Bankruptcy and Firm Finance

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Abstract

This paper analyzes how an enforcement mechanism that resembles a court affects firm finance. The court is described by two parameters that correspond to enforcement costs and the amount of creditor/debtor protection. We provide a theoretical and quantitative characterization of the effect of these enforcement parameters on the contract loan rate, the default probability and welfare. We show that when constraints bind, which give agents an incentive to default and pursue bankruptcy, the enforcement parameters have a sharply non-linear effect on finance and welfare. The results provide guidance on when models which abstract from enforcement provide good approximations and when they do not. The bankruptcy rule corresponds to firm liquidation.

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1 Introduction

The power to enforce rights and obligations in a society is essential. For simplicity, economists have focused on two extreme forms of enforcement: perfect ex-post enforcement of contracts by an exogenous unmodeled authority (a “court”) or contracts that are “self-enforcing.” The two approaches have been widely used to study the ability and willingness of borrowers to honor outstanding debt obligations. Models that assume perfect ex-post enforcement have focused on ability to pay — a borrower fails to repay only when assets are below the promised amount. Otherwise, the borrower honors the promise. In contrast, when judicial enforcement is not possible the borrower may simply be unwilling to repay. The nature of judicial enforcement in most economies embodies characteristics of both assumptions. Would the terms of finance differ when the problems of both willingness and ability to pay arise? Further, when do the predictions of a model with a “richer” enforcement structure differ from those of a model that abstracts from enforcement?

To answer these questions, we consider the intermediate case where enforcement is possible but not all assets can be seized. We characterize the theoretical effect of judicial enforcement on firm finance, and then use numerical examples to show that the outcome can sometimes be drastically different than in existing models. We model enforcement as a technology with two parameters.

1. Enforcement cost c is the amount paid to secure rights in court. This cost may vary across countries due to different institutions (e.g., legal and accounting systems and corruption).

2. Debtor protection η is the percentage of total assets that a court cannot seize; $1 - \eta$ is creditor protection. The amount of protection is determined by factors such as the level of exemptions permitted by the bankruptcy code, inflation, the length of bankruptcy proceedings, and a debtor’s ability to “hide” assets.[1]

An entrepreneur and lender write a contract to facilitate production that accounts for their differential information about return risk. A dynamic game underlies the contract problem in which agents make sequential decisions. In the initial period agents have common beliefs about the possible returns and write the contract. In the next period only the entrepreneur observes the return realization and optimally chooses to default or repay the debt. In the final period the lender optimally chooses whether to request enforcement. If enforcement occurs, which we interpret as bankruptcy, the realization is publicly revealed.

[1] The creditor and debtor have opposite interests. All else equal, the debtor is better protected by high exemptions, inflation and delay; the creditor is better protected by the reverse. La Porta et al. [14] construct an index of creditor rights which focuses primarily on governance (control of assets). The index measures whether (i) a country imposes restrictions such as creditor consent or minimum dividends for an entrepreneur to file for reorganization; (ii) secured creditors can take possession of the security during reorganization; (iii) secured creditors are first in line when the court distributes assets; (iv) the entrepreneur controls property pending reorganization. In contrast, our model focuses on asset liquidation and creditor/debtor protection.

We use the costly enforcement model to study the effect of enforcement parameters on the terms of firm finance, both theoretically and quantitatively (cf., Krasa and Villamil [13] and Krasa, Sharma and Villamil [12]). Given specifications for model primitives (preferences, endowments and firm and enforcement technologies), we show how the bankruptcy probability, debt face value and investor expected return change when parameters that describe the legal system, c and η, are altered. We also study the quantitative implications of the model and match the following facts.

- Boyd, Levine and Smith [6] find an inverse, non-linear relationship between sustained, predictable inflation and bank lending using data for up to 100 countries from 1960-1995.[2] In our model enforcement parameter η broadens the notion of real investment return to include inflation and legal factors (e.g., exemptions, delay, etc.) Our results imply that for some parameter values finance is not sensitive to the legal structure, hence perfect ex-post enforcement may be used as a simplifying assumption for local theoretical and computational comparative exercises. For other values, after a critical threshold is reached, finance is severely compromised (e.g., we show it is possible for a country like Mexico to fall in the critical region).

- Hillegeist, Keating, Cram and Lundstedt [11] document that the average annual bankruptcy rate for U.S. firms was 1% from 1980–2000, but it varied across industries. We show how the bankruptcy probability varies with legal parameters and firm characteristics. For example, the model bankruptcy probability is not sensitive to changes in the debt-equity ratio until it reaches a critical value of 2:1, after which the default probability increases rapidly. This result explains standard lending practices, such as the U.S. Small Business Administration debt-equity guideline of 2:1 or better for loans.

Our analysis provides a positive theory with quantitative implications that can explain the relationship between legal systems and firm finance. We take the legal system as given and consider the opportunity to relieve financial distress by dissolving the firm. For example, when liquidation occurs under Chapter 7 bankruptcy in the U.S.,[3] the debtor gives up all non-exempt property owned at the time the bankruptcy petition is filed. If the court grants a discharge, the debtor is not liable for any pre-bankruptcy debts and no claims can be made against future earnings. Thus, Chapter 7 simultaneously liquidates assets for the benefit of creditors and protects the insolvent debtor. We model this debtor protection via parameter η and the enforcement cost by c.

[2] Boyd, Levine and Smith [6] use two data sets, on banks and equity markets. The banking data set is relevant for our analysis as it measures the size of the formal lending sector. They compute the average inflation rate over the sample period, and examine the cross sectional relationship between inflation and the terms of finance (e.g., the effect of inflation on private credit availability).

[3] In the U.S. there are five types of bankruptcy, Chapters 7, 9, 11, 12 and 13. Chapter 11 is designed for corporations seeking to reorganize debts while continuing to operate, Chapter 12 is the analog for family farms and Chapter 9 is for government bodies. Chapter 13 requires debtors to repay creditors under a court approved plan, and is used when a debtor is better off repaying but needs more time than creditors will allow (e.g., if a debtor misses mortgage payments and faces foreclosure due to a temporary job loss, Chapter 13 allows the debtor three years to repay). Businesses can file Chapter 7 or 11.

Chapter 7 bankruptcy has been the focus of a number of researchers recently (cf., Athreya [4], Chatterjee, Corbae, Nakajima, Rios-Rull [8] and Livshits, MacGee, Tertilt [17]). These models examine consumer bankruptcy when agents face exogenous shocks, lending is unsecured, and there is risk of default. Risk averse agents wish to smooth consumption but cannot because markets are incomplete. The contract structure (i.e., debt) and market incompleteness are taken as given. In these models Chapter 7 bankruptcy introduces contingencies into non-contingent debt contracts, and hence increases agents' ability to smooth consumption.[4] In contrast, we focus on firm finance and show how the legal system affects agents' incentives to default and pursue bankruptcy. Our model differs from these insurance models of unsecured consumer lending in three ways: (i) Risk neutral agents write a complete contract that is a constrained optimal response to frictions – incomplete information, limited commitment, and costly enforcement. Thus, the optimal contract (debt) is derived. (ii) Default and bankruptcy are separate decisions that are part of the optimal contract.[5] (iii) Lending is secured by the firm's risky investment project.

Finally, there is a sizable literature on strategic default in incomplete contract models which consider dynamic games with renegotiation. Unlike our model with a stylized description of bankruptcy liquidation (Chapter 7), they assume an exogenous legal authority that solely assigns ownership rights. Bankruptcy is interpreted as a situation where "control" is transferred from the firm to creditors. Strategic default leads to debt forgiveness rather than bankruptcy, because it is Pareto improving for both parties to renegotiate. In contrast, we model the liquidation process and show how parameters η and c affect the incentive to default and pursue bankruptcy. In our model agents may choose to enter bankruptcy even if they could pay, which is consistent with empirical observation.[6]

2 The Model

Consider an economy with a risk-neutral entrepreneur and lender, where agents derive utility only from consumption in the final period. The entrepreneur owns a technology that requires one unit of input to produce an output described by the random variable X with realization $x \in [\underline{x}, \bar{x}]$. Ex-ante the agents have a common prior $\beta(x)$ over $[\underline{x}, \bar{x}]$, where $\beta(x)$ has a probability density function $f(x)$ that is differentiable and strictly positive on $[\underline{x}, \bar{x}]$. Assume that the entrepreneur has only $0 \leq 1 - d < 1$ units of input, and must borrow d units from the lender to produce. If the firm is financed, then d is the percent of debt and

[4] Bankruptcy also weakens agents' ability to commit to repay future debt, which limits the ability to borrow. The models are well suited to quantitative analysis of a rich set of tradeoffs, e.g., changes in bankruptcy law.

[5] Default is chosen by the borrower, and occurs when it is not optimal to make a voluntary payment. When default occurs, the lender chooses (optimally) whether to invoke bankruptcy proceedings to liquidate the firm.

[6] Testimony in the U.S. House Judiciary Committee in 2002 indicated that "about 25% of Chapter 7 debtors could have repaid at least 30% of their non-housing debts over a 5-year repayment plan, after accounting for monthly expenses and housing payments" and "about 5% of Chapter 7 filers appeared capable of repaying all of their non-housing debt over a 5-year plan." Under Chapter 7, debt is extinguished and never repaid.

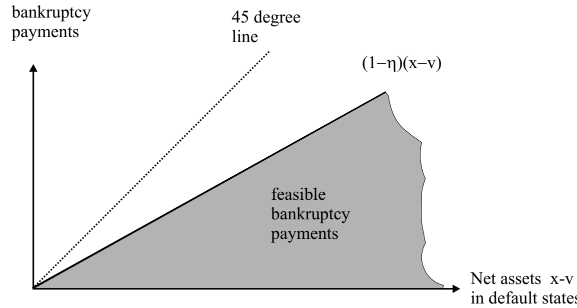

Figure 1: Feasible Bankruptcy Payments

$1 - d$ is firm equity. The timing of events is as follows:

t=0 Agents specify an *enforceable loan contract* $\ell(x, v)$, which is a payment schedule with state x determined by a court at $t = 2$, and payment $v \geq 0$ made by the entrepreneur at $t = 1$. If agents cannot agree, no loan is made.

t=1 The entrepreneur, but not the lender, privately observes project realization x and chooses a payment $v \geq 0$. Payment v is not enforceable by the court (though enforceable payment $\ell(\cdot)$ depends on v), but cannot be retracted once made. Because v is not enforceable, we refer to it as a *voluntary payment.*

t=2 The lender chooses whether to request costly enforcement by the court. If no enforcement is requested, the lender's payoff is v and the entrepreneur's payoff is $x - v$. If enforcement is requested, the lender pays cost c, the court determines the true state x, and payment $\ell(x, v)$ is transferred to the lender. The lender's payoff is $v + \ell(x, v) - c$ and the entrepreneur's payoff is $x - [v + \ell(x, v)]$.

We focus on two parameters to describe enforcement. First, c is a deadweight loss to the contracting parties. Ceteris paribus this cost is higher if accounting standards are poor, which implies a higher cost to determine entrepreneur assets, or if corruption exists, such as bribes paid to government officials or the court.[7] Second, η determines the amount of creditor versus debtor protection, measuring the percent of total entrepreneur assets the court cannot seize. η includes exemptions specified in the bankruptcy code, inflation, and the length of bankruptcy proceedings. The higher these factors are, the higher η, which means that creditor protection is weak (equivalently, debtor protection is strong). The maximum enforceable payment is given by $(1 - \eta)(x - v)$.[8]

[7] See Bond [5] for an analysis of how bribes affect the judicial agency problem.

[8] For example, consider an entrepreneur with equity in a principal residence. If bankruptcy occurs, in seven U.S. states all home equity is exempt while in other states the maximum equity that can be sheltered in a principal homestead is \$15,000 or less. See Lenhert and Maki [16]. η is relevant whether a firm is organized as a sole proprietorship (exemptions apply) or is incorporated (inflation, delay and asset diversion apply).

Figure 1 illustrates the effect of the legal system on contract payments. Suppose that the entrepreneur repays nothing (i.e., $v = 0$) and the lender requests enforcement. The shaded, cone-shaped area is the set of all feasible bankruptcy payments. The court cannot seize η percent of entrepreneur assets. Thus the maximum possible payment to the lender is $(1 - \eta)x$. By an appropriate choice of ℓ, any payment in the cone can be obtained.

Definition 1 *Payment schedule $\ell(x, v)$ is legally enforceable if, for all x, v with $x \geq v$, $0 \leq \ell(x, v) \leq (1 - \eta)(x - v)$.*

The investment problem is a dynamic game with imperfect information because beliefs are allowed to vary endogenously as information changes during the game. We focus on pure strategy equilibria that are Pareto efficient in the set of all perfect Bayesian Nash equilibria (PBNE) of the game (see Krasa and Villamil [13] and Krasa, Sharma and Villamil [12] for conditions under which pure strategies are optimal even when mixed strategies are admissible). In Pareto problem 1, a planner maximizes the lender's expected payoff (1), given entrepreneur utility constraint (2), by choosing:

- $\mathscr{v}(x)$: an entrepreneur strategy to select voluntary payment v.
- $\ell(x, v)$: a legally enforceable payment function.
- $\mathscr{e}(v)$: a lender enforcement strategy, where if $\mathscr{e}(v) = 1$ the lender requests enforcement of $\ell(x, v)$ and if $\mathscr{e}(v) = 0$ the lender does not request enforcement.
- $\beta(x|v)$: the lender's updated belief about the return at $t = 2$.

(1) and (2) are equivalent to maximizing a weighted sum of the two agents' utilities. Varying reservation utility $\bar{u}_E$ gives the entire Pareto frontier, where $\bar{u}_E = (1 + r_E)(1 - \mathscr{d})$ is the entrepreneur's utility if endowment $1 - \mathscr{d}$ is invested in an alternative investment with return $1 + r_E$. Constraints (3)–(5) require the solution to be a PBNE: (3) ensures optimality of $\mathscr{v}$, (4) ensures optimality of $\mathscr{e}$, and (5) requires belief $\beta(x|v)$ to be consistent. (6) requires payment $\ell(x, v)$ to be enforceable (feasible), see Definition 1.

Problem 1 *At $t = 0$, choose $\{\mathscr{v}(x), \ell(x, v), \mathscr{e}(v), \beta(x|v)\}$ to maximize*

$$E_0[u_L(x)] = \int \Big[\mathscr{v}(x) + \mathscr{e}\big(\mathscr{v}(x)\big)\Big(\ell\big(x, \mathscr{v}(x)\big) - c\Big)\Big] \, d\beta(x) \tag{1}$$

subject to

$$E_0[u_E(x)] = \int \Big[x - v(x) - e\big(v(x)\big)\ell\big(x, v(x)\big) \Big]\, d\beta(x) \geq \bar{u}_E \tag{2}$$

$$v(x) \in \arg\max_{v \geq 0} \Big[x - v - e\big(v(x)\big)\ell\big(x, v(x)\big) \Big] \tag{3}$$

$$e(v) = 1 \textit{ if and only if} \int \big[\ell(x, v) - c\big]\, d\beta(x|v) \geq 0 \tag{4}$$

$$\beta(x|v) \textit{ is derived from } \beta(x) \textit{ using Bayes' rule whenever possible} \tag{5}$$

$$\ell(x, v) \textit{ is enforceable} \tag{6}$$

At first glance, it may seem unusual to specify beliefs as part of the contract problem. However, this natural extension of the well established Pareto approach allows for dynamic information revelation. In the contract literature it is standard to assume ex-ante (before information is revealed) that a "planner" coordinates agents on actions and a contract to attain an efficient allocation, subject to constraints. We also consider a planner who coordinates agents to achieve efficient outcomes, but the lender's off-equilibrium path beliefs $\beta(x|v)$ matter in our dynamic game because different beliefs give rise to different equilibrium payoffs. Thus, the planner must coordinate agents on payment function $\ell(x, v)$, strategies $v(x)$ and $e(v)$ where payment v can reveal information, and beliefs that could arise if the entrepreneur were to deviate from the equilibrium strategy.[9]

3 The Equilibrium Contract

Let $\bar{v}$ denote the face value of the contract (principal and interest). To characterize the solutions of problem 1 we use Lemma 1 and Theorem SDC which are stated formally and proved in the Appendix. The Lemma implies that we can restrict attention to payments that are either 0 or $\bar{v}$ on the equilibrium path, i.e., only no payment or full payment occur in equilibrium. Default occurs if and only if $v = 0$ and payment $\bar{v}$ corresponds to no default. Theorem SDC establishes that a simple debt contract solves problem 1. The key characteristic of simple debt is that when enforcement occurs the firm is liquidated and the legally enforceable amount, $(1 - \eta)(x - v)$, of assets are transferred to the creditor up to the amount owed, $\bar{v}$. Thus, enforcement corresponds to bankruptcy. We assume that the bankruptcy rule followed by the court is liquidation, transfer of the legally enforceable amount, and full discharge of all remaining debt (e.g., Chapter 7 of the U.S. Bankruptcy Code). Let x^* be the lowest non-bankruptcy state.

[9] Many different off equilibrium path beliefs $\beta(x|v)$ support efficient outcomes. We admit any belief that supports an allocation on the Pareto frontier (where payoffs are maximized). In the quantitative analysis in section 5.4 we derive an empirical bound on those off-equilibrium path beliefs that support efficient allocations. Our approach differs from the refinements literature in game theory that may provide equilibria where the lender gets a lower payoff.

Definition 2 *$\{\ell(x, v), \mathcal{v}(x)\}$ is a simple debt contract if there exists $\bar{v}$ and $x^* \in [\underline{x}, \bar{x}]$ with $x^* \geq \bar{v}$ such that*

$$\ell(x, v) = \begin{cases} \min\{(1-\eta)x, \bar{v}\} & \text{if } x < x^*,\ v = 0; \\ 0 & \text{if } v \geq \bar{v}; \\ (1-\eta)(x-v) & \text{otherwise;} \end{cases} \qquad \mathcal{v}(x) = \begin{cases} \bar{v} & \text{if } x \geq x^*; \\ 0 & \text{if } x < x^*; \end{cases}$$

We first note that the classic costly state verification (CSV) model is contained in problem 1 if we choose the enforcement parameter $\eta = 0$ and eliminate the dynamic structure (i.e., remove the PBNE constraints (3)–(5)). In order to understand the effect of legal parameter η, which determines the amount of assets that can be seized in bankruptcy, consider the following example. Suppose a debtor owes $\bar{v} = \$100,000$, has home equity of $\$50,000$, private property of $\$80,000$, and retirement savings of $\$100,000$. The total value of the debtor's assets, $x = \$230,000$, therefore exceeds $\bar{v}$. If the debtor files for bankruptcy in Texas, under state law all equity in a homestead and pension/retirement accounts are exempt, as is personal property up to $60,000. Chapter 7 specifies that exempt assets cannot be used to satisfy creditor claims. The court can seize only $(1-\eta)x = \$20,000$. This amount is transferred to creditors (net of c), and the case is discharged. The debtor is "protected" from paying the remaining $\$80,000$. Given a particular bankruptcy code, it may therefore be optimal for a debtor to default even if assets x exceed debt $\bar{v}$. We refer to such a default as "willful," which is represented by region B in figure 2, a region that does not occur in the CSV model. In contrast, in region A, which occurs in the CSV model, debtor assets are less than the amount owed, $x < \bar{v}$ and the entrepreneur is unable to pay. We also show that η generates important quantitative differences relative to the CSV model. Note that the existence of region B and the quantitative effects of η do not require a dynamic game.

Our dynamic game together with sequential rationality ensure that the investor is willing to enforce when the entrepreneur defaults. In contrast, in the CSV model the sole concern is to minimize expected bankruptcy costs; there is no need to provide an incentive to enforce and default occurs if and only if the entrepreneur is unable to pay (Gale and Hellwig [10], Townsend [18] or Williamson [19], [20]). To understand the role of sequential rationality in our model, suppose that $c = \$20,500$ in the above example. Because the creditor expects to be able to recover only $20,000, it is not rational for the creditor to enforce, even if she had threatened to do so ex ante. The creditor's unwillingness to enforce induces the entrepreneur to default more frequently. This generates region C in figure 2.

The second panel of figure 2 illustrates why simple debt is optimal. Consider a simple debt contract with face value $\bar{v}_D$ and an arbitrary debt contract with face value $\bar{v}_A$ (in the proof, the "arbitrary" contract need not be debt). The lender's expected payment under contract $\bar{v}_A$ is area $b+c+d+e$. We next find a simple debt contract with face value $\bar{v}_D$ such that the lender's expected payment is the same as under the original contract, i.e., $a+b+e = b+c+d+e$. This implies that $a+b > b+c$, where $b+c$ is the bankruptcy area under the alternative contract and $a+b$ is the bankruptcy area under the simple debt contract. If bankruptcy

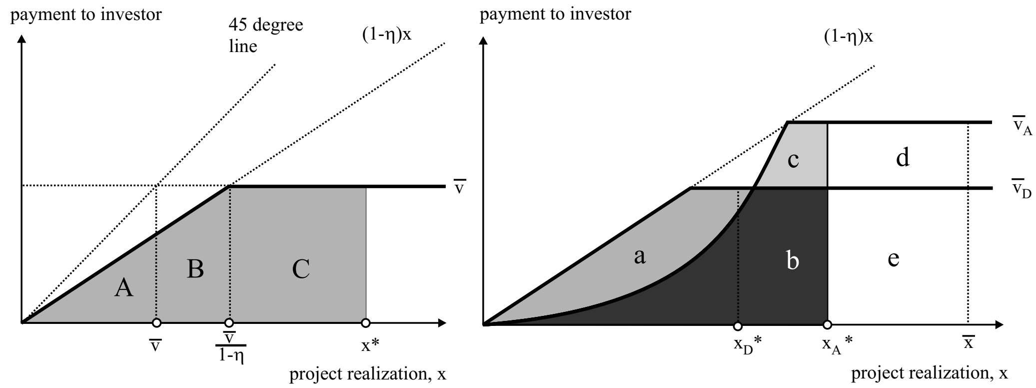

Figure 2: Simple Debt Contracts with Enforcement

occurs for all states $x < x_A^*$ in both contracts, then the lender's expected bankruptcy payment is strictly higher under simple debt contract $\bar{v}_D$. This implies that constraint (4) is slack. The size of the bankruptcy set for the simple debt contract can then be reduced to x_D^*, thereby decreasing expected enforcement costs, which increases the lender's expected payoff.

Because simple debt contracts are completely described by default cutoff x^* and face value $\bar{v}$, problem 1 can be simplified as follows:

Problem 2 *At $t = 0$, choose $\bar{v}$ and x^* to maximize*

$$E_0[u_L(x)] = \int_{\underline{x}}^{\frac{\bar{v}}{1-\eta}} (1-\eta)x \, d\beta(x) + \int_{\frac{\bar{v}}{1-\eta}}^{\bar{x}} \bar{v} \, d\beta(x) - \int_{\underline{x}}^{x^*} c \, d\beta(x) \tag{7}$$

subject to

$$E_0[u_E(x)] = \int_{\underline{x}}^{\frac{\bar{v}}{1-\eta}} \eta x \, d\beta(x) + \int_{\frac{\bar{v}}{1-\eta}}^{\bar{x}} (x - \bar{v}) \, d\beta(x) \geq \bar{u}_E \tag{8}$$

$$\frac{\bar{v}}{1-\eta} \leq x^* \tag{9}$$

$$\int_{\underline{x}}^{\frac{\bar{v}}{1-\eta}} (1-\eta)x \, d\beta(x|x < x^*) + \int_{\frac{\bar{v}}{1-\eta}}^{x^*} \bar{v} \, d\beta(x|x < x^*) - c \geq 0 \tag{10}$$

There exist $x_v^e \leq \bar{x}$ such that $(1-\eta)(x_v^e - v) - c \geq 0$, for all $0 < v < \bar{v}$. (11)

Objective (7) and constraint (8) correspond to (1) and (2) of problem 1. Constraint (9) follows from (3) and specifies that default must occur at least in all states x with $x < \frac{\bar{v}}{1-\eta}$, which implies $\frac{\bar{v}}{1-\eta} \leq x^*$ (see figure 2). Constraint (4) implies (10) and (11), where (10) considers the case where payment occurs on the equilibrium path and (11) considers off-equilibrium path payments v. In (11) x_v^e is the project realization the investor expects if a partial payment v was received, i.e., $x_v^e = \int x \, d\beta(x|v)$. Under a simple debt contract,

(4) implies $(1-\eta)(x_v^e - v) - c \geq 0$ for all $0 < v < \bar{v}$, i.e., the investor will enforce unless full repayment occurred. Finally, (5) and (6) of problem 1 are satisfied by construction. Existence of a solution follows from standard compactness and continuity arguments.

4 Enforcement and Entrepreneur Finance

We have constructed a model of enforcement where the legal system is described by parameters, η and c, and the lender has an incentive to request enforcement (because of constraints (10) and (11)). We now analyze how the enforcement parameters affect the solution to the contract problem. Theorems 1 and 2 provide complete characterizations of the effect of c and η on the default probability and the loan rate. The face value (principal plus interest) is related to the loan rate by $\bar{v} = d(1+r)$. The default probability is $\beta[\underline{x}, x^*]$.

Theorem 1 analyzes the effect of c on finance. The size of c measures the efficiency of bankruptcy procedures. Assume that $\beta(x)$ has a density function $f(x)$ that is differentiable.

Theorem 1

1. *Assume that c is increased. Then the lender's expected payoff is decreased. The decrease is strict if the bankruptcy probability is strictly positive.*

2. *When c changes, the effect on the loan rate and the bankruptcy probability is characterized by four distinct parameter regions.*

   Region 1 *If (8) binds, but (10) and (11) do not bind, which occurs for small c, the bankruptcy probability and the loan rate do not depend on c.*

   Region 2 *If (8), (10) and (11) do not bind, which may occur for intermediate values of c, the bankruptcy probability and the loan rate are decreasing in c.*

   Region 3 *If (10) binds but (11) does not bind, which occurs for larger values of c, the bankruptcy probability and the loan rate increase. If (8) holds with equality, the loan rate is constant.*

   Region 4 *If c is sufficiently large, the bankruptcy probability is zero. The loan rate is constant, unless (11) binds, in which case it decreases.*

Figure 3 illustrates Theorem 1.[10] In region 1, the entrepreneur's participation constraint binds. Therefore, the face value does not change with c, which means that the bankruptcy probability is constant. In

[10] The parameter values in the figures are $d = 0.5$, $\bar{u}_E = 0.503$, $f(x)$ is a normal distribution with mean $\mu = 1.1$ and standard deviation $\sigma = 0.2$, and $\eta = 0.4$ in figure 3 and $c = 0.1$ in figure 4. These parameters were chosen solely to illustrate the four regions of the Theorems. We discuss empirical parameters in section 5.

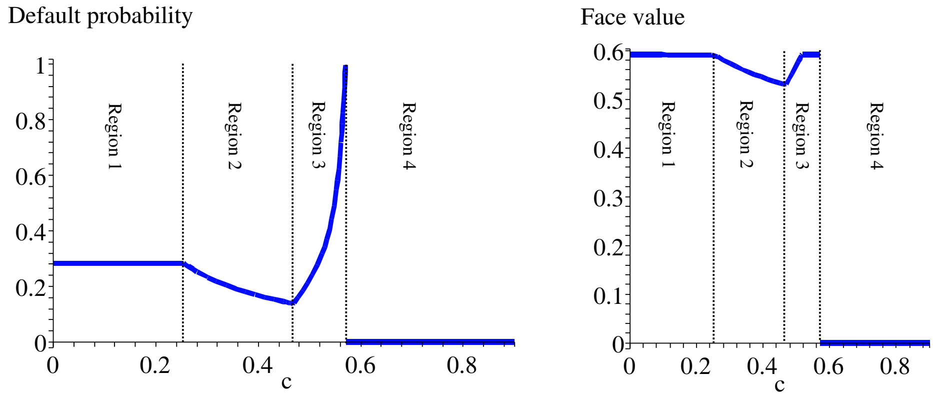

Figure 3: The Four Regions of Theorem 1

region 2, c is sufficiently high that it becomes optimal to reduce face value $\bar{v}$. Reducing $\bar{v}$ lowers the bankruptcy probability and saves expected bankruptcy costs. For the lender, this saving compensates for the lower face value. In region 3, (10) binds. This means that x^* must be increased to give the lender an incentive to enforce (recall region C in figure 2). The resulting rapid increase in the default probability also generates a steep decline in the investor's return, which can be seen in figure 6. Once c is sufficiently large it is not optimal to provide finance, or to invest solely in projects fully collateralized by $\underline{x} > 0$. The inability of entrepreneurs to obtain finance is a significant problem in many emerging markets. Our result indicates that high enforcement costs can easily be a source of credit market failure. In practice, cost c includes payments to accountants, lawyers, and the court to establish the size of the entrepreneur's assets, x, payments to liquidate assets, and bribes to expedite the case or influence the outcome. The government can play an important role in determining the size of c by requiring a high level of disclosure and routine accounting practices, and by policies to deter corruption.

One may think it is possible to weaken constraint (10) and reduce the steep increase in the default probability by increasing the loan, thereby raising $\bar{v}$. However, formalizing this intuition requires rather strong assumptions. For example, one would need to penalize the entrepreneur for prepaying part of the "excess loan" from the additional assets borrowed. In the U.S. firms typically have access to a line of credit but cannot be forced to draw more credit than they wish. Even if firms could be forced to take on "excess credit," as long as there are no prepayment penalties they would simply repay just enough from their excess funds, to undermine the incentive effect of the excess loan. The formal proof of this intuition is in Proposition 1 in the Appendix.

Parameter η determines the percent of total assets that the court cannot seize, for example due to exemptions in the legal code or because inflation lowers the real value of creditor claims. Theorem 2 investigates the impact of η on the optimal contract.

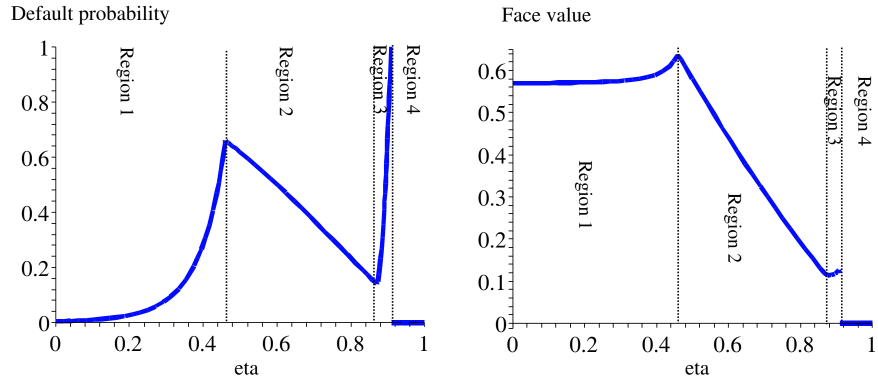

Figure 4: The Four Regions of Theorem 2

Theorem 2

1. *Assume that η is increased. Then the lender's expected payoff decreases. The decrease is strict if $c > 0$ and if bankruptcy occurs with positive probability.*

2. *When η changes, the effect on the loan rate and the bankruptcy probability is characterized by four distinct parameter regions.*

Region 1 *If (8) binds but (10) and (11) do not, which occurs if η and c are not too large, the loan rate and bankruptcy probability are increasing in η.*

Region 2 *If (8), (10) and (11) do not bind, which occurs for intermediate values of η, the loan rate and bankruptcy probability are decreasing in η.*

Region 3 *If (10) binds, which occurs for larger values of η, the bankruptcy probability is increasing in η. The loan rate is increasing in η if (8) also binds.*

Region 4 *If η is sufficiently close to 1, the bankruptcy probability is 0. The loan rate is constant unless (11) binds, in which case it decreases.*

Figure 4 illustrates Theorem 2 for the baseline parameters. In region 1 as η increases, the entrepreneur retains more assets in bankruptcy. In order to make up for this, the lender raises the face value. The increase in the face value is small until η is close to region 2, but the increase in the bankruptcy probability is more rapid because the bankruptcy cutoff $x^* = \frac{\bar{v}}{1-\eta}$ is increasing in both η and $\bar{v}$.

In region 2 an increase in η, ceteris paribus, would further increase the bankruptcy probability. However, at the end of region 1 it is inefficient to increase the bankruptcy probability further because expected bankruptcy costs are large. In order to keep the bankruptcy probability at least constant, the face value must be decreased.[11] However, as η gets larger it becomes optimal to actually decrease the bankruptcy probability. In region 2, $x^* = \frac{\bar{v}}{1-\eta}$ (cf., figure 2). At the optimum the marginal loss to the lender of lowering the face value by $\Delta\bar{v}$ must equal the marginal gain of a decreased bankruptcy probability. If $\bar{v}$ is decreased by $\Delta\bar{v}$, then x^* decreases by $\frac{\Delta\bar{v}}{1-\eta}$, which is the lender's gain from less bankruptcy. This benefit increases as η increases. Therefore, a larger η results in a lower x^* and hence a lower bankruptcy probability. This decrease of x^* accelerates the drop in the face value because to keep the bankruptcy probability constant, we must lower $\bar{v}$. Hence to lower the bankruptcy probability, $\bar{v}$ must decline at an even faster rate. This also leads to a rapid drop of the investor's return as figure 6 will show.

Region 3 occurs when η is relatively large and (10) binds. In figure 2 this means that x^* is increased. The bankruptcy probability quickly increases to a level where it is no longer optimal to provide finance, which leads to region 4.

5 Quantitative Analysis

We now evaluate how the model performs on two important dimensions observed in the data, when key parameters are varied. We show that:

1. The model produces the negative and highly non-linear relationship between real investment returns and financial activity documented by Boyd, Levine and Smith [6].

2. The default probability is in the 1% range observed in the U.S. data by Hillegeist, Keating, Cram and Lundstedt [11]. We also examine how the default probability varies in response to the model parameters.

Section 7.2 in the Appendix describes the computation algorithm.

The baseline parameters are summarized in the table below.

Preferences	r_E	d	$f(x)$	μ	σ	c	η
risk neutral	0.07	0.5	normal and t	1.1	0.25	0.1	0.1

We choose the parameters as follows. First, both agents are risk neutral, thus utility is linear.[12] Second, the entrepreneur's reservation utility is given by $\bar{u}_E = (1-d)(1+r_E)$, where r_E is the entrepreneur's opportunity cost of funds, i.e., the minimum return the entrepreneur requires to invest in his/her own project. We assume $r_E = 0.07$ and $d = 0.5$. We choose $r_E = 0.07$ because the compound annual real return on a diversified portfolio of common stock in the U.S. is 6.9% over the period 1802-2001. We choose $d = 0.5$ as a baseline

[11] Recall from region 1 that increasing η, keeping $\bar{v}$ constant, increases the bankruptcy probability.

[12] We focus on firm finance and abstract from the insurance aspect of bankruptcy that has been a focus of consumer bankruptcy.

because this places the firm within the bounds to get a loan from the Small Business Administration—their lending limit is a debt-equity ratio of 2:1. We choose $\mu = 1.1$ for the mean, and $\sigma = 0.25$ for the standard deviation of the return distribution. The return of 10 percent is slightly higher than the real return on the S&P500, as is the standard deviation (which is 18 percent for the S&P500). We take these slightly higher values to account for the fact that we have individual investments rather than an index.

For project return distribution $f(x)$ we consider the normal distribution as a benchmark and the following t distribution density:

$$f_{\mu,\sigma,n}(x) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\sigma\sqrt{(n-2)\pi}\,\Gamma\left(\frac{n}{2}\right)}\left(1 + \frac{(x-\mu)^2}{\sigma^2(n-2)}\right)^{-\frac{n+1}{2}},$$

where μ is the mean, σ the standard deviation, and n controls the excess kurtosis of the distribution.[13] We choose $n = \infty$ as one benchmark, because this corresponds to the normal distribution, and $n = 2.5$ as the other benchmark, to generate a t distribution with large excess kurtosis.

Excess kurtosis is defined as $\gamma_2 = \frac{\mu_4}{\sigma^4} - 3$, where μ_4 is the normalized fourth moment, i.e., $\mu_4 = \int (x-\mu)^4 f_{\mu,\sigma,n}(x)\,dx$.[14] Intuitively, positive excess kurtosis means that returns are farther from the mean than in the normal distribution, which has zero excess kurtosis. Figure 5 compares the two distributions. The t distribution is more peaked and has fatter tails relative to the normal distribution. For example, the probability that a return greater than 100% will occur is about 24 times larger for the t than the normal distribution (i.e., $P(x > 2)$ is 0.377% for the t distribution and 0.016% for the normal distribution.) The two distributions have the same mean, and in our experiments they also have the same variance (i.e., the greater peakedness exactly offsets the fatter tails).

Empirical magnitudes for the legal parameters are taken from a study of U.S. Chapter 7 business bankruptcies by Lawless and Ferris [15]. They document that about 40% of the book value of assets reported by debtors in bankruptcy filings are not distributed to creditors.[15] The direct costs of bankruptcy (mostly attorney and other professional fees) are about 6% of total reported assets. The remaining loss of value may be due to inefficient liquidation (e.g., immediate asset liquidation at "fire sale" prices), which is reflected in c, or a decline in asset value (e.g., due to depreciation), which is reflected in η. We set $c = 0.1$ to be consistent with standard estimates (see Boyd and Smith [7]). We set $\eta = 0.1$, and compute the cutoff for the bankruptcy set x^*. We then compute the expected asset value given bankruptcy, which is 0.45 for the baseline parameters. Thus, c is about 22% of these bankruptcy assets. This measure of c and $\eta = .1$ account

[13] $\mu = 0$ and $\sigma = \sqrt{n/(n-2)}$ give the standard (student) t-distribution with n degrees of freedom.

[14] The excess kurtosis of the t distribution is given by $\gamma_2 = \frac{3(n-2)}{n-4} - 3$, if $n \geq 4$ and $\gamma_2 = \infty$ otherwise.

[15] Lawless and Ferris [15] Table 2 reports median total debtor assets of $107,602 and in Table 3 total distributions to creditors of $65,615. Thus, only about 60% of claimed assets are available for distribution. They measure distributions broadly, including transfers to secured creditors through abandonment and relief from automatic stay. On p. 7 they note, "In most cases, nothing was distributed to unsecured creditors." Bankrupt firms had almost 6 times as much debt as liquidated assets.

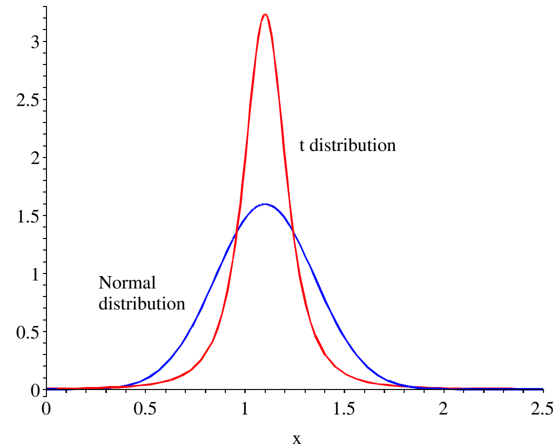

Figure 5: The Normal and t Distributions

for 32% of the assets that are not transferred to the debtor. The remaining 8% difference with the Lawless and Ferris loss of 40% is easily covered by the difference between book and economic values for firms in distress.

5.1 Financial Crises: Non-linear Firm Finance

Figure 6 shows how the lender's expected payoff varies with the enforcement parameters, c and η. The most striking result in both panels are the existence of a region over which changes in c and η have little effect on investor return, and a sharp transitional region. The intuition for the sharp decline in investor return is discussed after Theorems 1 and 2. Our quantitative result is consistent with Boyd, Levine and Smith [6], who find empirical evidence of an inflation threshold; when inflation exceeds 15% there is a discrete drop in financial sector activity in their data set of about 100 countries. To understand this result in our model, consider a country like Mexico before bankruptcy reform in 1996 where contracts could not be indexed for inflation. Further note that the value of η implied by an inflation rate of π over n years is given by $1 - \eta = (1 - \pi)^n$. In Mexico the average duration of a bankruptcy case was 6 years. The average inflation rate of 16% for the last 10 years reported by the bank of Mexico compounded over 6 years would lower the value of creditor claims significantly, yielding an η of 0.65. This indicates that attaining an η in or above the critical range is a legitimate concern in many economies.

The first panel of figure 6 is a quantitative comparison our model and the CSV model. For sufficiently small parameter values the predictions of the two models coincide, but once c reaches a critical threshold they differ dramatically. The second panel shows the transition as η changes. Although η is not present in the CSV model, the models deliver very similar predictions for small parameter values and again differ dramatically at higher values. Antinolfi and Huybens [2] and [3] show that capital markets can also "crash"

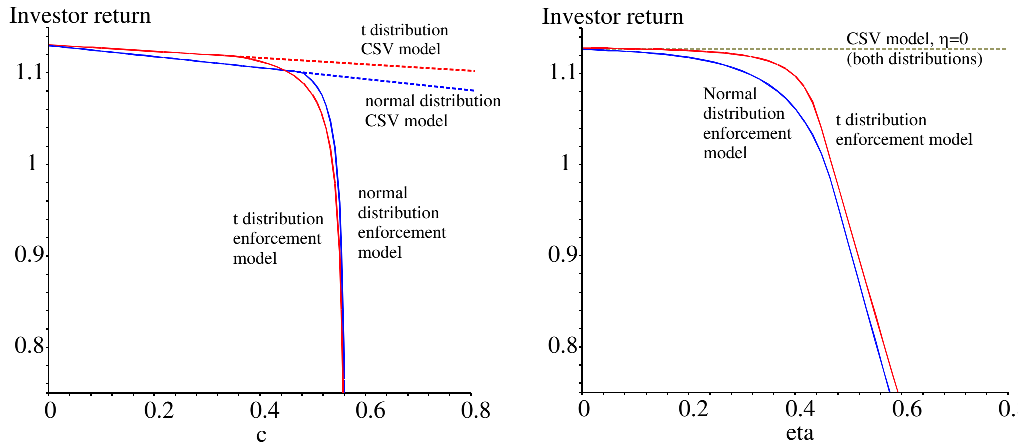

Figure 6: The Effect of c and η on the Investor's Expected Return

or oscillate depending on parameter values in an overlapping generations model with capital accumulation and a CSV friction. However, the reason is different. In their model multiple steady states occur due to the interaction between the real exchange rate and the CSV friction. In contrast, Theorems 1 and 2 and figure 6 show that countries in the critical range may experience rapid and severe "financial crises" due to a small change in fundamentals, c or η, such as bribery or accounting scandals or severe inflation. Our model predicts that this phenomenon would not be observed in countries with low parameter values for c and η (e.g., the U.S.). In contrast, countries with very high parameter values (e.g., sub-Saharan Africa) would have low expected returns, and therefore would receive little private investment unless c and/or η were lowered substantially.

5.2 Default

Hillegeist, Keating, Cram and Lundstedt [11] use Moody's Default Risk Services' Corporate Default database and SDC Platinum's Corporate Restructuring database to construct the percent of bankrupt firms in the U.S. from 1980–2000 by year and by industry (using the Fama and French [9] classification by SEC code). Their study indicates that the average annual bankruptcy rate for these firms during the sample period is about 1% (i.e., 0.97 from Table 1). The first column in figure 7 shows that the default probability in the model is close to the value observed in the data when $c < 0.3$ and $\eta < 0.25$ for the t distribution, and that the default probability is quite sensitive to η once it exceeds this critical value. The figure indicates a low value of η is consistent with the observed U.S. default probability.

The fourth panel of figure 7 and the second panel of figure 6 suggest two different critical values of η: the bankruptcy rate increases rapidly at $\eta = 0.25$ and the investor return drops rapidly at $\eta = 0.40$. Is a high default probability per se sufficient to deter investors, or does only investor return matter? To answer

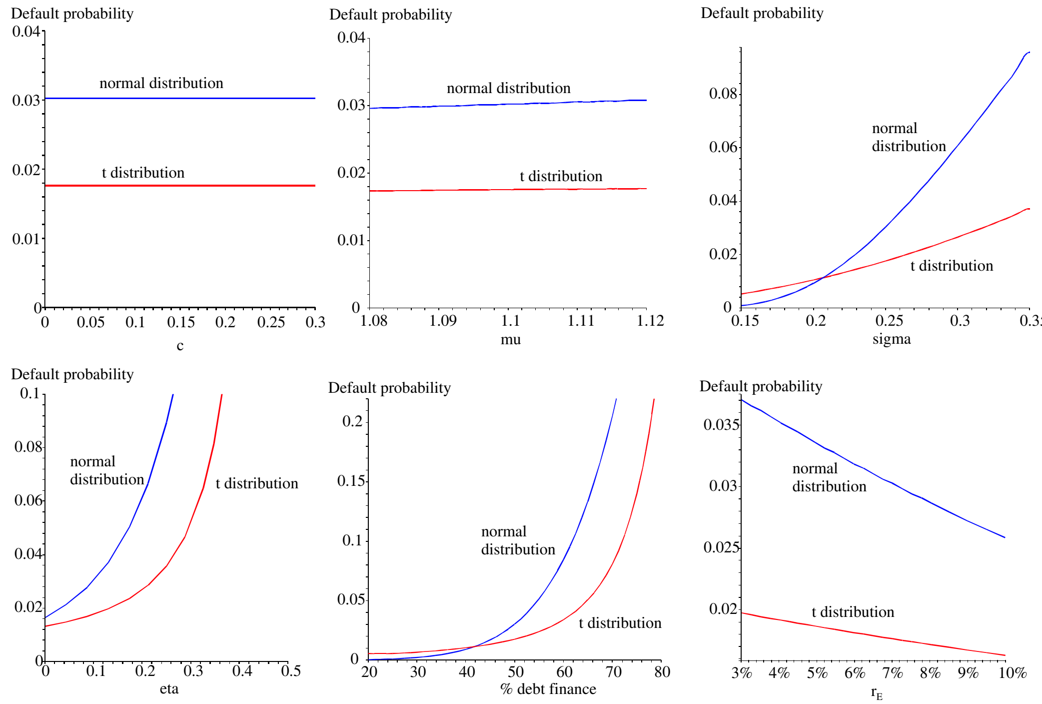

Figure 7: Sensitivity of the default probability with respect to model parameters

this question, consider the two major sources of debt finance for firms: banks and private investors. Banks are often subject to regulations that prevent excessive risk taking. For example, in the U.S. banks must increase capital to offset more risky loans, which often makes such risky loans unattractive. A project with a high default probability will therefore be unlikely to attract bank finance. If a firm only has access to bank finance (and the default probability matters for regulatory reasons), then finance can be compromised at the lower value of η. In contrast, if the firm is able to issue bonds to private investors directly, then only expected investor return is likely to matter and higher default rates may be observed. For example, Altman and Bana [1] report that in 1990, 1991, and 2002, default rates on bonds exceeded 10%, and in the last quarter of 2002 the default rate reached 15%. Thus, the prediction of the model that default rates of 10% to 15% can sometimes be observed is consistent with observation.

The remaining panels show the effect on the default probability of variations in the mean about $\mu = 1.1$, the standard deviation about $\sigma = 0.25$, the entrepreneur's opportunity cost of funds about $r_E = 7\%$ and the percent of debt finance about $d = 50\%$. Figure 7 shows that for the low enforcement parameter values that characterize the U.S., the default probability is 1.8% for the t distribution, with relatively low sensitivity except when the d exceeds $2/3$, which is the U.S. Small Business Administration debt-equity guideline of 2:1 for firm loans. At this limit, the default probability is 5.9%, and it is interesting to note that the default probability begins to rise rapidly precisely in the range where the SBA lending limit is reached.

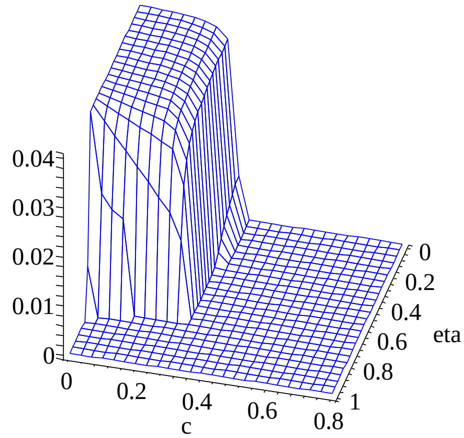

Figure 8: The Joint Effect of c and η on Net Surplus

The default figures also show that the default probability can be matched successfully with the t distribution, but not with the normal distribution. First, the normal distribution generates a default probability of 3% for the baseline parameters, which is somewhat too high. Second, and more importantly, the normal distribution is very sensitive to the debt-equity ratio even when this ratio is 1:1 (i.e., 50% debt finance). The default probability is also sensitive to small increases in the project's standard deviation σ for the normal distribution, but not for the t distribution. These results indicate that in order to match key characteristics about firm default one must use a distribution with high excess kurtosis. Less technically, the firm's return distribution must have more weight in the "tails, " i.e., there must be a significantly higher chance of large losses and great successes than a normal distribution allows.

5.3 Welfare

We now evaluate how net-surplus changes when the legal parameters η and c change. Because agents are risk neutral net-surplus is equivalent to consumer welfare, which is investor payoff plus entrepreneur payoff less the total opportunity cost of funds $1 + r_m$ if the project is undertaken.[16] If the project is not financed, net-surplus is zero. In order to fund the project, each agent's payoff must cover at least the opportunity cost of funds, i.e., $(1 + r_m)(1 - d)$ for the entrepreneur and $(1 + r_m)d$ for the investor. Figure 8 shows how the legal parameters jointly affect net-surplus. Again, striking non-linearities are evident. For small η and c net-surplus is not sensitive to small parameter changes. However, there is a rapid transitional region. When the parameters are sufficiently large it is no longer optimal to fund the project.

Figure 8 shows that for small values of η and c, production yields a 4% net-surplus. If, instead, η and

[16] We assume that $r_E = r_I = r_m$ is the return on an outside investment option available to both agents.

c are large this surplus disappears because investors will not fund production, but rather will invest in the outside alternative. As we have already noted, the moderate inflation rate of 16% experienced by Mexico in the 1990s together with 6 year delay in resolving bankruptcies, easily generates a value of η that is above the critical threshold. Thus, moderate rates of inflation together with an inefficient legal system can generate a welfare loss of about 4% per unit of investment. If investment is roughly one fifth of GDP (as is the case in Mexico) then the welfare loss is about 0.8% of GDP. In contrast, if the legal system is well developed and delay does not occur, η and c are small and the same inflation rate will not generate this type of welfare loss. These simple computations show that there can be non-trivial gains from either lowering moderate levels of inflation or legal reform. As a consequence, legal institutions are important for understanding the impact of macroeconomics policies on welfare.

5.4 Beliefs

As we discussed at the outset, a new feature of our analysis is to specify beliefs as part of the contract problem. We argued that this natural extension of the Pareto approach, where the planner coordinates agents to achieve an efficient outcome, allows for dynamic information revelation. The planner coordinates agents on payment function $\ell(x, v)$, strategies $\mathscr{v}(x)$ and $\mathscr{e}(v)$ where payment v can reveal information, and beliefs which would arise if the entrepreneur were to deviate from the equilibrium strategy. These off-equilibrium path lender beliefs $\beta(x|v)$ matter because different beliefs give rise to different equilibrium payoffs. We admit any belief that supports an allocation on the Pareto frontier, where payoffs are maximized. A useful feature of our approach is that we can derive an empirical bound on these beliefs for the baseline parameters in order to understand the implications of the belief constraint.

Recall from problem 2 that the project realization the investor expects if a partial payment v was received is $x_v^e = \int x \, d\beta(x|v)$. This value is important because it determines whether belief constraint (11) binds, and hence could affect the outcome. For off equilibrium path partial payments $0 < v < \bar{v}$, recall that (11) is $(1-\eta)(x_v^e - v) - c \geq 0$. Substituting $\eta = 0.1$ and $c = 0.1$ yields $x_v^e - v \geq 0.11$. This is a very weak requirement on beliefs. In particular, given payment v, the investor will enforce as long as she believes that remaining firm assets are at least 11% of ex-ante assets. In general, model sensitivity to off equilibrium path beliefs is problematic because the predictions then depend on a parameter (beliefs) that is inherently unobservable. Our computational results show that for all but very extreme beliefs the model is not sensitive to the specification of these beliefs.

6 Concluding Remarks

Our model of judicial enforcement sheds light on why firms may experience difficulty raising finance, especially when institutions are poor. Parameters η and c describe institutional features of contract enforcement, with η measuring the percent of assets a firm retains in bankruptcy and c measuring the deadweight cost of enforcement. The model indicates that through its effect on η even moderate levels of inflation can have significant, non-linear effects on production and welfare. Further the losses are consistent with the non-linearity and threshold effects observed in the data. In contrast, when η and c are low they affect borrowing and lending only minimally. For such cases, models that abstract from enforcement may provide good approximations.

Our analysis also indicates that a low default rate may not indicate that economic conditions are favorable. In particular, when the enforcement system is weak, firms may be unable to obtain outside finance because investor returns are too low. Firms must primarily self-finance, making start up more difficult and production and welfare lower. In the baseline model this welfare loss can be 4% per unit invested, even for moderate levels of inflation, when the judicial systems permits delay and creditors' claims are not indexed for inflation (as was the case in Mexico). In contrast, if the legal system protects the value of creditors' claims, then inflation may not have an impact. Our results therefore show that an otherwise benign level of inflation can interact with a poorly structured legal system to generate significant welfare costs.

7 Appendix

7.1 Proofs

Lemma 1 *Without loss of payoff, we can restrict attention to strategies $\mathscr{v}(x)$ which assume at most two values, $\bar{v}$ and 0, a payment function $\ell(x, v)$ and enforcement strategy $\mathscr{e}(v)$ with the following properties:*

1. *If $v \geq \bar{v}$ then $\ell(x, v) = \mathscr{e}(v) = 0$;*

2. *If $0 < v < \bar{v}$ then $\ell(x, v) = (1 - \eta)(x - v)$.*

3. *If $0 \leq v < \bar{v}$ then $\mathscr{e}(v) = 1$.*

Proof of Lemma 1. Consider any solution to problem 1. Let $X_N = \{x | \mathscr{e}\big(\mathscr{v}(x)\big) = 0\}$ and $X_D = \{x | \mathscr{e}\big(\mathscr{v}(x)\big) = 1\}$. Then X_N and X_D partition the set of all possible realizations $[\underline{x}, \bar{x}]$ into a set of non-bankruptcy and a set of bankruptcy states. Note that $\mathscr{v}(x)$ is constant on X_N. Assume by contradiction that there exist x, x' such that $v = \mathscr{v}(x) < \mathscr{v}(x') = v'$. Then the entrepreneur's payoff could be increased in state x' by switching from payment v' to payment v, a contradiction to (3). Let $\bar{v}$ be the entrepreneur's payment on X_N.

Consider the following alternative contract.

$$\mathscr{v}_A(x) = \begin{cases} \bar{v} & \text{if } x \in X_N \\ 0 & \text{if } x \in X_D. \end{cases} \qquad \mathscr{e}_A(v) = \begin{cases} 0 & \text{if } v \geq \bar{v} \\ 1 & \text{if } v < \bar{v}. \end{cases}$$

$$\ell_A(x, v) = \begin{cases} 0 & \text{if } v \geq \bar{v} \\ (1 - \eta)(x - v) & \text{if } 0 < v < \bar{v} \\ \mathscr{v}(x) + \ell(x, \mathscr{v}(x)) & \text{if } v = 0, \end{cases}$$

and the new belief $\beta_A(x|0)$ given by $d\beta_A(x|0) = \int d\beta(x|\mathscr{v}(y))\, d\beta(y|\mathscr{v}(y) < \bar{v})$. It follows immediately that the payoffs to both parties under the alternative contract are the same as under the original contract. In particular, if $x \in X_N$ then payment $\bar{v}$ occurs under both contracts. If $x \in X_D$ then under the original contract payment $\mathscr{v}(x)$ was made and the court enforced payment $\ell(x, \mathscr{v}(x))$. The total payment was $\mathscr{v}(x) + \ell(x, \mathscr{v}(x))$, which is the same if the debtor were to pay 0 in all states in X_D under the alternative contract. Hence, it is optimal for the debtor to choose $\mathscr{v}(x) = 0$ for all $x \in X_D$.

It remains to show that the constraints are all satisfied. (2) holds because the payments are the same under both contracts. (3) is automatically satisfied for $v = \{0, \bar{v}\}$. Next, it is not optimal for the entrepreneur to choose a payment $v > \bar{v}$. Now assume that the entrepreneur chooses $0 < v < \bar{v}$. Let $\beta(x|v)$ be the belief in the original equilibrium. Recall from the previous paragraph that under the proposed new solution

$v(x) = 0$ or $\bar{v}$ for all $x \in X_D \cup X_N$. Because a v with $0 < v < \bar{v}$ is never paid in our new solution, the PBNE allows the creditor to have any belief if v were to be paid. Let such beliefs be the same as those under the original solution, i.e., $\beta(x|v)$. Then $\ell_A(x, v) \geq \ell(x, v)$ implies

$$\int \left[\ell_A(x, v) - c\right] d\beta(x|v) \geq \int \left[\ell(x, v) - c\right] d\beta(x|v) \geq 0,$$

The last inequality follows from (4). Therefore, it is optimal for the lender to enforce.

Finally, when the entrepreneur selects $v = 0$, then

$$\int \left[\ell_A(x, 0) - c\right] d\beta_A(x|0) \geq \int \left[\ell(x, v(x)) - c\right] d\beta_A(x|0)$$
$$= \int \int \left[\ell(x, v) - c\right] d\beta(x|v(y))\, d\beta(y|v(y) < \bar{v}) \geq 0.$$

The last inequality follows from (4) and the fact that $v(x) = 0$ for all $x \in X_D$ in our candidate solution. Hence (4) is satisfied for the alternative contract. No enforcement occurs if the entrepreneur pays $\bar{v}$. Therefore payment $\ell_A(x, \bar{v})$ can be assumed to be 0. ■

Theorem SDC. *Simple debt contracts solve problem 1.*

Proof of Theorem SDC. Assume by way of contradiction that an arbitrary contract, which is not simple debt, $\{v(x), \ell(x, v), e\}$, solves problem 1. Because of lemma 1 we can assume that $v(x)$ is either 0 or $\bar{v}$. Choose x_D^* such that

$$\beta\big([\underline{x}, x_D^*]\big) = \beta\big(\{x|v(x) = 0\}\big). \tag{12}$$

Constraint (6) implies $\ell(x, 0) \leq (1 - \eta)x$. Constraint (3) implies that $\ell(x, 0) \leq \bar{v}$ for all x with $v(x) = 0$. Therefore, $\ell(x, 0) \leq \min\{(1 - \eta)x, \bar{v}\}$. Thus,

$$\int_{\{x|v(x)=0\}} \ell(x, 0)\, d\beta(x) + \int_{\{x|v(x)=\bar{v}\}} \bar{v}\, d\beta(x) \leq \int_{\underline{x}}^{\bar{x}} \min\{(1 - \eta)x, \bar{v}\}\, d\beta(x).$$

Therefore there exist $\bar{v}^* \leq \bar{v}$ such that

$$\int_{\{x|v(x)=0\}} \ell(x, 0)\, d\beta(x) + \int_{\{x|v(x)=\bar{v}\}} \bar{v}\, d\beta(x) = \int_{\underline{x}}^{\bar{x}} \min\{(1 - \eta)x, \bar{v}^*\}\, d\beta(x). \tag{13}$$

Let

$$\ell^*(x, 0) = \min\{(1 - \eta)x, \bar{v}^*\}, \quad \text{and} \quad v'(x) = \begin{cases} 0 & \text{if } x < x_D^*; \\ \bar{v}^* & \text{if } x \geq x_D^*. \end{cases}$$

Let $e^*(v) = 1$ if and only if $v < \bar{v}^*$. By construction the lender's total payment and hence the entrepreneur's expected utility under $\{\ell^*, v', e^*\}$ is same as under $\{\ell, v, e\}$. Moreover, because the bankruptcy probability does not change, the lender's payoff is unchanged. We show that (4) is slack, if $\bar{v}^* < \bar{v}$.

Note that $\bar{v}^* < \bar{v}$ and (12) imply $\int_{x_D^*}^{\bar{x}} \bar{v}^* \, d\beta(x) < \int_{\{x|v(x)=\bar{v}\}} \bar{v} \, d\beta(x)$. Therefore, (13) implies $\int_{\underline{x}}^{x_D^*} \ell^*(x,0) \, d\beta(x) > \int_{\{x|v(x)=0\}} \ell(x,0) \, d\beta(x)$. (12) gives $\int_{\underline{x}}^{x_D^*} \big[\ell^*(x,0) - c\big] \, d\beta(x) > \int_{\{x|v(x)=0\}} \big[\ell(x,0) - c\big] \, d\beta(x)$. Thus,

$$\begin{aligned}
\int \big[\ell^*(x,0) - c\big] \, d\beta(x|v'(x) = 0) &= \frac{1}{\beta([\underline{x}, x_D^*])} \int_{\underline{x}}^{x_D^*} \big[\ell^*(x,0) - c\big] \, d\beta(x) \\
&> \frac{1}{\beta([\underline{x}, x_D^*])} \int_{\{x|v(x)=0\}} \big[\ell(x,0) - c\big] \, d\beta(x) \\
&= \frac{1}{\beta(\{x|v(x) = 0\})} \int_{\{x|v(x)=0\}} \big[\ell(x,0) - c\big] \, d\beta(x) \\
&= \int \big[\ell(x,0) - c\big] \, d\beta(x|v(x) = 0) \geq 0,
\end{aligned}$$

which implies that (4) is slack.

Note that $(1-\eta)x_D^* > \bar{v}^*$. Otherwise, if $(1-\eta)x_D^* \leq \bar{v}^*$ then $x_D^* < \bar{v}/(1-\eta)$. This would imply $v(x) = 0$ for all x with $0 < x < \bar{v}/(1-\eta)$, which contradicts (12). Now decrease x_D^* marginally to x^* and define

$$v^*(x) = \begin{cases} \bar{v}^* & \text{if } x \geq x^*; \\ 0 & \text{if } x < x^*. \end{cases}$$

The lender's expected payment is unchanged, therefore the entrepreneur's payoff is unaffected. Because $\beta([\underline{x}, x^*]) < \beta(\{x|v(x) = 0\})$, there are less bankruptcies under $\{\ell^*, v^*, e^*\}$, thereby strictly increasing the lender's payoff. Next, (3) holds by definition. (4) is satisfied because it was shown to be slack for contract $\{\ell^*, v', e^*(v)\}$ and because x^* is only marginally smaller than x_D^*. (5) holds because $\bar{v}^* < \bar{v}$. (6) holds by construction. As a consequence, $\{\ell^*, v^*, e^*\}$, fulfills all constraints of problem 1 and increases the investor's payoff. This contradicts the proposed optimality of $\{\ell, v, e\}$. ■

Proof of Theorem 1.

Statement 1. Assume by contradiction that the lender's payoff increases if costs are increased from c to c'. Let $\bar{v}'$, $x^{*\prime}$ be the solution to problem 2 when costs are c'. Then $\bar{v}'$, $x^{*\prime}$ fulfills all constraints of problem 2 when costs are $c < c'$. However, the expected bankruptcy costs decrease. Therefore, $\bar{v}'$, $x^{*\prime}$ dominates the contract that is optimal when costs are c, a contradiction.

Statement 2, Region 1. (8) determines face value $\bar{v}$. If (10) does not bind, then $x^*(1-\eta) = \bar{v}$. Therefore, the face value and bankruptcy probability do not change in this region.

Statement 2, Region 2. If (8), (10) and (11) are slack, the first and second order conditions are given again by (18) and (19). Taking the derivative with respect to c yields

$$\frac{dx^*(c)}{dc} = -\frac{f(x^*(c))}{(1-\eta) f(x^*(c)) + cf'(x^*(c))}. \tag{14}$$

Also note that $x^*(c)(1-\eta) = \bar{v}(c)$. Therefore (19) implies that the bankruptcy set and the face value decrease.

Statement 2, Region 3. If (8) binds, $\bar{v}$ is independent of c and the face value is constant. As c is increased, (10) becomes tighter. As a consequence, x^* must be increased, thereby increasing the bankruptcy probability. Now assume that (8) is slack. Then (9) must bind. Otherwise, we could increase the lender's payoff by increasing $\bar{v}$. Therefore, (10) implies $\frac{1}{\beta(x<x^*)}\int_{\underline{x}}^{x^*}\left(1-\eta\right)x\,d\beta(x) = c$. Thus if c is increased, x^* must be increased, thereby increasing the bankruptcy probability. Since $x^*(c)(1-\eta) = \bar{v}(c)$, the face value and loan rate increase as well.

Statement 2, Region 4. If c is sufficiently large, then (10) is only satisfied if bankruptcy never occurs. The face value remains constant as long as (11) does not bind. If (11) binds for some $0 < \hat{v} < \bar{v}$, then $x^e_v = \bar{x}$. Further, in order for (11) to bind $(1-\eta)(\bar{x}-\bar{v}) - c = 0$. Increasing c therefore decreases $\bar{v}$ and the loan rate. ■

Proof of Theorem 2.

Statement 1. It follows immediately that the lender's payoff is non-increasing in η as the constraint set becomes smaller when η is increased. We now show that the decrease is strict if bankruptcy occurs with positive probability. First, assume that constraints (10) and (11) are slack. Then (9) binds. As a consequence, increasing η increases x^* and the expected bankruptcy costs, which strictly decreases the lender's expected payoff. Next, assume that (10) binds. Then increasing η again increases x^*, making the lender strictly worse off. Finally, if (11) binds, then the lender is worse off because the face value is lowered.

Statement 2, Region 1. Constraint (8) holds with equality, i.e.,

$$\int_{\underline{x}}^{\frac{\bar{v}(\eta)}{1-\eta}} \eta x\,d\beta(x) + \int_{\frac{\bar{v}(\eta)}{1-\eta}}^{\bar{x}} (x-\bar{v}(\eta))\,d\beta(x) = u_E \tag{15}$$

Taking the derivative of (15) with respect to η and solving for $\frac{d\bar{v}(\eta)}{d\eta}$ yields

$$\frac{d\bar{v}(\eta)}{d\eta} = \frac{\int_{\underline{x}}^{\frac{\bar{v}}{1-\eta}} x\,d\beta(x)}{\beta\left(\left[\frac{\bar{v}}{1-\eta},\bar{x}\right]\right)} > 0. \tag{16}$$

Recall that the loan rate is given implicitly by $\bar{v} = (1+r)d$. Therefore, (16) implies that the face value and the loan rate are strictly increasing in η.

If (10) and (11) are slack then constraint (9) must bind. Taking the derivative of (9) with respect to η and solving for $\frac{d\bar{v}(\eta)}{d\eta}$ yields

$$\frac{d\bar{v}(\eta)}{d\eta} = \frac{dx^*(\eta)}{d\eta}(1-\eta) - x^*(\eta), \tag{17}$$

This and (16) imply that $\frac{dx^*(\eta)}{d\eta} > 0$, i.e., the lowest bankruptcy state and therefore the bankruptcy probability are increasing in η.

Statement 2, Region 2. If (8), (10) and (11) do not bind, then (9) binds. The first order condition is

$$(1-\eta)\int_{x^*}^{\bar{x}} f(x)\,dx - cf(x^*) = 0. \tag{18}$$

The second order condition is

$$-(1-\eta)\,f(x^*) - cf'(x^*) \leq 0. \tag{19}$$

Taking the derivative in (18) with respect to η, and solving for $\frac{dx^*(\eta)}{d\eta}$ yields

$$\frac{dx^*(\eta)}{d\eta} = -\frac{\beta(x \geq x^*(\eta))}{(1-\eta)\,f(x^*(\eta)) + cf'(x^*(\eta))}. \tag{20}$$

Therefore, (19) implies that the bankruptcy probability is decreasing. Finally, (17) implies that $\frac{d\bar{v}(\eta)}{d\eta} < 0$ if $\frac{dx^*(\eta)}{d\eta} \leq 0$, i.e., the implied loan rate is decreasing.

Statement 2, Region 3. Assume that (10) binds. First, assume that $\bar{v} < (1-\eta)x^*$, i.e., (9) is slack. It follows immediately that (8) binds. Assume by contradiction that (8) is slack. Now raise $\bar{v}$. We can lower x^* because the lender's expected payment in bankruptcy states is increased. Therefore, the bankruptcy probability is decreased. This and the increase in $\bar{v}$ makes the lender strictly better off, a contradiction.

Because (8) binds, we get (15). Therefore, (16) implies that the face value $\bar{v}(\eta)$ is increasing in η. Since (9) is assumed to be a strict inequality, we get $\frac{\bar{v}(\eta')}{1-\eta'} < x^*(\eta)$, for some η' which is marginally larger than η. Let ℓ and ℓ' be the optimal contracts given η and η' respectively. Because (8) binds and $\bar{v}(\eta') > \bar{v}(\eta)$, we get $\int_{\underline{x}}^{x^*(\eta)} \ell(x,0)\,d\beta(x) > \int_{\underline{x}}^{x^*(\eta)} \ell'(x,0)\,d\beta(x)$. In order for (10) to be satisfied, $x^*(\eta') > x^*(\eta)$, i.e., the bankruptcy probability increases.

Now assume that (9) holds with equality, i.e., $\bar{v}(\eta) = (1-\eta)x^*(\eta)$. Then (10) implies

$$\int_{\underline{x}}^{x^*(\eta)} \big(1-\eta)x\,d\beta(x) = c\beta(x < x^*(\eta)). \tag{21}$$

Taking the derivative with respect to η and solving for $\frac{dx^*(\eta)}{d\eta}$ yields

$$\frac{dx^*(\eta)}{d\eta} = \frac{1}{f(x^*(\eta))\big((1-\eta)x^*(\eta) - c\big)}\int_{\underline{x}}^{x^*(\eta)} x\,d\beta(x),$$

which is strictly positive, because (21) implies $c < (1-\eta)x^*$.

Statement 2, Region 4. If η is close to 1, (10) cannot hold if bankruptcy occurs with positive probability. Therefore, as long as (11) is slack, $\bar{v}$ and the loan rate do not depend on η because no bankruptcy occurs. If

(11) binds then as in the proof of statement 2, region 4 of theorem 1 it follows that $(1-\eta)(\bar{x}-\bar{v})-c=0$. Thus, the face value and the loan rate decrease as η is increased. ■

We now prove it is optimal to either take no loan or a loan of size d. The only relevant case is where loan L is larger than d; a loan less than d is not sufficient to run the project. Because the firm only requires one unit of input, any loan amount that exceeds d is invested by the entrepreneur in an outside option, which earns a safe rate of return, r_E. Suppose that funds invested in the outside option are observable by the lender. The firm's total assets are now $x = x_E + x_S$, where x_E is described by $\beta(\cdot)$ and where $x_S = (1+r_E)(L-d)$ is the firm's return on funds invested in the outside option. Since x_E is private information, we must distinguish between payments v_E and v_S, which are the respective payments made from the entrepreneur's return realization x_E and from the outside investment x_S, respectively. Thus, ℓ is now given by $\ell(x_E, v_E, v_S)$ (note that $\ell(\cdot)$ does not depend on x_S, because x_S is fixed). Further, $v(x)$ consists of payments $v_E(x)$ and $v_S(x)$. Finally, the enforcement decision is $e(v_E, v_S)$.

Assume the following restrictions on the equilibrium:

Assumption 1 $\ell(x, v_E, 0) = v_S + \ell(x, v_E, v_S)$ for all v_S.

Assumption 2 $\beta(x_E|v_E, v_S) = \beta(x_E|v_E, v_S')$ for all v_S that are on the equilibrium path and for all v_S' that are off the equilibrium path.

Assumption 1 means that the entrepreneur cannot be penalized for using funds from the safe investment to repay the debt, i.e., the timing of payment v_S is irrelevant, because the total payment to the investor is the same. Assumption 2 means that off-equilibrium path beliefs are not affected by payments from the safe investment. This is the case because the outside investment is observable and not correlated with the project's return.

Proposition 1 *Suppose that $r_E = r_m$, assumptions 1 and 2 hold, and investor return is at least $1+r_m$. Then without loss of generality we can restrict attention to contracts where the loan size is either zero or d.*

Proof of Proposition 1. The result follows immediately if $L \le d$. Thus, assume that $L > d$.

We first show that $e(v_E, v_S)$ is decreasing in v_S. Thus, let $v_S < v_S'$. Consider the expected payment from enforcement from (4). Assumptions 1 and 2 imply

$$(v_S' - v_S) + \int [\ell(x, v_E, v_S') - c]\, d\beta(x|v_E, v_S') = \int [\ell(x, v_E, v_S) - c]\, d\beta(x|v_E, v_S').$$

$\int[\ell(x, v_E, v_S) - c]\, d\beta(x|v_E, v_S) > \int[\ell(x, v_E, v_S') - c]\, d\beta(x|v_E, v_S')$. Constraint (4) then implies $e(v_E, v_S') \le e(v_E, v_S)$, i.e., the entrepreneur is more likely to enforce for the lower payment v_S.

We next use (3) and the monotonicity of $e(v_E, v_S)$ to determine the optimal repayment v_S. Recall that $x_S = (L - d)(1 + r_E)$, and $v_S = x_S$ is the maximum possible repayment from funds invested in the safe asset. Consider the three possible cases for enforcement:

1. If $e(v_E, 0) = 0$, then $e(v_E, v_S) = 0$ for all $v_S \leq x_S$. (3) implies that it is optimal to choose $v_S = 0$.
2. If $e(v_E, x_S) = 1$, then $e(v_E, v_S) = 1$ for all $v_S \leq x_S$. (3) implies that any $0 \leq v_S \leq x_S$ is optimal.
3. If $e(v_E, x_S) = 0$ and $e(v_E, 0) = 1$, then the optimal v_S is the lowest payment at which $e(v_E, v_S) = 0$.

Thus, for a given v_E, the entrepreneur makes a repayment v_S such that $e(v_E, v_S) = e(v_E, x_S)$. Total surplus gross of the investment input (i.e, entrepreneur plus investor payoff) is given by $\int x \, d\beta(x) - c \int e(v(x)) \, d\beta(x)$. Thus, surplus does not depend on whether v_S or x_S is repaid.

Now suppose that $0 < v_S < x_S$ and $e(v_E, v_S) = 0$. The first part of the argument of Lemma 1 immediately implies that $\bar{v} = v_S + v_E$ must be the same for all v_E, v_S with $e(v_E, v_S) = 0$. Further $\bar{v} > x_S$. Otherwise, the investor would not receive the market return on her total investment L. Therefore the entrepreneur's payoff remains the same if he selects $v_S = x_S$ or $v_S = \bar{v} - x_S$. Because total surplus is unaffected, the investor's payoff does not change as well. As a consequence, the following strategies are equivalent: the entrepreneur (i) takes an "excess loan," $L - d > 0$, and repays $x_S = (1 + r_E)(L - d)$ at the riskless interest $1 + r_E$; or (ii) takes a loan of size $L = d$. ■

7.2 Explanation of the Quantitative Analysis

To compute solutions, consider a discrete distribution with realizations x_i, $i = 1, \ldots, n$. First, assume the entrepreneur's participation constraint binds and compute $\bar{v}$. Next, check whether the enforcement constraint (10) is satisfied. Then check whether (11) holds. If both constraints hold, this is a candidate optimum. If (11) is violated, replace the previous $\bar{v}$ by the value at which (11) binds. If (10) holds for this new value of $\bar{v}$ this is a candidate optimum. If (10) is violated, increase the number of bankruptcy states until (10) binds.[17]

Next, consider the case where the participation constraint (8) is slack. In such a case it is optimal to choose $\bar{v} = (1 - \eta)x_i$ for one of the realizations x_i. If (10) is violated, again increase the number of bankruptcy states. Each realization x_i, $i = 1, .., n$ gives a candidate optimum. The true optimum provides the highest payoff to the lender among all candidate optima.

[17] We cannot simply raise $\bar{v}$ to get (10) to hold as this would violate either (8) or (11).

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