The homework is due on Wednesday, December 7. Each question is worth 0.8 points.

Question 1 Suppose there are two types of consumers, type 1 and 2, with demand functions $P(Q_1) = 20-2Q_1$ and $P(Q_2) = 20-4Q_2$. Suppose that the firm's cost function is $C(Q) = 4Q$. The firm wants to offer two pricing schedules: (F_1, Q_1), (F_2, Q_2), where F_i is the price the consumer must pay to get Q_i units of consumption. The pricing schedules are structures such that type 1 consumers choose (F_1, Q_1) and type 2 consumers (F_2, Q_2).

Recall that the optimal Q_1 and Q_2 solve $P_1(Q_1) = \text{MC}$ and $P_2(Q_2) = MC + \frac{n_1}{n_2}(P_1(Q_2) - P_2(Q_2))$, where n_1 and n_2 are the number of type 1 and type 2 consumers, respectively.

(a) Suppose that $n_1 = 2,000$ and $n_2 = 1,000$. Determine Q_1, Q_2, F_1 and F_2 and the firm's profit.

(b) Now suppose that the firm offers the efficient level of consumption also to type 2 costumers (n_1 and n_2 remains the same as in (a)). Determine Q_1, Q_2, F_1 and F_2 and the firm's profit (Note that type 1 consumers should still be better off choosing contract 1).

(c) Now suppose that $n_1 = 2,000$ and $n_2 = 9,000$. Determine Q_1, Q_2, F_1 and F_2. What happens to the distortion in Q_2 — does it increase of decrease compared to (a)? Determine which types are better off and which are worse off compared to (a).

Question 2 Suppose that potential subscribers to a video rental service have the following valuations, v_C, v_O, for CD and online rental, respectively.

Type A: $v_C = 6$, $v_O = 4$.

Type B: $v_C = 3$, $v_O = 7$.

Type C: $v_C = 2$, $v_O = 10$.

Type D: $v_C = 5$, $v_O = 5$.

Type E: $v_C = 7$, $v_O = 0$.

The potential subscriber's type is private information — the video rental firm only knows that are 100 consumers of each type.

(a) Suppose that the firm charges separately for video rental and for online rental. Determine the optimal prices for each service and the firm's revenue.

(b) Now suppose that the firm only offers a bundled video rental plus online rental service. Determine the optimal price and the firm's revenue.

(c) Can you increase revenue further by offering both bundled pricing and individual pricing?

Question 3 Suppose there are three firms producing the same product. The firm's cost functions are $C_1(Q) = 10Q$, $C_2(Q) = 18Q$ and $C_3(Q) = 12Q$. Demand for the product is given by $Q(P) = 1,000 - 10P$. Determine the equilibrium price, each firm's market share, and each firm's profit.

Question 4 Suppose there are two firms. The demand for firm 1's product is given by $Q_1(P_1, P_2) = 10 - P_1 + 0.5P_2$, and the demand for firm 2's product is $Q_2(P_1, P_2) = 10 - P_2 + 0.5P_1$. Both firms have cost functions $C(Q) = 2Q$.

(a) Suppose that each firm charges a pricer per unit that maximizes profits taken the price of the other firm as given. Determine P_1, P_2, Q_1, Q_2 and each firm's profit.

Hint: Determine the price elasticity of demand for each firm, and then use the formula that MC $= P_i(1 + 1/\epsilon_P^i)$, where ϵ_P^i is the price elasticity of demand for firm i's product and P_i is the price firm i charges. You get two equations in two unknowns, P_1, and P_2 which you can solve.

(b) Now suppose that both firms use two-part-pricing, i.e., each firm i charges a fixed fee F_i and a price per unit P_i that is equal to marginal costs (charging a price equal to marginal cost is still optimal in this setting). Determine F_i, P_i, and the firm's profit.

Question 5 There are 50 firms producing an output given a cost function $C(Q) = 0.5Q^2$. Suppose there are 100 consumers with utility functions $u(Q, m) = 10Q - Q^2 + m$, where Q is the quantity of the firm's product and m money spent on other items. Determine the equilibrium price in a competitive market, and each consumer's utility (which is a function of m).

Can the firms improve upon this outcome by forming a cartel, i.e., by agreeing upon a production quota? If firms form a cartel, what happens to consumer utility? Find an example of a production quota that would make firm's better off.