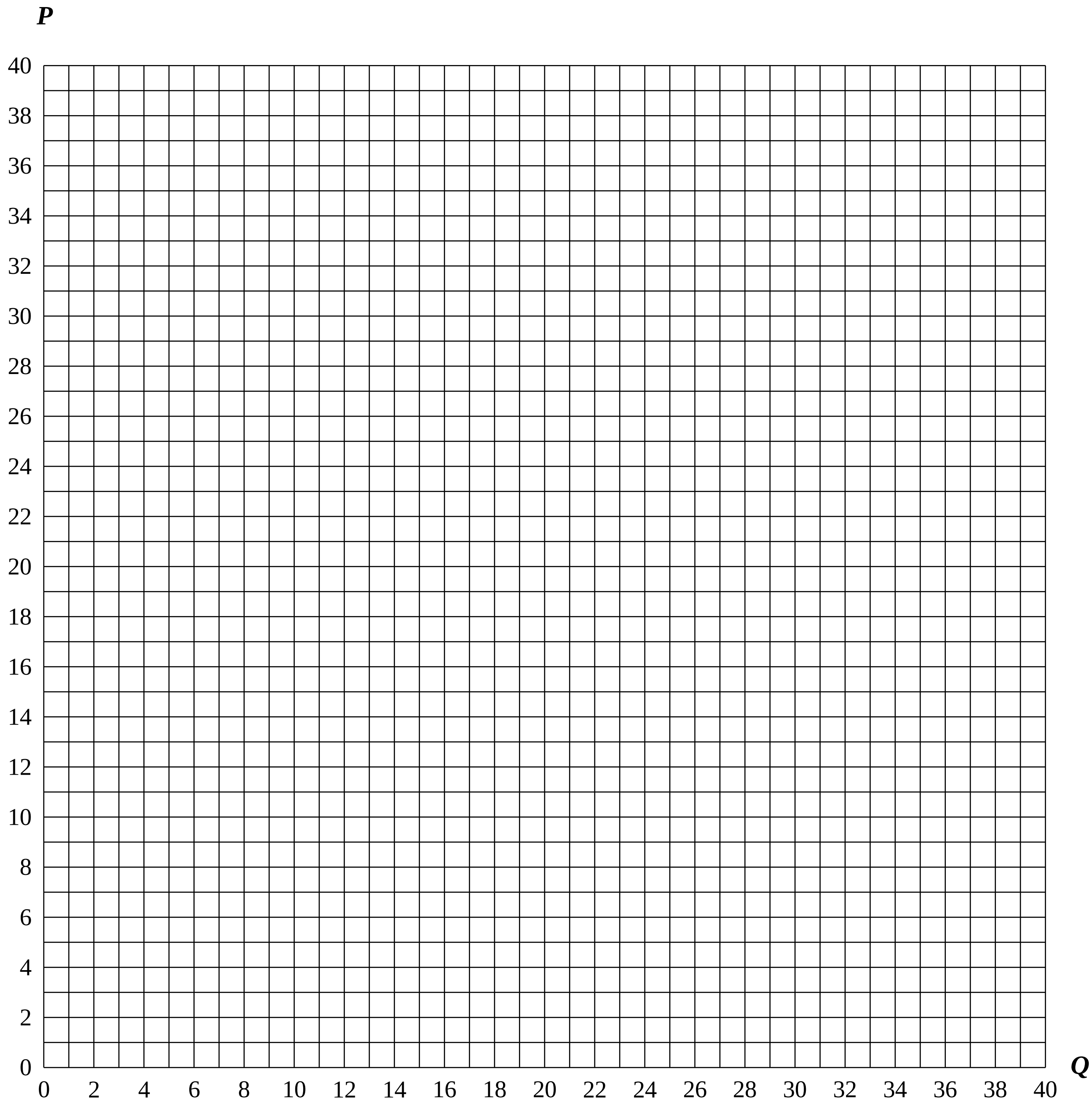
(c) Now suppose that the wealth increases to 140. This changes constraint (i) to $2x_1 + 4x_2 \leq 140$. Determine again the optimal values of x_1, and x_2 and the the percentages of income the person spends on each good. Because wealth is now 140, these percentages are given by

$$100 \frac{2x_1}{140}, \text{ and } 100 \frac{4x_2}{140}.$$

(d) Now suppose that the wealth is again 140, but that the price of good 1 increases to 4. This changes constraint (i) to $4x_1 + 4x_2 \leq 140$. Determine again the optimal values of x_1, and x_2 and the the percentages of income the person spends on each good. Now the percentages are given by

$$100 \frac{4x_1}{140}, \text{ and } 100 \frac{4x_2}{140}.$$

(e) Any guess what the percentage will be if you choose for example the budget line $8x_1 + 2x_2 = 200$?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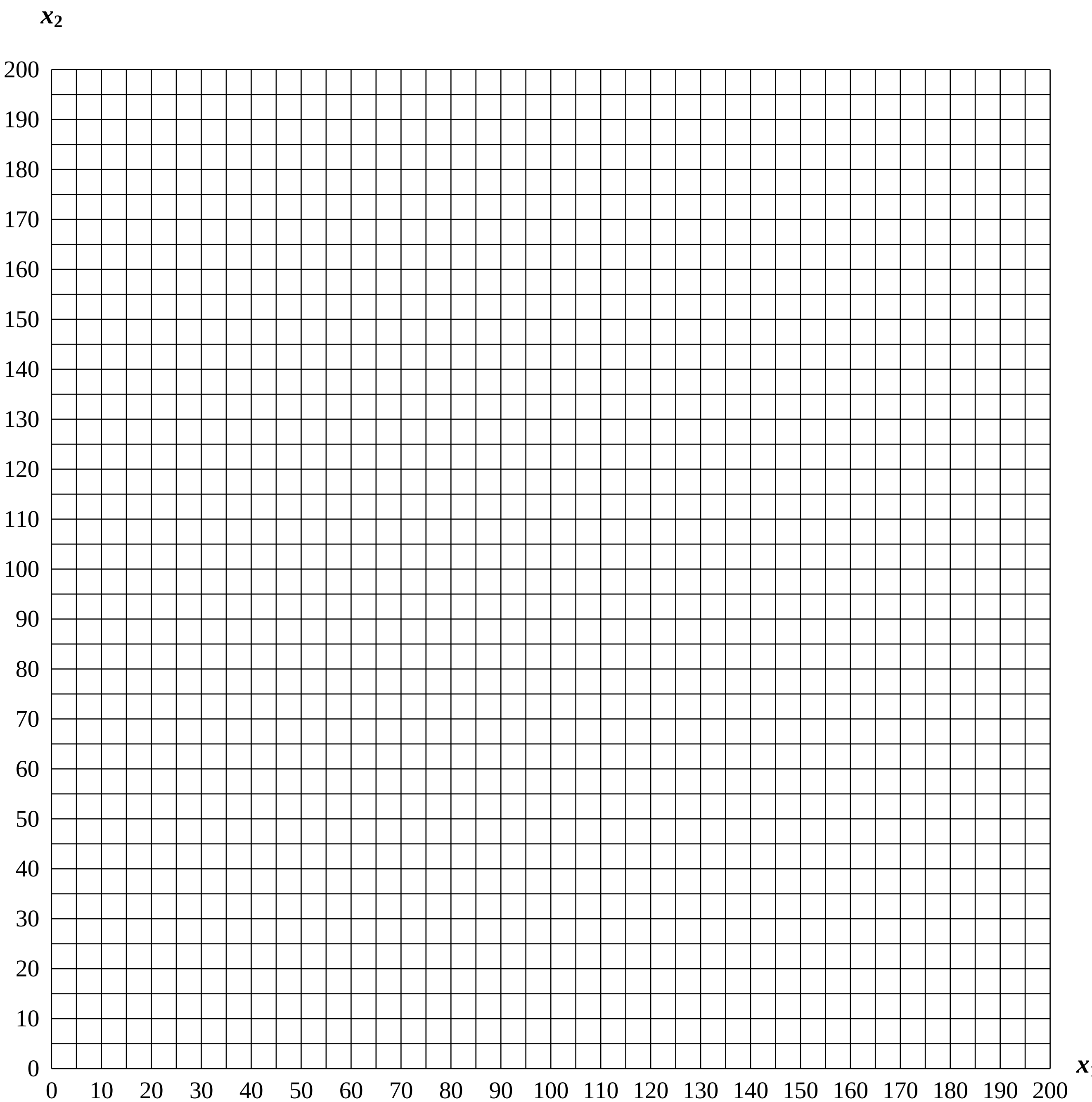

x_2
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10
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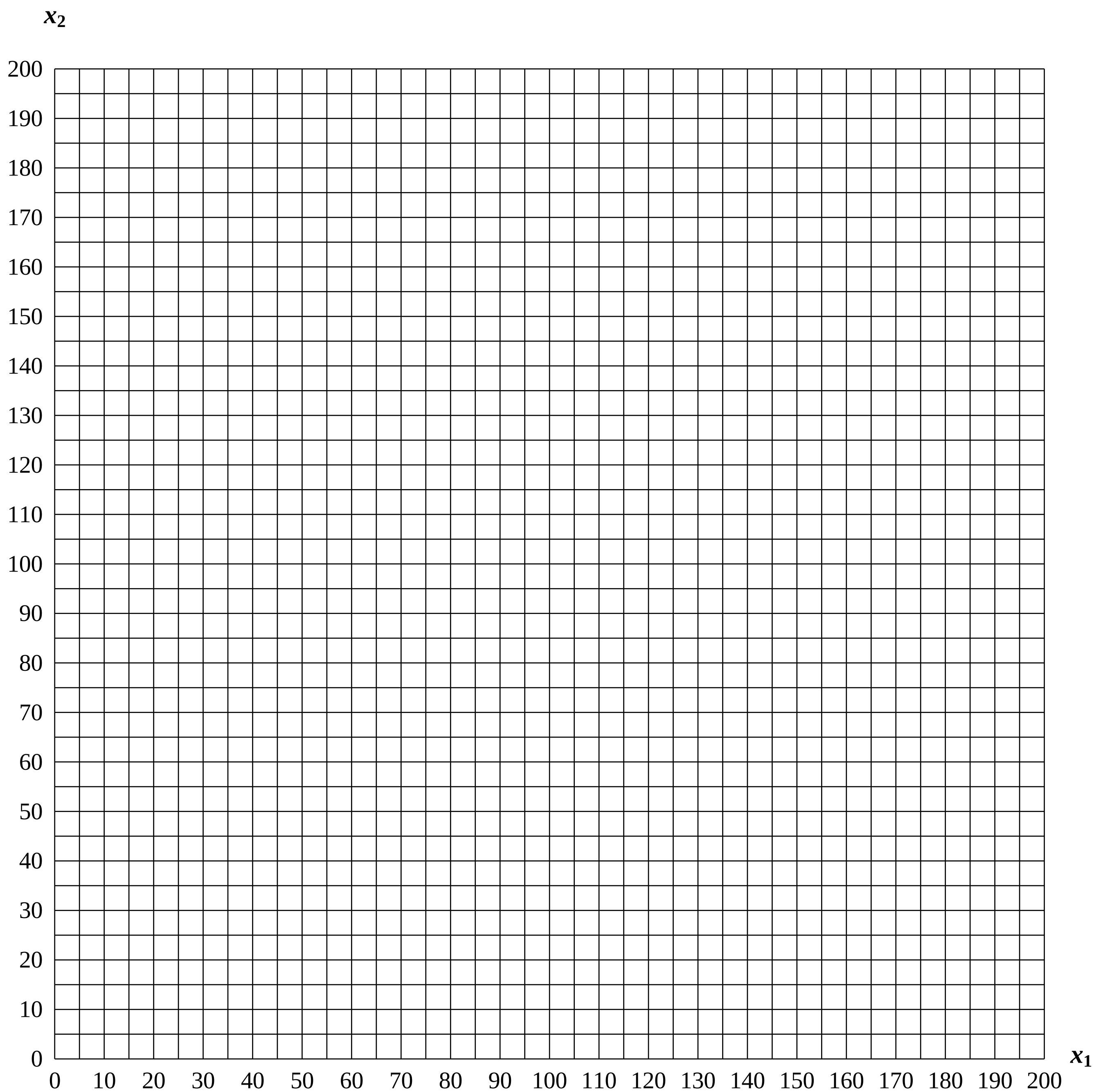
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200
x_1



x_2
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x_1