On Credible Monetary Policy and Private Government Information

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Credible and optimal monetary policies are considered in environments in which the government observes a signal that is correlated with the state of the economy. When the signal is public information it is optimal for monetary policy to be conditioned upon it. The extent to which such conditioning should occur when the signal is the private information of the government depends upon the government’s incentives to misrepresent information. It is shown that in some cases the Ramsey policy is incentive compatible, in others it is not. In the latter cases, policy must be constrained to be incentive compatible. This may result in “penalty phases” along the equilibrium path following apparent “mistakes” by the policy maker; it may result in the optimal monetary policy making no use of the signal.

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1. INTRODUCTION

A credible monetary policy is one such that after every possible history the government has no incentive to deviate from it and implement some publicly observable alternative policy. Credible government policies have received much attention in the literature. In particular, recent papers have reworked earlier reduced form game theoretic models of credible policy to include richer specifications of the non-government private sector. Methodologically, progress has been made by making explicit use of recursive equilibrium concepts. This has led to new insights and new computational strategies. In contrast, the incentive for a government to exploit any private information that it might have about the economy in its implementation of monetary policy has received much less attention.

1 Seminal contributions in the new literature include Chari and Kehoe [10], Stokey [26], and Chang [8]. See Barro and Gordon [5] for an influential reduced form game theoretic model.

2 However, see Canzoneri [7] for an important exception. A number of papers have considered environments in which the government has private information about its own preferences and these preferences do not change; see, e.g., Cho and Matsui [12].
However, such private information may be important. Romer and Romer [23] show that private Federal Reserve forecasts are significant predictors of commercial forecast errors. This suggests that the Federal Reserve may indeed possess information that others do not. Romer and Romer also provide some evidence that the Fed acts upon this information. Borrowing from the contract theory literature, monetary policies that have been designed to eliminate the incentives for a government to exploit its private information by misrepresenting it will be referred to as incentive compatible. As argued below incentive compatibility places additional constraints on monetary policy. These may be useful in understanding the structure of monetary policy and in understanding recent episodes in monetary history.

To make the idea of credible and incentive compatible monetary policies concrete, consider a macroeconomy with the following features: (i) a population of households that must use cash to make goods purchases and (ii) a population of monopolistically competitive firms that set their prices in each period before the government selects the period's monetary growth rate. Such an environment is fairly standard and has been analyzed by Chari et al. [9], and Ireland [15, 16]. Assume an upper bound on feasible monetary growth rates, and, for the moment, complete information. Then the ex ante optimal policy will (roughly speaking) prescribe a low monetary growth rate. However, once firms have set their prices, the government can reduce the distortion that stems from the imperfectly competitive goods sector and can increase welfare by raising the monetary growth rate above that prescribed by the ex ante optimal policy and anticipated by firms. In the absence of any private information such deviations are automatically observable. A monetary policy is credible if the government has no incentive to take such an observable deviation.

Now consider adding a productivity shock and providing the government with a privately observed signal correlated with this shock. Suppose that the timing of events is as follows. First, firms set their prices; then the government receives the signal. Having received the signal, it chooses a monetary growth rate, and, finally, the productivity shock occurs. If the signal were publicly observed, the optimal policy would prescribe increasing the monetary growth rate in response to a good signal and, conversely, reducing it in response to a bad signal. However, when the signal is not publicly observable the government may face an incentive to deviate from the optimal policy. In particular, it might be tempted to implement the higher monetary growth rate associated with a good signal on the receipt of a bad signal. In doing so it can reduce the distortion associated with the

A signal will be referred to as good (bad) if it indicates an increased (reduced) likelihood of a high productivity shock.
imperfectly competitive goods sector. If challenged it can always lie and claim that, in fact, it did receive the good signal. Such deviations are inherently unobservable. A monetary policy is incentive compatible if the government has no incentive to take such an unobservable deviation.

Credibility is best formulated as an equilibrium concept. As earlier contributions have noted, there are usually many credible equilibria supporting different credible monetary policies. There have been two responses to this multiplicity. One is to select equilibria that induce outcomes that correspond to empirical events (see, e.g., [9]). Another is to select equilibria on a priori grounds. In particular, the best credible equilibrium (that which gives the highest payoff to a benevolent government) is often selected. When the government has no private information and is sufficiently patient the optimal monetary policy (i.e., optimal absent credibility or incentive compatibility considerations) can be supported as part of an optimal credible equilibrium. Indeed, some commentators have suggested that patient governments can “solve” the credibility problem since their preferences are consistent with an equilibrium that supports the optimal monetary policy; see Canzoneri [7]. However, it is not clear that such optimal monetary policies provide a good description of the actual policies that are used. Canzoneri [7] characterizes the former as being too stable and the latter as having “episodic periods of breakdown.” Introducing private government information leads to the imposition of additional incentive compatibility constraints on monetary policy. These constraints may ensure that the optimal monetary policy cannot be supported by any credible, incentive compatible equilibrium even if the government is very patient. This suggests the following question: Do the best credible, incentive compatible equilibria provide approximate descriptions of observed monetary policies?

The paper takes a first step in answering this question and, more generally, analyzing the nature of credible, incentive compatible monetary policies. It proceeds in stages. First, it considers the Ramsey policy (the optimal policy of a benevolent government unconstrained by credibility or incentive compatibility requirements). Sufficient conditions for this policy to fail to be incentive compatible are provided. Second, it considers the set of incentive compatible (but not necessarily credible) policies. Third, it analyzes policies that are both credible and incentive compatible (CIC). A recursive formulation for each of these problems is obtained. These formulations make heavy use of the techniques of Abreu et al. [2]. CIC equilibrium sets of payoffs and shadow prices are computed and some (numerical) characterization of “optimal” CIC equilibria across different parametric versions of the model is provided.

The games analyzed have some similarity to those considered by Green and Porter [14].
Several results are obtained. To begin with, it is shown that incentive compatibility tends to constrain the government to make less use of its signals. It may be that the optimal incentive compatible policy makes no use of them at all. When this policy is signal contingent incentive compatibility may require that the government be “punished” for making mistakes, i.e., for undertaking high monetary growth ex ante when ex post circumstances suggest a low rate was appropriate. Since the government’s signal is noisy such mistakes (and the consequent punishments) are inevitable even if the government adheres to its strategy. In general, these (optimal) punishments do not resemble the severe continuation equilibria associated with grim trigger strategies in complete information repeated games. One way of implementing them is through lotteries that attach a small probability to very adverse conditions. The paper tentatively concludes that these punishment phases may explain the “episodic periods of breakdown” described by Canzoneri.

It is also shown that the extent to which an optimal CIC equilibrium permits the government to make use of its private information depends on the government’s incentives to misrepresent its signal. These in turn depend on the degree of goods market distortion. Thus, an imperfectly competitive goods market can be doubly bad. First, it is a distortion in itself and, in particular, it distorts the optimal competitive allocation away from the first best. Second, it may distort the allocation implemented by the optimal CIC equilibrium away from the optimal competitive allocation by increasing the government’s incentive to misrepresent its private signal.

1.1. Related Literature

As noted above there is a large literature on the credibility problem. Early contributions, including the seminal work of Kydland and Prescott [18] and Calvo [6], emphasized the time consistency problem of optimal policy, i.e., the failure of government policy problems to satisfy Bellman’s principle of optimality. Later work took a game theoretic approach to the policy problem and modeled credibility as an equilibrium concept in a repeated game (see, e.g., Barro and Gordon [5]). More recently, analysts have embedded a strategic government into an explicit general equilibrium model inhabited by private agents. Several of these analysts have built on the game theoretic insights of Abreu, Pearce, and Stacchetti (APS) [2]. APS developed set-valued dynamic programming techniques for solving for the (sequential) equilibrium payoff sets of certain classes of repeated game. Chang [8], Phelan and Stacchetti [20], and Sleet [24] showed how APS methods could be applied to government policy games that incorporate a large strategic player (the benevolent government) and a fringe of competitive players. Building on an earlier insight of Kydland and Prescott [19], their innovation was to keep track of the Lagrange multipliers from
the competitive players’ choice problems rather than their payoffs. This paper takes a similar approach, but goes beyond these contributions in incorporating private government information.

In contrast, Canzoneri [7] does incorporate private government information and raises similar issues to those considered here. This paper extends Canzoneri’s by replacing his reduced form model of the behavior of private sector agents with a fully articulated general equilibrium model. This permits an analysis of the relationship between preference and technology parameters and the set of incentive constrained monetary policies. Furthermore, the application of the APS machinery permits a complete solution of the set of credible, incentive constrained equilibria in the monetary policy game.

2. COMPETITIVE EQUILIBRIA IN NON-STRATEGIC SETTINGS

This section considers an environment in which the government can commit and has no private information. Both assumptions are relaxed in later sections. The economy is a simple dynamic new Keynesian one. It is substantively the same as those described in Chari et al. [9] or Ireland [15]. These papers may be consulted for further details. There are three types of infinitely lived agents: firms, households, and a monolithic government. The timing of moves within each period is as follows. At the beginning of the period monopolistically competitive firms set their prices. The government then receives a signal concerning the productivity of firms. After receiving this signal, but before the realization of a productivity shock, the government issues additional cash (money), giving it to households. The representative household adds this to its portfolio, which also consists of cash, bonds, and shares inherited from the previous period. The productivity shock then occurs, perturbing the production functions for firms. At this point, the bond market opens and households are able to trade bonds with one another. Since households operate under a cash in advance constraint they must have sufficient cash for their current period expenditures. Firms hire labor and use this labor to produce goods. Households buy these goods with the money that they are holding. At the end of the period, firms pay their workers out of their cash receipts and distribute their net profits to households. Finally, the stock market opens, allowing households to buy and sell shares. The details follow.

Let \( \{s_t\}_{t=0}^{\infty} \) denote an \( S \)-valued stochastic process that describes the signals received by the government and let \( \{\theta_t\}_{t=0}^{\infty} \) denote a \( \Theta \)-valued stochastic process that describes productivity shocks. \( S \) and \( \Theta \) are finite subsets of \( \mathbb{R}_+ \). To simplify the analysis assume that the \( \{\theta_t\} \) are i.i.d. and let \( f \) denote the joint density for \( \theta_t \) and \( s_t \). Furthermore, assume that the
stochastic kernel associated with $f$ satisfies a monotone likelihood ratio assumption; i.e., if $\theta_i, \theta_j \in \Theta$, $\theta_i > \theta_j$ and $\delta_i, \delta_j \in S$, $\delta_i > \delta_j$ then $f(\theta_i | \delta_i)/f(\theta_j | \delta_j) > f(\theta_i | \delta_i)/(f(\theta_j | \delta_j)$. Let $x_t$ denote the government's choice of a monetary growth rate at date $t$. The government's $t$-period history, $h^x_t$, is a realized sequence of productivity shocks and past monetary growth choices from date 0 through to $t-1$ and a $t$th period signal. The set of $t$ period government histories will be denoted $H^x_t$. A monetary policy, $x = \{x_t\}_{t=0}^\infty$, is a fixed collection of functions with $x_t: H^x_t \to \mathbb{R}$, that describe the rate of monetary growth in each period given the government’s history. If the rate of monetary growth is “too small,” then a monetary policy may not be consistent with any competitive equilibrium. On the other hand, if there is no upper bound on monetary growth there will be no monetary policies consistent with credible government behavior (in this model). To avoid these difficulties the following bounds are placed on monetary policies: $x_t \in [\varepsilon - 1, \hat{x}]$, $\varepsilon > 0$ and small and $E_{t-1}(1 + x_t) \leq 1/\beta$. [9] provides further discussion and some justification for the upper bound. Let $H^x_t$ denote the set of possible $t$-period histories of productivity shocks and money growth rates induced by a fixed pair $(\theta, x)$ and let elements of $H^x_t$ be denoted $h^x_t$. Such sequences of productivity shocks and money growth rates will be referred to as “public histories.” The following sections describe the components of a competitive equilibrium.

2.1. The Household’s Choice Problem

Let $M_t$ denote the period $t$ aggregate money supply. Let $c_t$, $l_t$, $B_t$, and $a_t$ denote the representative household’s period $t$ consumption, labor supply, and nominal bond and share holding choices. It will be convenient to normalize the nominal variable $B_t$ in the period $t$ money supply. Thus, the variable $b_t = B_t/M_t$ is used. Similarly, let $m_t$ be the normalized stock of money chosen and held by the representative household at the beginning of $t$.

In period $t$, after receiving the normalized monetary transfer from the government ($x = (M_{t+1} - M_t)/M_t$) and observing current goods prices ($p_t$), the current wage ($w_t$), and the current interest rate on bonds ($r_t$), the representative household makes its bond holdings $b_{t+1}$, consumption $c_t$, and labor supply $l_t$ choices. At the end of the period, having observed share prices ($q_t$) and received dividends ($d_t$) it undertakes trades on the stock market and chooses $a_t$. The nominal variables $p_t$, $q_t$, and $d_t$ are also normalized in the period $t$ money supply. The household’s choices are feasible if they satisfy the following budget and cash-in-advance constraints for all $t$, $h^x_{t+1} \subseteq H^x_{t+1}$:

\[ c_t \text{ is used to denote the consumption of the good produced by the } i \text{th firm at period } t. \]
\[ c_t \text{ denotes the consumption of all goods in period } t. \]
for realized values of $c$

interpreted as consumption of the representative good at $t$

petitively determined prices. Such a $z$

random variables describing private sector choices, dividends and com-

labor supply according to the functional

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|29x377| x
|33x301| At equilibrium all firms charge the same prices. Hence, abusing notation, $c_t$ will be interpreted as consumption of the representative good at $t$ in the remainder of the paper.
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Thus, $V(c, l, x | h_t)$ denotes a household’s continuation payoff after a history $h_t$ from a triple $(c_t, l_t, x_t, x)$, $t = 0, \ldots, \infty$. Given a market process $x_t$ and a monetary policy $y_t$, a household solves:

$V^*(x, x_t | h_0) = \sup_{E_{h_0} x} \sum_{i=0}^{\infty} \beta^i u(c_i, l_i)$ subject to (1) and (2).

The first order conditions for this problem include

\[ c(h_{t+1}) = \frac{1}{\lambda_i(h_{t+1})} \]

\[ w(h_{t+1}) = \frac{1}{\lambda_i(h_{t+1})}, \quad r(h_{t+1}) = \frac{\mu_i(h_{t+1})}{\lambda_i(h_{t+1})} \]

\[ (1 + x_i) \lambda_i(h_{t+1}) = \beta \sum_{h_{t+2}} [\lambda_i(h_{t+2}) + \mu_i(h_{t+2})] d\pi(h_{t+2} | h_{t+1}) \]

\[ q(h_{t+1}) = \frac{\beta}{\lambda_i(h_{t+1})} \sum_{h_{t+2}} \lambda_i(h_{t+2}) \]

\[ \times [q(h_{t+2}) + d(h_{t+2})] d\pi(h_{t+2} | h_{t+1}). \]

At equilibrium all firms charge the same prices. Hence, abusing notation, $c_t$ will be interpreted as consumption of the representative good at $t$ in the remainder of the paper.
and the budget and cash in advance constraints (1) and (2). In the above equations, \( \pi \) is the probability measure over public histories induced by \((\theta, x)\), \( \lambda_t \) and \( \mu_t \) are the multipliers on the \( t \)th period’s budget and cash in advance constraints, respectively.

2.2. Firms

Firms choose their (normalized) prices before the government selects the money growth rate. Thus, the representative firm’s choices are described by a sequence of functions \( p_t : H_x^* \to \mathbb{R}_+ \). Firms use only labor to produce and do so according to the production function: \( j(\theta, l_t) = \theta l_t \). Firms are assumed to maximize their stock market valuation. Substituting from the household’s first order conditions into the firm’s problem it may be shown that the representative firm sets a price given by:

\[
 p_t(h_{x+1}^*) = \frac{\phi}{\phi - 1} \sum_{h_{x+1}}^{1} \frac{d\pi(h_{x+1}^* | h_{x}^*)}{\partial[\lambda_t + \mu_t]} \sum_{h_{x+1}}^{\lambda_t} \frac{d\pi(h_{x+1}^* | h_{x}^*)}{\partial[\lambda_t + \mu_t]}. \tag{9}
\]

The term \( \phi/(\phi - 1) \) gives the firm’s markup. Both it and the associated product market distortion are decreasing in \( \phi \).

2.3. The Cash in Advance Constraint

In the sequel attention will be restricted to histories along which the cash in advance constraint holds with equality. Such equality occurs at \( t \) after \( h_{x+1}^* \) if \( r(h_{x+1}^*) > 0 \), but it need not occur if \( r(h_{x+1}^*) = 0 \). In the former case, the cash in advance constraint binds; in the latter case it does not. The assumption that the constraint holds with equality (even when it does not bind) is frequently made in analyses of optimal and credible monetary policy to make the assumption.

In analyses of optimal policy this represents a restriction on the set of monetary policies that can be used to implement the optimal allocation. In the context of credible policy games it can be thought of as a restriction on the beliefs of the private sector agents, and in the game considered in this paper, with private government information, it will restrict the set of allocations that can be implemented. The assumption is made to ensure that there is some gain to the government from conditioning monetary policy on the current signal it has received.

\[\text{For example, [9, 11] make the assumption; [15] belongs to the smaller set of papers that do not.}\]

\[\text{For example, it is consistent with the restriction that firms believe that households will spend all of their cash.}\]
This assumption coupled with the fact that bonds are in zero net supply implies that in equilibrium
\[ c_t(h^x_{t+1}) = (1 + x_t)/p(h^x_t). \]  
(10)

Equation (5) then yields
\[ \lambda_t(h^x_{t+1}) + \mu_t(h^x_{t+1}) = 1/(1 + x_t). \]  
(11)

This equation has two implications. First, it implies that \( \psi_t \equiv \lambda_t + \mu_t \) is \((h^x_t, x_t)\)-measurable (rather than \(h^x_{t+1}\)-measurable). Second, coupled with the non-negativity of \( \lambda_t \) and \( \mu_t \) and the bounds on monetary policies described earlier, it implies that the sequence of multipliers is uniformly bounded. In particular, this means that there exist compact sets \( A \) and \( M \) such that \((\lambda_t, \mu_t) \in A \times M \forall t\). The boundedness of these multipliers guarantees that the transversality condition associated with the household's optimization holds along any path satisfying the first order conditions of the households and of the firms.

2.4. Competitive Equilibria

A competitive equilibrium is a monetary policy and a process for household and firm choices and for competitively determined prices that is consistent with household and firm optimality and with market clearing. More precisely:

**Definition 2.1**. A competitive equilibrium, \( e \), is a monetary policy \( x \) and a market process \( z_x \) such that:

1. There exists a multiplier sequence \( (\lambda_t, \mu_t) \equiv \{\lambda_t, \mu_t\}_{t=0}^{\infty} \) satisfying (7) for all \( h^x_t \), taking values in \( A \times M \), and
2. \( \forall t, h^x_{t+1}, \) the \( \{c_t, b_{t+1}, a_{t+1}, l_t, m_{t+1}\}_{t=0}^{\infty} \) that describe the household's choices, satisfy (1), (2) and (5).
3. \( \forall t, h^x_t \), the \( \{p_{t+1}\}_{t=0}^{\infty} \) that describe the firm's choices satisfy (9).
4. \( \forall t, h^x_{t+1} \), wages, interest rates, and share prices satisfy (6) and (8).
5. At all dates and after all histories, markets clear:

\[ c_t(h^x_{t+1}) = \theta_t l_t(h^x_{t+1}) \quad \text{Goods Market Clearing} \]  
(12)
\[ c_t(h^x_{t+1}) = \frac{1 + x_t}{p_t(h^x_t)} \]  
(13)
\[ a_t(h^x_t) = 1 \quad \text{Stock Market Clearing} \]  
(14)
\[ b_{t+1}(h^x_{t+1}) = 0 \quad \text{Bond Market Clearing.} \]  
(15)
A sequence of functions \((c, l) = \{c_t, l_t\}_{t=0}^\infty\) describing consumption and labor supply choices will be referred to as a competitive allocation if it is supported by (i.e., is part of) some competitive equilibrium, \(e\). Similarly a competitive allocation, monetary policy pair that are part of the same competitive equilibrium will be said to be supported by that equilibrium. The set of such pairs will be denoted \(\mathcal{E}\).

Any competitive equilibrium can be recovered from a multiplier sequence \((\lambda_t, \psi_t)\) that satisfies the following conditions:

\[
\frac{\lambda_t(h^s_{t+1})}{\psi_t(h^s_t, x_t)} = \beta \sum_{h^s_{t+1}} \left[ \psi_{t+1}(h^s_{t+1}, x_{t+1}) \right] \, d\pi(h^s_{t+1}, x_{t+1} | h^s_{t+1})
\]

\[
\psi_t - \lambda_t \geq 0, \quad \lambda_t \geq 0, \quad \psi_t \in \left[ \frac{1}{1+x}, \frac{1}{1+x} \right], \quad E_{t-1} \psi_t \leq 1/\beta.
\]

(16)

The first of these equations is obtained from (7), (11) and the definition of \(\psi\); the second follows from the monetary policy bounds. Let \(\mathcal{E}(\zeta)\) denote the set of competitive allocation and monetary policy pairs that are supported by competitive equilibria with associated multiplier sequences satisfying \(E\psi_0 = \zeta\). In the sequel any \((c, l, x) \in \mathcal{E}(\zeta)\) will be said to be a competitive allocation and monetary policy consistent with \(\zeta\). \(\mathcal{E}(\zeta)\) will be referred to as an expected multiplier. Note that, by (11), \(\zeta = E1/(1+x_0)\). The bounds on monetary policies imply that \(\zeta \in \mathbb{E} = [1/1+x, 1/\beta]\). Notice that associated with each \(\zeta \in \mathbb{E}\) there exists a stationary competitive equilibrium with \(1+x = 1/\zeta\) and the other elements of the equilibrium defined using the above first order conditions. Thus, \(\mathcal{E}(\zeta) \neq \emptyset\), \(\zeta \in \mathbb{E}\).

Suppose that \((c, l, x)\) is a competitive allocation, monetary policy pair. Suppose also that it is part of a competitive equilibrium with an associated multiplier sequence \(\{\lambda_t, \psi_t\}\). Then note that after any history \(h^s_t\) the continuation sequence of consumption and labor supply choices \(\{c_{t+1}, h^s_{t+1}, l_{t+1}, \cdots\}_{t=0}^\infty\) and the continuation monetary policy \(\{x_{t+1}, h^s_{t+1}, \cdots\}_{t=0}^\infty\) is also a competitive allocation monetary policy pair belonging to: \(\mathcal{E}((\lambda_t, (h^s_t)))/ (\beta \psi_{t-1}(h^s_t))\).

3. INCENTIVE INCOMPATIBILITY OF RAMSEY EQUILIBRIA

A Ramsey equilibrium is an optimal competitive equilibrium, i.e., a competitive equilibrium that maximizes the representative household’s

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9 See Ireland [15] for a detailed derivation of this fact in a similar setting.
preferences. Hence a Ramsey equilibrium supports a competitive allocation-monetary policy pair that solves:

\[ (c^*, l^*, x^*) \in \arg \max_{(c, l, x) \in \mathcal{A}} V(c, l, x). \]

For the remainder of the paper it will be assumed that the government is benevolent and shares the same preferences as the households. Thus, the previous optimization may be thought of as the choice problem of a government that can commit to future plans and is unable to misrepresent its signals. Proposition 3.1 below characterizes the Ramsey equilibrium. It extends arguments in [9, 15] to the present economic environment with government signals in a straightforward way. Its proof is omitted.

**Proposition 2.** Suppose that for all \( s \):

\[ \sum_{\theta} \frac{1}{\beta} f(\theta | s) \leq \left[ \frac{1}{1 + \frac{1}{\alpha}} \right]. \]

Then a Ramsey equilibrium is such that along any realized outcome path

\[ l_t(h_{t+1}^*) = \phi \left( \frac{1}{\phi} \sum_{\theta} \frac{1}{\beta} f(\theta | s_t) \right)^{-1} \quad (17) \]

\[ c_t(h_{t+1}^*) = \phi \left( \frac{1}{\phi} \sum_{\theta} \frac{1}{\beta} f(\theta | s_t) \right)^{-1} \quad (18) \]

\[ \lambda_t(h_{t+1}^*) = \psi_t(h_t^*, s_t) \quad (19) \]

\[ \sum \lambda_t(h_{t+1}^*) d\pi(h_{t+1}^* | h_t^*) = \frac{1}{\beta} \quad (20) \]

Observe that Eq. (19) implies that \( \mu_t = 0 \) and that the cash in advance constraint does not bind. Equations (17) and (18) indicate that both the labor supply and consumption increase when the government receives a signal indicating that productivity is likely to rise. More precisely, these equations indicate that the labor supply and consumption rise when \( E(1/\theta | s_t) \) falls. If the cash in advance constraint holds with equality such increases will require an accommodating increase in the money supply. It is in this sense that the monetary policy used in a Ramsey equilibrium (the Ramsey policy) is signal contingent.

It is worth comparing these results with those that would obtain if the government could directly observe the current \( \theta_t \) shock before it set the
current period’s money supply.\footnote{This is the case considered in [15].} In this case, the labor supply would be a constant \( \frac{\Delta s}{s} \) and consumption would rise or fall depending upon the value of the shock. Under the assumption that the cash in advance constraint holds with equality, this would imply that the money growth rate responds contemporaneously to the productivity shock. Specifically, the money growth rate would be increased in response to a “favorable” productivity shock and reduced otherwise.

It is well known that, in general, when a government can not commit to implementing policies that are contingent upon publicly observed histories then the Ramsey policy is time inconsistent. However, when the government has private information on the appropriate conduct of policy then the Ramsey policy may be incentive incompatible as well. To see this consider two signals \( s_H \) and \( s_L \), with \( s_H > s_L \). The Ramsey equilibrium implies that the government selects a monetary policy that implements \( \psi_H \) and \( \psi_L \), where

\[
\psi_i = \frac{\phi}{\phi - 1} \frac{E(1/\theta | s_i)}{p}, \quad i = \{L, H\}
\]

Notice that once firms have set their prices constraint (9) acts as a restriction on continuation equilibria and continuation monetary policies. Consider relaxing this restriction after the setting of a price \( p \). The optimal (relaxed) continuation equilibrium entails a value of \( \psi_L \) of \( \psi_L^* = E(1/\theta | s_L)/p \). If \( \psi_L > \psi_H \geq \psi_L^* \) then, after receiving the low signal, the government obtains a higher current payoff from implementing the high signal policy. In doing so it reduces the current distortion that stems from the imperfectly competitive product market and raises its current payoff. Since the Ramsey equilibrium is stationary, and the government receives the same expected continuation payoff along the equilibrium path regardless of its current actions, the government increases its total payoff by deviating in this way. It follows that if inequality (*) holds the Ramsey equilibrium is not incentive compatible. The above argument establishes the following proposition:

**Proposition 3.2.** (Incentive Incompatibility of Ramsey Equilibria). If there exists a pair of signals \( s_j > s_i \) such that:

\[
\frac{\Delta s}{s} E(1/\theta | s_j) \geq E(1/\theta | s_i)
\]

then the Ramsey equilibrium is not incentive compatible.

### 4. INCENTIVE COMPATIBLE EQUILIBRIA

This section considers in more detail an environment in which (1) the signal is the private information of the government and (2) the government
can commit not to take a publicly observable defection from a monetary policy. In other words the government can commit to choosing only those actions that are in the “support of a monetary policy.” The ultimate objective of this paper is to study cases in which the government has private information and lacks any ability to commit. The rationale for studying the environment of this section (with commitment) is that it facilitates understanding of the no commitment case. Roughly speaking, if the government is sufficiently patient then the optimal policy obtained in this section will remain optimal when the government’s ability to commit is removed. A “folk theorem” is at work.\(^{11}\)

Although the \(\{s_t\}\) process is not publicly observed, by the revelation principle, there is no loss of generality in restricting attention to equilibria that provide the government with the necessary incentives to truthfully reveal its private information. Thus, as before \(c_t, l_t\) etc. can be taken to be functions of \(h^*_t\) and the definition of a competitive equilibrium in (2.1) suffices for the analysis in this section.

Let \(x = \{x_t\}\) denote a particular monetary policy. Another monetary policy \(x^*\) will be said to be publicly indistinguishable from \(x\) at \(h^*_t\) if for all \(s \in S, x'_t(h^*_t, s) \in \{z \in R \mid z = x_t(h^*_t, s')\}\) some \(s' \in S\). \(x^*\) will be said to be publicly indistinguishable from \(x\) if it is publicly indistinguishable from \(x\) for all \(t, h^*_t\). Let \(X(x)\) denote the set of monetary policies publicly indistinguishable from \(x\).

Suppose that \((\eta, x)\) is a competitive allocation-monetary policy pair. \((\eta, x)\) will be said to be incentive compatible if the government is better off implementing \(x\) than any other element of \(X(x)\). Notationally,

\[
E \left( \sum_{t=0}^{\infty} \beta^{t-t} u(c_t, l_t) \mid \eta, x \right) \geq E \left( \sum_{t=0}^{\infty} \beta^{t-t} u(c_t, l_t) \mid \eta, x^* \right) \quad (21)
\]

for each \(x^* \in X(x)\). Let \(v_t: H_t^* \rightarrow R\) denote the continuation payoff to the government from the competitive allocation \(\eta\) after a history \(h_t^*\). Consider the temporary incentive compatibility constraint

\[
\sum [u(c_t(h_t^*, x_t(h_t^*, s_t), \theta), l_t(h_t^*, x_t(h_t^*, s_t), \theta))
+ \beta v_t(h_t^*, x_t(h_t^*, s_t), \theta)] f(\theta \mid s_t)
\geq \sum [u(c_t(h^*_t, x'_t(h^*_t, s'), \theta), l_t(h^*_t, x'_t(h^*_t, s'), \theta))
+ \beta v_t(h^*_t, x'_t(h^*_t, s'), \theta)] f(\theta \mid s_t),
\]

\(11\) Fudenberg et al. [13] obtain folk theorems that ensure that efficient outcomes (absent private information) can be obtained in the presence of private information if players are sufficiently patient. However, these must be modified before they can be applied to this problem since the government is benevolent and its objective coincides with that of the representative household.
where $x'_t$ is the $t$th period element of a monetary policy indistinguishable from $x$ at $h^*_t$. It may be shown that a competitive equilibrium satisfies (21) if and only if it satisfies a temporary incentive compatibility constraint for all $t$, $h^*_t$, $s_t$, and all $x'$ publicly indistinguishable from $x$. The argument is similar to those given in the dynamic contracting literature (e.g., Atkeson and Lucas [4] or Phelan and Townsend [21]) and is omitted.$^{12}$

Define $\psi^*(\xi)$ to be the set of incentive compatible allocations and monetary policies consistent with $E\psi_0 = \xi$. The following lemma makes the recursive nature of incentive compatible allocations and monetary policies clear. See the Appendix for the proof of this and other results.

**Lemma 4.1.** Let $x = \{x_{it}\}_{t=0}^\infty$ be a monetary policy. Let $(c, l) = \{c_t, l_t\}_{t=0}^\infty$ be a sequence of functions describing household consumption and labor supply choices. Let $(c, l)(h^*_t)$ denote the continuation of $(c, l)$ after a history $h^*_t = (\theta_0, \xi_0(s_0), \xi_1(s_0), \ldots)$, i.e., $(c, l)(h^*_t) = \{c_t(h^*_t), l_t(h^*_t), \ldots\}_{t=0}^\infty$. Define, $x(h^*_t)$ analogously as the continuation of $x$ after a history $h^*_t$. Then $(c, l, x) \in \psi^*(\xi)$ if and only if there exist three functions $\lambda_0: \Theta \times R_+ \rightarrow A$, $\psi_0: \Theta \times R_+ \rightarrow M$ and $v_0: \Theta \times R_+ \rightarrow R$ with $E\psi_0 = \xi$, $\psi_0 \in [1/1 + x, 1/\varepsilon]$, $\psi_0 - \lambda_0 \geq 0$ and:

1. For all $(\xi_0, \theta_0)$, the date 0 elements of $(c, l, x)$ and the two functions $\lambda_0$ and $\psi_0$ satisfy (5), (9), (12) with $\mu_0 = \psi_0 - \lambda_0$;
2. For all $h^*_t$, $(c, l, x)(h^*_t) \in \psi^*(\xi_0(s_0))$ and $v_0(h^*_t) = V(c, l, x | h^*_t)$;
3. For all $(\xi_0, \theta_0)$, the date 0 elements of $(c, l, x)$ and the function $v_0$ satisfy (22), the temporary incentive compatibility constraint.

Let $\mathcal{\Psi}^*$ denote the set of pairs of payoffs $(v)$ and expected multipliers $(\xi \in \Xi)$ induced by incentive compatible competitive equilibria. The competitive equilibrium described in Section 2 as existing for each $\xi \in \Xi$ is incentive compatible so $\mathcal{\Psi}^* = \{v: (\xi, v) \in \mathcal{\Psi} \} \neq \emptyset$. The one period payoff to the government is bounded above, consequently, the set $\mathcal{\Psi}^*$ is bounded above by some $\bar{v}$, each $\xi \in \Xi$. The bounds on the set of monetary policies given previously ensure that the sets $\mathcal{\Psi}^*(\xi)$ are also bounded below by some finite $v$. Thus, $\mathcal{\Psi}^* \subset \mathcal{\Psi} \equiv \Xi \times [\underline{v}, \bar{v}]$.

The recursiveness of the set of allocations and policies is now exploited to provide an algorithm for calculating the set $\mathcal{\Psi}^*$. The method draws on earlier results of Kydland and Prescott [19] and APS [2] and the application of these ideas by Chang [8], Phelan and Stacchetti [20] and Sleet [24]. Let $D$ be a compact subset of $\mathcal{\Psi}^*$.

**Definition 4.2.** A three-tuple of functions, $\psi: S \rightarrow R_+^+$, $\lambda: S \times \Theta \rightarrow A$, and $v: S \times \Theta \rightarrow R_+$, are said to be $I$-admissible w.r.t. $D$ if:

$^{12}$ It is included in the working paper version of this paper, which is available on request.
1. \( \left( \frac{\lambda(s, \theta)}{\beta \psi(s, \theta)}, \psi(s, \theta) \right) \in D, \)

2. \( \lambda(s, \theta) \leq \psi(s), \quad \psi(s) \in \left[ \frac{1}{1 + \delta}, \frac{1}{\varepsilon} \right], \sum_{s} \psi(s) f(\theta, s) \leq 1/\beta, \)

and

3. \[
\sum_{\theta} \left[ \ln \left( \frac{1}{p\psi(s)} \right) - \frac{1}{\partial \psi(s)} + \beta \psi(s) \right] f(\theta | s) \geq \sum_{\theta} \left[ \ln \left( \frac{1}{p\psi(s)} \right) - \frac{1}{\partial \psi(s)} + \beta \psi(s) \right] f(\theta | s)
\]

where

\[
p = \frac{\sum_{s} \frac{1}{\psi(s)} f(\theta, s)}{\phi - 1 - \sum_{s} \frac{\lambda(s, \theta)}{\psi(s)} f(\theta, s)}.
\]

The first step in understanding this definition is to interpret elements of the set \( D \) as candidate equilibrium “expected multiplier” and payoff pairs. The next step is to define a function \( \psi(s) \). If \( \lambda \) and \( \psi \) are interpreted as “current” multiplier functions then it is natural to interpret \( \psi \) as a “future” expected multiplier function. These interpretations are consistent with the Euler Eq. (16). Suppose that \( \psi' \) is thought of as a function describing continuation payoffs in the next period. Then the first condition in the \( \psi \)-admissibility definition can be viewed as one that requires the expected multiplier function \( \psi \) and the continuation payoff function \( \psi' \) to take their values in the set of candidate equilibrium expected multipliers and payoffs. The second condition in the above definition ensures that \( \lambda \) and \( \psi \) are consistent with the bounds on the multipliers \( \mu \) and \( \lambda \). The third condition may be interpreted as a temporary incentive compatibility constraint. It requires that the government is better off implementing the prescribed policy than deviating to an alternative one legitimized by some other value of the signal.

Let \( \mathcal{X} \) denote the space of compact subsets of \( \mathbb{W} \). Consider the operator \( B: \mathcal{X} \rightarrow 2^\mathbb{W} \) defined as follows:

\[
B(D) = \left\{ (\tilde{\lambda}, \psi) \mid (\psi, \tilde{\lambda}, \psi') \text{ is admissible w.r.t. } D \text{ and } \tilde{\lambda} = \sum_{s, \theta} \psi(s) f(\theta, s) \right\}
\]

where \( \tilde{\lambda} \) is defined as in (24).
The operator $B$ is somewhat analogous to the Bellman operator in standard dynamic programming. Thus, if $D$ is a set of candidate equilibrium expected multipliers and payoffs, then $B(D)$ is the set of expected multipliers, payoffs, and prices that can be supported by $D$ in a manner consistent with current market clearing, private sector optimality and government incentive compatibility conditions.

Notice that it is an immediate corollary of Lemma 4.1 that $B(\mathcal{V}) = \mathcal{V}$. Additionally, $B$ satisfies the following proposition.

**Proposition 4.3.** $B$ satisfies the following properties:

(Compactness). $B: \mathcal{X} \to \mathcal{X}$.

(Monotonicity). Suppose $V, V' \in \mathcal{X}$ and $V \subseteq V'$. Then $B(V) \subseteq B(V')$.

Furthermore, if $\mathcal{V} \subseteq W_0 \in \mathcal{X}$, then $\mathcal{V} = \lim_{n \to \infty} B^n(W_0) \subseteq B^n(W_0) \subseteq W_0$.

Finally, $\mathcal{V} \in \mathcal{X}$.

The proposition has two useful implications. The first is the compactness of $\mathcal{V}$ which is used in arguments given below. The second is the iterative procedure for obtaining $\mathcal{V}$ given in the last part of the proposition. This is used heavily in the later section on CIC equilibria.

5. OPTIMAL INCENTIVE COMPATIBLE EQUILIBRIA

This section provides some characterization of optimal incentive compatible equilibria. It shows that when the product market distortion is mild the Ramsey policy is incentive compatible; when it is severe the optimal policy makes no use of the government’s signal. In both these situations policy is simply repeated period after period. However, as shown below, intermediate cases entail some conditioning of optimal policy on the signal. This conditioning requires that the government is “punished” for apparent mistakes and these punishments induce optimal fluctuations into policy.

The characterization is obtained by analyzing a “simple optimization” problem. Before formulating this problem the preliminary issue of the dependence of the optimal incentive equilibrium on the initial expected multiplier $\mathcal{z}_0$ is taken up. First, note that a low value of $\mathcal{z}_t$ tends to imply a high expected rate of monetary growth at $t$. Such a high expected rate of monetary growth is costly if it is anticipated at $t - 1$. Recall that workers

13 A related $B$-operator was introduced by Abreu et al. It was applied to a repeated monetary policy game by Chang [8], to a dynamic game with a physical state variable by Atkeson [3], and to dynamic policy games with physical state variables by Phelan and Stacchetti [20] and Sleet [24].
can not spend money wages that they receive at \( t - 1 \) until \( t \). Thus, increases in expected money growth rates at \( t \) tend to raise the marginal cost of labor at \( t - 1 \). This translates into higher prices at date \( t \) and, ceteris paribus, lower consumption. Since there is no period \(-1\) in the model it is tempting to conclude that the value of \( \xi_0 \) is irrelevant for the payoff obtained from optimal incentive compatible equilibrium. This is not quite correct, because the pointwise bounds imposed previously restrict the government’s ability to vary the monetary growth rate across different signal states when \( \xi_0 \) is close to the end points of \( \Xi \). To avoid this complication the bounds on monetary policies are altered in this section. For all \( t \), a monetary policy is required to satisfy (1+\( x \)) and for \( Q > E(1/\theta) E(\theta) > 0 \):

\[
E_{t-1} 1/(1+x) \in [1/(1+\bar{x}), 1/\beta] \\
E_{t-1} 1/(1+x) \leq Q.
\]

All earlier results go through with this change in bounds (coupled with the finiteness of the number of signals and shocks). Abusing notation slightly, notation used earlier in the paper will be used to describe the corresponding objects when monetary policies are subject to these altered bounds. Let \( v^*(\xi) = \sup_{V(c, l, x)} V(c, l, x) \) denote the payoff to the optimal incentive compatible equilibrium consistent with \( \xi \). By the compactness of \( V \) there exists a competitive equilibrium that attains this payoff. In the remainder of this section several results are proven that characterize optimal incentive compatible equilibria. To simplify the analysis \( S \) and \( \Theta \) will be assumed to have two values. Thus, \( S = \{s_1, s_2\} \) and \( \Theta = \{\theta_1, \theta_2\} \), where \( s_2 > s_1 \) and \( \theta_2 > \theta_1 \). Additionally, let \( f_{ji} = f(\theta_j | s_i) \) and \( x_i = \sum(1/\theta_j) f_{ji} \) (so that, by the monotone likelihood ratio (MLR) assumption, \( x_2 > x_1 \)) and let \( q_i \) be the probability of signal \( i \). In addition to the MLR assumption, assume that \( 1 > f_{12}/f_{11} \) —bad shocks are more likely than good given a bad signal. The following lemma shows that, under the altered monetary policy bounds, \( v^*(\xi) \) does not depend on \( \xi \).

**Lemma 5.2.** For all \( \xi \in \Xi \), \( v^*(\xi) \) equals a constant value.

In the sequel \( v^* \) will be used to denote this constant. We now consider the first period choices of an optimal incentive compatible equilibrium. To determine these choices we proceed indirectly by analyzing the “simple” problem (Program P)

\[
\sup \sum_{i=1,2} \left[ a_i + \beta \sum_{\theta_j} v(\theta_j, s_i) f_{ji} \right] q_i,
\]
where the maximization is subject to
\[ \beta e^* \geq \beta e(\theta_i, s_j), \quad i = 1, 2, \quad j = 1, 2 \]  
\[ 1 \geq \frac{\phi}{\phi - 1} \sum_{i} c_i s_i q_i \]  
\[ u_i \leq \ln c_i \alpha_i c_i, \quad i = 1, 2 \]
and the following “upwards incentive compatibility” constraint:
\[ u_1 + \beta \sum_i v'(\theta_i, s_j) f_{ij} \geq u_2 - (\alpha_1 - \alpha_2) c_2 + \beta \sum_i v'(\theta_i, s_j) f_{ij}. \]  

The $u_i$ variables may be interpreted as the government's current utility in each state. The introduction of the $u_i$ variables and the relaxation implied by the constraint (29) helps to ensure that (P) is a concave program. The objective is continuous and sup-compact on the constraint set, so a solution exists. A comparison of this problem with that described in the proof of Lemma 5.2 reveals that this is a relaxed (and slightly redefined) version of a program that gives the first period elements of an optimal allocation-policy pair. Lemma 5.3 gives some initial characterization of (P).

**Lemma 5.3.** $v(\theta_i, s_i) = v^*, i = 1, 2, v(\theta_2, s_2) = v^*, v'(\theta_1, s_1) \leq v^*.$

Consider dropping constraint (30) in the above program. The solution to this relaxed problem is characterized by $c_i = (\phi - 1)/(\alpha_i \phi),$ and $v'(\theta_i, s_i) = v^*.$ This solution satisfies the upwards incentive compatibility constraint if and only if
\[ \left(\frac{\phi - 1}{\phi}\right) \begin{bmatrix} \alpha_1 - \alpha_2 \\ \alpha_2 \end{bmatrix} \geq \ln \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \]  
\[ (31) \]
Hence, for each pair $(\alpha_1, \alpha_2)$ there exists a $\phi > 1$ such that for $\phi \geq \phi$ the upward incentive compatibility constraint does not bind. For $\phi$ below $\phi$ it does.

**Lemma 5.4.** The solution to (P) is such that:
1. The constraint (28) binds.
2. The constraints (29) bind.
3. The solution satisfies the following downwards incentive compatibility constraint:
\[ u_2 + \beta \sum_j v'(\theta_j, s_j) f_{j2} \geq u_1 - (\alpha_2 - \alpha_1) c_1 + \beta \sum_j v'(\theta_j, s_j) f_{j2} \]  
\[ (32) \]
Comparison of Program (P) with the optimization used to obtain an optimal incentive compatible equilibrium in the proof of Lemma 5.2 reveals that (P) differs from the latter in the following regards: (a) the constraint $u_i = \ln c_i - x_i$ is relaxed, (b) the downwards incentive compatibility constraint is not imposed, (c) the restriction that $(\zeta'(\theta_j, s_i), \nu'(\theta_j, s_i)) \in \mathcal{V}$ is relaxed, and (d) the restriction that $E[\nu(1/c_i)] < Q$ is not imposed. As the previous lemma shows, the solution to program (P) satisfies (a) and (b). By (27) the continuation payoffs satisfy the upper bounds on payoffs implied by $\mathcal{V}$. The fact that (28) binds, coupled with the definitions of $\zeta$ and $\zeta'$ and Eqs. (9), (10), and (16), imply that program (P) is only consistent with values of $(\zeta'(\theta_j, s_i) = 1/\beta$ for each $(\theta_j, s_i)$. It follows that the solution to Program (P) will correspond to the first period of an optimal incentive period compatible payoff if the value of $Q$ is large enough in (26) and if the worst incentive compatible equilibrium payoff in $\mathcal{V}(1/\beta)$ is low enough.

**Proposition 5.4.** Suppose that the solution to Program (P) is consistent with the monetary policy bound (26) and has a continuation payoff in state $(s_2, \theta_1)$ that lies within $\mathcal{V}(1/\beta)$. Then the solution to program (P) corresponds to the first period of an optimal incentive compatible equilibrium.

It follows that if the conditions of Proposition 5.4 hold then, from the discussion preceding Lemma 5.4, the incentive constraints do not bind if $\phi \geq \bar{\phi}$. Furthermore, when they do bind it is clear that the optimal incentive compatible equilibrium relies on intertemporal incentives—reducing the government’s continuation payoff in state $(s_2, \theta_1)$. The remainder of this section considers the possibility that the first period of an optimal incentive compatible equilibrium does not permit the government to make any use of the signals it receives. In this case the optimal incentive equilibrium, like the Ramsey equilibrium, is stationary or repeated. The optimal incentive compatible equilibrium trades off the gain from making use of the signal against the cost of providing the incentives necessary to ensure incentive compatibility. Two factors are important in this trade-off: the value of the information that the signal contains and the extent of the product market distortion. Condition M below is useful in describing the former. Roughly speaking, the function $g$ can be thought of as a measure for describing how “close” the two signals are.

**Condition M.**

$$g(s_1, s_2) = \frac{f_{12}}{f_{11}} \left[ \frac{1}{\alpha_1 q_1} + \frac{1}{\alpha_2 q_2} \right] q_2 - \left( \frac{1}{\alpha_2} - \frac{1}{\alpha_1} \right) > 0.$$ 

The next result says that when the signals are close in the sense of condition (M) and $\phi$ is small enough (the product market distortion large
Lemma 5.5. Suppose that the conditions of Proposition 5.4 hold. Assume that condition (M) holds. Then there exists a \( \tilde{\phi} \in (1, \phi) \) such that for \( \phi \in (1, \tilde{\phi}) \) \( c_1 = c_2, v(\theta_1, s_2) = v^* \) and the government makes no use of its signal.

Proof. Fix \( \phi < \tilde{\phi} \). Since the conditions of Proposition 5.4 hold the solution to Program (P) corresponds to the first period of an incentive compatible equilibrium. Consider solving the program (P) subject to the additional constraint that \( c_1 = c_2 \) and \( v(\theta_1, s_2) = v^* \). Let \( c \) denote the current consumption that solves this problem. It may be verified that \( c \) is rising in \( \phi \) with \( c \downarrow 1 \). Consider adjusting this solution in a feasible way by reducing \( c_1 \) by \( e/(q_1 s_1) \), raising \( c_2 \) by \( e/(q_2 s_2) \) and resetting \( u_i = u_i(e) \equiv \ln c_i(e) - \sigma_i c_i(e) \), where \( c_i(e) \) denotes the \( i \)th state's current payoff associated with this choice. This raises the current expected payoff by approximately \( Au(e) \):

\[
Au(e) = \frac{1}{\sigma_1} \left( \frac{1}{c} \right) e + \frac{1}{\sigma_2} \left( \frac{1}{c} \right) e = \frac{1}{\sigma_1} \left( \frac{1}{c} - \frac{1}{\sigma_1} \right) e.
\]

In order to ensure incentive compatibility \( v(\theta_1, s_2) \) must be reduced by \( \delta(e)/\beta \), where:

\[
\left( \frac{1}{c} - \frac{1}{\sigma_1} \right) \frac{1}{\sigma_1 q_1} e + \left( \frac{1}{c} - \frac{1}{\sigma_2} \right) \frac{1}{\sigma_2 q_2} e \approx f_{11} \delta(e).
\]

Denote the altered continuation payoff in the \( i \)th state, \( j \)th shock state by \( v_{ij}'(e) \). Under condition (M): \( Au(e) - \delta(e) f_{12} q_2 < 0 \) for \( c \) small enough. Thus, there exists a \( \tilde{\phi} \in (1, \phi) \) such that for all \( \phi \in (1, \tilde{\phi}) \) the government’s payoff is reduced by this alteration. Suppose that \( \phi < \tilde{\phi} \). Now by the proof of Lemma 5.4 any solution to program (P) satisfies (28) with equality and entails \( c_2 \geq c_1 \). So if there is a feasible choice for the government that gives a strictly higher payoff than the solution to the more constrained problem (with \( c_1 = c_2 \)), then there must exist a choice that (1) satisfies (28) with equality, (2) satisfies \( c_2 > c > c_1 \), and (3) satisfies constraints (29) that also does so. Let \( c_{ij}' \) and \( u_i' \) denote, respectively, the \( (i,j) \)th state’s continuation payoff and the \( i \)th state’s current payoff associated with this choice. Suppose \( \phi \) and \( c \) are sufficiently small so that: (A) the adjusted choice \( \{ c_i(e), u_i(e), \{ c_{ij}'(e) \}_{j=1,2} \}_{i=1,2} \) gives a lower payoff than the more constrained choice described initially and (B) \( c_2 > c > c_1 \) > \( c_1 \) > \( c_2 \) > \( c_1 \). Since \( c, c' \), \( i = 1, 2 \) and \( c_i(e) = 1, 2 \) all satisfy (28) with equality \( c_i(e) = q c + (1 - q) c' \) each \( i, q \in [0, 1] \) and \( u_i(e) = u_i' = q u_i + (1 - q) u_i' \). Define \( v_{ij}' = q v_i' + (1 - q) v_{ij}' \) and \( v_{ij}' = q v_i' + (1 - q) v_{ij}' \). Now, the combination \( \{ u_i', c_i(e), \{ v_{ij}' \}_{j=1,2} \}_{i=1,2} \) gives a higher payoff...
than \( \{u_i, c, \{v^*_j\}_{i=1,2}\}_{i=1,2} \). Consider raising \( u_2 \) by \( \kappa \) and reducing \( v^*_2 \) by \( \kappa/(f(f_1)) \). This alteration raises the government’s payoff and leaves \( v^*_1, v^*_2 \) unchanged. Hence, \( \{u_i, c, \{v^*_j\}_{i=1,2}\}_{i=1,2} \) delivers a higher payoff than \( \{u_i, c, \{v^*_j\}_{i=1,2}\}_{i=1,2} \) and, hence, higher than \( \{u_i, c, \{v^*_j\}_{i=1,2}\}_{i=1,2} \). But this contradicts (A).

6. CREDIBLE INCENTIVE COMPATIBLE EQUILIBRIA

This section extends the analysis of earlier sections by dropping the assumption that the government can commit to implementing a particular (incentive compatible) competitive equilibrium. Instead it will be assumed that the government will only implement a continuation competitive equilibrium if it is in its interests to do so. To analyze the latter sort of government behavior the optimal policy problem will be recast in game theoretic terms and the notion of credibility will be developed as an equilibrium concept in a game. This requires some additional definitions and notation which are now provided.

As before, the notion of a public history of shocks and money growth rates will be used. In the previous section both the shock process \( \theta \) and the monetary policy were fixed and attention was then restricted to those public histories induced by the process and the policy. In the strategic environment it is necessary to work with the complete set of possible public histories induced by the fixed process \( \theta \) and any monetary policy. The \( t \)th period public history will be denoted \( h_t \in H \).

A strategy, \( g \), for the government will be identified with a monetary policy as defined in the previous sections. The following notation will be used: \( g_s = \{x_t\}_{t=s}^\infty \) and \( \sigma^g_s = \{x_t\}_{t=s}^\infty \). The latter will be called a continuation strategy. A market profile, \( \sigma^p \), in this section will be similar to the market process of the previous section except that the former will be defined for the larger set of public histories described above. A pair \( (\sigma^g, \sigma^p) \) will be called a strategy profile.

The credible, incentive compatible (CIC) equilibrium concept defined below restricts the government’s behavior to be consistent with the maximization of its preferences after all public histories. As before the government is assumed to be benevolent. Formally:

**Definition 6.3.** A credible incentive compatible (CIC) equilibrium is a market profile \( \sigma^p \) and a government strategy \( \sigma^g \) such that:

14 Hereafter the superscript \( \tau \) will be dropped from the history notation since the set of public histories no longer depends on the government’s policy choice.

15 We revert to using the bounds imposed before Section 5.
1. After any history $h_t \in H$, the monetary policy induced by $\sigma^g$ and the market process induced by $\sigma^p$ form an incentive compatible competitive equilibrium.

2. Let $(c, l, x)(\sigma^g, \sigma^p)$ denote the competitive allocation–policy pair induced by a strategy profile $(\sigma^g, \sigma^p)$. For all $h_t \in H$, and all continuation government strategies, $\sigma^g_{t-1}$,

$$V((c, l, x)(\sigma^g, \sigma^p) | h_t) \geq V((c, l, x)(\sigma^g_{t-1}, \sigma^g_t, \sigma^p) | h_t).$$  \hspace{1cm} (33)

Let $\mathcal{V}^c$ denote the set of CIC equilibrium payoffs ($v$) and initial period expected multipliers ($\lambda$) implied by the strategies that form a CIC equilibrium. Clearly $\mathcal{V}^c \subset \mathcal{V}$ since any CIC equilibrium must support an incentive compatible equilibrium along its equilibrium path. A CIC equilibrium $(\sigma^g, \sigma^p)$ will be said to be optimal if it induces an initial expected multiplier equal to $\lambda$ and if $V((c, l, x) \sigma^g, \sigma^p | h_0) = \max \{w : (\lambda, w) \in \mathcal{V}^c\}$. Similarly, the worst CIC equilibrium is that which minimizes the government’s payoff, i.e., delivers a payoff of $w = \inf \{w : (\lambda, w) \in \mathcal{V}^c\}$. The following lemma provides a condition under which the worst equilibrium is associated with maximal monetary growth and price inflation forever.

**Lemma 6.6.** Suppose that for all $s \in S$

$$\beta - 1 \frac{E \left( \frac{1}{\theta} \right)}{E \left( \frac{1}{\theta} \right)} (1 + \bar{x}) > 1;$$  \hspace{1cm} (34)

then in the worst CIC equilibrium $x_t(h_t) = \bar{x}$ and $p_t(h_t) = \beta \frac{\phi}{\bar{\theta}} E(\frac{1}{\theta}) (1 + \bar{x})^2$.

A CIC equilibrium has the feature that after any public history the induced continuation government strategy profile and the continuation market profile themselves form a CIC equilibrium. Hence, after any public history, $h_t$, the expected multiplier–payoff pair $(\lambda, v(h_t))$ associated with the induced continuation profiles also lies in $\mathcal{V}^c$. In this sense CIC equilibria are recursive. As in the earlier sections the APS approach is taken. To begin with the definition of admissibility is extended to include a “recursive credibility constraint”:

**Definition 6.4.** A three-tuple of functions, $\psi : S \to \mathcal{M}_{++}$, $\lambda : S \times \Theta \to A$, and $v' : S \times \Theta \to \mathcal{M}_{++}$, are said to be C-admissible w.r.t. a compact set $D \subset \mathcal{M}^2$ if they are 1-admissible w.r.t. $D$ and
\[
\sum_{\theta} \left[ \ln \left( \frac{1}{p\psi(s)} \right) - \frac{1}{\partial p\psi(s)} + \beta\ell' (s, \theta) \right] f(\theta | s) \\
\geq \max_{\{\theta, \psi \in \psi(s), \forall s\}} \min_{(\zeta, \psi) \in D} \sum_{\theta} \left[ \ln \left( \frac{1}{p\psi} \right) - \beta\ell = \beta\ell' (s, \theta) \right] f(\theta | s), \tag{35}
\]
where \( p \) is defined as in (24).

The new inequality of the C-admissibility definition can be interpreted as requiring that the government has no incentive to deviate to some arbitrary money growth rate that is not legitimized by any signal that the government might have received. More precisely, the inequality says that the government receives a higher payoff from implementing the prescribed monetary growth rate than from undertaking a recognizable deviation and then receiving the lowest feasible continuation payoff from \( D \).

The definition of C-admissibility can be used to construct an alternative \( B \)-operator that incorporates the recursive credibility constraint. For \( D \in \mathcal{K} \):

\[
B^c(D) = \left\{ (\zeta, v) \mid (\psi, \lambda, v') \text{ is C-admissible w.r.t. } D \right\}
\]

where \( \zeta = \sum_{\theta, s} \psi(s) f(\theta, s) \)

\[
v = \sum_{\theta, s} \left[ \ln \left( \frac{1}{p\psi(s)} \right) - \frac{1}{\partial p\psi(s)} + \beta\ell' (s, \theta) \right] f(\theta, s),
\]

where \( p \) is defined as in (24).

The pair \( (B^c, \varepsilon^c) \) may be shown to possess similar properties to the pair \( (B, \varepsilon) \). For completeness, we merely state that:

**Proposition 6.5.**

1. \( B^c \) is monotone and compactness preserving (i.e., \( B^c : \mathcal{K} \rightarrow \mathcal{K} \)).
2. \( \varepsilon^c = B^c(\varepsilon^c) \).
3. If \( \varepsilon^c \in W_0 \in \mathcal{K} \) then \( \lim_{n \rightarrow \infty} B^c(W_0) = \varepsilon^c \).

The set \( \varepsilon^c \) is of interest in itself, but additionally, it is valuable because it can be used to recover the allocations implemented by particular CIC equilibria. Proposition 6.5 is useful since it provides a basis for computing the set \( \varepsilon^c \). The main practical obstacle to implementing this procedure numerically concerns the representation of these sets on a computer. Judd, Yeltekin, and Conklin (JYC) [17] provide practical techniques for implementing such an iteration and these are applied below.
JYC recommend “convexifying” the game as outer and inner approximations to convex sets can be described by a finite vector of parameters and, thus, may be represented on a computer. An analogue of the JYC procedure is followed here. At a theoretical level this requires modifying the game described above to incorporate lotteries. The details are given in Sleet [25] which also gives precise definitions for $V^\circ$ and $B^\circ$, the analogues of $V^c$ and $B^c$ for this case. The basic idea, however, is that at the beginning of each period a lottery is held over a state space $\Omega$. Player strategies are then assumed to condition player choices on histories of lottery outcomes as well as histories of money growth rates and shocks. The lottery probabilities are themselves determined by a profile of functions that map histories into probability distributions on $\Omega$. These are common knowledge, as, of course, are the lottery outcomes. It may be shown that this formulation’s multiplier–value set $V^\circ = \text{convex hull}(V^c)$.

7. SOME NUMERICAL EXAMPLES

This section reports the results obtained from applying the numerical procedures of JYC to the CIC policy problem of the previous section. The computations undertaken are intended to be indicative of possible economic behaviors rather than precise exercises in calibration. The economy outlined in previous sections is described by the following vector of parameters: $\{\Theta, S, f, \beta, \bar{\xi}, \phi\}$. The examples will differ only in their $\phi$ values. The $\phi$ values examined are $\{1.05, 2.5, 20.0\}$. The value 2.5 will be taken as the baseline. The values of the other parameters are common across the two examples and are as follows:

1. The set of productivity shocks: $\Theta = \{1, 2\}$.
2. The set of signals: $S = \{0, 1\}$.
3. The kernel, $f = (0.8 \ 0.2 \ 0.2 \ 0.8)$.
4. The discount factor: $\beta = 0.945$.
5. The upper bound on money growth rates: $\bar{\xi} = 1.0$.

7.1. Multiplier–Value Sets

The graphs in Fig. 1 show multiplier–value sets for several alternative values of $\phi$. The first is drawn for the baseline value of $\phi$, 2.5. The second and third are drawn for values of 20 and 1.05 respectively. Several observations are immediate. First, as $\phi$ is reduced and the mark up increased, the set of equilibrium payoffs shifts downwards. This is unsurprising since reductions

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16 An inner approximation to a set is one contained within the set; an outer approximation is one that contains the set.
in the value of $\phi$ imply greater distortion in the product market. Second, the figures are consistent with the result of Lemma 6.6: when condition (34) holds the worst equilibrium is associated with maximal monetary growth. With the parameterization described above, maximal monetary growth corresponds to a value of $\xi$ of 0.5. The worst equilibrium $(\xi, v)$ pair is marked on the graph for $\phi = 2.5$. Third, the range of equilibrium payoffs (as measured by the gap between the maximum and minimum payoffs) is falling in $\phi$ (although the range of equilibrium outputs, consumptions, and labor supplies is rising). Finally, all of the diagrams have approximately horizontal upper surfaces. The upper surface is exactly horizontal for $\phi = 1.05$ (the third panel); the other values of $\phi$ admit a small amount of curvature. The latter stems from the use of the pointwise bounds in the analysis of CIC equilibria.

7.2. Policy Functions for Optimal CIC Equilibria

Figures 2-4 show the $\psi$-policy functions for the first period of an optimal CIC equilibrium.\(^{17}\) Recall that $\psi = 1/(1 + x)$ so that these figures also give the government's monetary growth rate choices in the first period of the

\(^{17}\) They are drawn under the assumption that a lottery is not used in the first period of the equilibrium.
FIG. 2. $\psi$ function: $\phi = 2.5$.

FIG. 3. $\psi$ function: $\phi = 20$.

FIG. 4. $\psi$ function: $\phi = 1.05$. Note: The curves lie on top of one another.
initial expected multiplier $\xi$ and the government’s signal $s$. The figures are drawn for different values of the parameter $\phi$. Figure 2 is drawn for $\phi = 2.5$. For a given $\xi$, this graph indicates that $\psi$ is set to a low value after a low realization of the signal and a high value otherwise. Thus, the government operates a signal contingent monetary policy, raising the rate of monetary growth when its information suggests that productivity is more likely to be high. As $\phi$ is increased to 20, the signal contingency appears to increase with greater variations in $\psi$ across signal realizations. On the other hand, when $\phi$ is reduced to 1.05, variations in $\psi$ are entirely eliminated and there is no signal contingency in monetary policy.

Figure 5 illustrates the continuation value functions $v'$ for $\phi = 2.5$. It shows that $v'$ takes the same value when the government receives a low signal (independent of the subsequent productivity shock) and when it receives a high signal followed by high productivity shock. However, $v'$ is reduced after the realization of a high signal and a low productivity shock. $v'$ takes a constant value independent of the realizations of shocks and signals when $\phi = 1.05$ or 20.

7.3. Discussion of Results

The above figures are consistent with the theoretical results obtained for optimal incentive compatible equilibria and with the intuitive idea that it is constrained optimal for a government to collect and use private information about the state of the economy only when its incentives to misuse that information are small. In an economy with $\phi = 1.05$, the distortion arising from the monopolistic sector is large and the government will face a strong temptation to exploit any signal contingency in monetary policy. While there may be CIC equilibria that do implement a signal contingent monetary policy, they must be such that the government faces future penalties for implementing high rates of money growth when, ex post,
economic circumstances do not appear to indicate that they were warranted. Since the government's signal is noisy, it is inevitable in such a regime that the government will make "mistakes" and that such costly penalties will be implemented even if the government does not deviate from its strategy. With $\phi = 1.05$ such costs dominate the gains from a signal contingent monetary policy and the optimal CIC equilibria considered above allows the government no discretion. Since there is no signal contingency in policy, no variation in continuation payoffs is required to ensure incentive compatibility.

As $\phi$ rises, however, the distortion arising from the imperfectly competitive goods sector is much reduced and more modest future penalties are sufficient to ensure that the government does not misrepresent its signal. Consequently, at $\phi = 2.5$, the government operates a signal contingent policy at all $\xi$ values except 0.5. In order to render this policy incentive compatible the government's continuation payoffs are reduced after the realization of a high valued signal (and consequent increase in the money growth rate) and a low valued productivity shock. Since low productivity shocks are more likely after low signals this discourages the government from misreporting the signal value.

As $\phi$ rises further a threshold is crossed above which the Ramsey equilibrium is incentive compatible. In particular, for $\phi = 20$ the solution to the optimal CIC equilibrium problem described above implements a signal contingent monetary policy that does not rely on penalties for high money growth-low productivity shock realizations. The government has no incentives to misrepresent its information.

7.4. Optimal Markov Equilibria

By the convexity of $\mathcal{Y}$ any point $(\xi, w) \in \mathcal{Y}$ is equal to some convex combination of the extreme points of $\mathcal{Y}$ (Rockafellar [22, p. 166]). Thus any initial expected multiplier-payoff pair $(\xi, w)$ can be obtained by holding a lottery over the extreme points of $\mathcal{Y}$, ext $\mathcal{Y}$, that delivers $(\xi, w)$ in expectation. In a different context, APS [2] provide conditions under which the optimal recursive equilibria of a repeated game necessarily induce a stochastic process over the extreme points of the equilibrium payoff set. These results suggest that the incentive compatibility of an optimal CIC equilibrium might be achieved by occasional visits to elements of ext $\mathcal{Y}$ that entail very low payoffs. More generally, it may be obtained by rare, but very adverse events. Thus these equilibria might explain the "episodic periods of breakdown" described in [7]. This idea is explored below in the more specialized context of optimal Markov equilibria.

A CIC equilibrium will be described as Markov if it induces a Markov process for multipliers and firm prices along its equilibrium path. Let $P$ be the set of prices induced by CIC equilibria, then:
Definition 7.5 (Markov Equilibrium).

1. Provided the government has not deviated from its strategy in any previous period, the government’s $t$-period optimal choice is described by a function $x$ that maps from $P \times S$ to $X$.

2. Provided the government has not deviated from its strategy in any previous period, the private sector profile is described by a group of functions that map from $P \times X \times \Theta$.

Thus, the equilibrium is Markov in the sense that the stochastic process that it induces over outcomes is Markov when the expected multiplier “promise” inherited from the previous period and the price set by firms at the beginning of the current period are treated as state variables. In this section, the focus will be on the optimal Markov equilibrium.\textsuperscript{18}

The optimal Markov equilibrium is a solution to the following recursive program. Define the defection value function $v^d$ as follows:

$$v^d(p, s) = \max_{(\hat{\psi}, \hat{\phi})} \min_{(\zeta, \delta, \beta)} \sum_{\theta} \left[ \ln \left( \frac{1}{p \psi(s)} \right) - \frac{1}{\theta \phi(s)} + \beta \hat{\psi} \right] f(\theta | s).$$

Then the optimal Markov payoff function $v$ solves:

$$v(\zeta, p) = \sup_{(\hat{\psi}, \hat{\phi})} \sum_{\theta} \left[ \ln \left( \frac{1}{p \psi(s)} \right) - \frac{1}{\theta \phi(s)} + \beta \hat{\psi} \left( \frac{\hat{\psi}(s, \theta)}{\hat{\phi}(s, \theta)}, p'(s, \theta) \right) \right] f(\theta, s)$$

subject to $\hat{\psi}(s, \theta)/\theta \phi(s, \theta) \in \bar{\Xi}$, $\hat{\psi}(s, \theta) \leq \psi(s)$, $\hat{\phi}(s, \theta) \in \bar{A}$, $\psi(s) \in [1/(1 + \xi), 1/\xi]$, $\xi = \sum_{s} \psi(s) f(\theta, s)$ and

$$\sum_{\theta} \left[ \ln \left( \frac{1}{p \psi(s)} \right) - \frac{1}{\theta \phi(s)} + \beta \hat{\psi} \left( \frac{\hat{\psi}(s, \theta)}{\hat{\phi}(s, \theta)}, p'(s, \theta) \right) \right] f(\theta | s)$$

\begin{align*}
\geq & \sum_{\theta} \left[ \ln \left( \frac{1}{p \psi(s)} \right) - \frac{1}{\theta \phi(s)} + \beta \hat{\psi} \left( \frac{\hat{\psi}(s, \theta)}{\hat{\phi}(s, \theta)}, p'(s, \theta) \right) \right] f(\theta | s) \\
&\quad \text{subject to } \hat{\phi}(s, \theta) \in \bar{A}, \hat{\psi}(s, \theta) \leq \psi(s), \hat{\psi}(s, \theta) \in [1/(1 + \xi), 1/\xi], \xi = \sum_{s} \psi(s) f(\theta, s). 
\end{align*}

\textsuperscript{18} Atkeson [3] applies the techniques of APS [2] to analyze a game played between lenders and sovereign borrowers. He shows in this context that the optimal CIC equilibrium payoff is obtained by an optimal Markov equilibrium. This result relies on there being some physical state in which the optimal CIC equilibrium payoff is very low (perhaps, coinciding with the worst CIC equilibrium payoff). As the earlier results indicate, this is not the case here. In general, therefore, the optimal Markov equilibrium will be inferior to the optimal CIC equilibrium.
where $p$ is defined as in (24), and
\[
\sum_{\theta} \left[ \ln \left( \frac{1}{\hat{p}(s)} \right) - \frac{1}{\hat{p}(s)} \right] + \beta \mathbb{E} \left[ \frac{\hat{\xi}(s, \theta)}{\theta \hat{p}(s, \theta)} \cdot p'(s, \theta) \right] f(\theta | s) \geq \psi'(p, s).
\]
These conditions are analogous to those for their optimal CIC equilibrium counterparts.

7.5. Numerical Illustration

As in the earlier analysis of general CIC equilibria, public lotteries are introduced. These convexify the problem and facilitate computation. Unlike earlier sections the problem is discretized and reformulated as a linear programming problem. The numerical parameters used in the analysis of CIC equilibria above are used here. In particular, $\phi$ is set to 2.5.

Figure 6 shows the policy function for the $\psi$ variable. The two graphs in the figure are drawn for the low and high values of the signal respectively. In each graph $\psi$ is drawn as a function of the price and the expected multiplier. As the figure illustrates $\psi$ is set to a lower value contingent on the high realization of the signal.

The final figures show the results of a simulation of the optimal Markov equilibrium. The first panel of Fig. 7 shows the realized path for $\psi = 1/\xi + x$, the second the path for the expected multiplier $\xi$ and the third the path for the price variable. These graphs illustrate that the price variable is usually roughly constant. Occasionally, however, there are large upward jumps in prices and corresponding large upward jumps in expected money growth (as indicated by the downward jumps in $\xi$). These represent punishment phases that occur with some positive probability after high monetary growth–low productivity shock realizations. Figure 8 shows histograms associated with

\[\text{CREDIBLE MONETARY POLICY}\]

19 Such an approach is standard in the dynamic contracting literature; see, for example, Phelan and Townsend [21].
FIG. 7. Simulation paths.

FIG. 8. Simulation histograms.
the realization illustrated in Fig. 7. These again indicate (Panel 4) that the price variable is constant and low for much of the time, but occasionally jumps upwards during punishment phases. The $\psi$ variable has two large spikes (Panel 1). The higher value is associated with low signal realizations. Low signal realizations result in a low monetary growth (high $\psi$) unless the economy is in a punishment phase. The lower valued large spike is associated with high signal-high shock realizations. The probability mass at still lower values is associated with punishment phases that are initiated with a high signal-low shock occurrence.

8. CONCLUDING COMMENTS

This paper has considered environments in which the government has some private information about the appropriateness of different monetary policies. In general the government may have incentives to misrepresent its information. In the model developed here the strength of these incentives depends on the extent of a distortion in the product market (which can be mitigated through an unexpected monetary expansion). When the incentives to misrepresent are large optimal CIC monetary policy may involve little or no conditioning of policy on the government's private information. Historically, macroeconomists have been interested in the issue of “macroeconomic rules” versus “macroeconomic discretion.” Recasting this language in terms of the current model, when the government's incentives to misrepresent information are high it is given little discretion in the optimal CIC equilibrium. Here “discretion” refers to the discretion of the government to condition its policy on its private information without automatic consignment to a low continuation payoff equilibrium.

When the government’s incentives to misrepresent information are smaller the optimal CIC equilibrium involves some amount of discretion in the sense defined above. However, in this case, as long as there is some temptation to misrepresent, penalties must be imposed contingent on those combinations of outcomes that are more likely if misrepresentation has occurred. One way that these (optimal) penalties may be introduced is through rare, but rather drastic adverse penalty phases. This was illustrated via a simulation of an optimal Markov policy equilibrium which had these features. There are, of course, many credible equilibria. Chari et al. [9] used this degree of freedom to select equilibria that appeared to correspond historical episodes. In particular, they focused on equilibria that were consistent with the high-inflation period of the 1970's. In the analysis described here equilibria are selected to satisfy certain optimality properties rather than to correspond to historical episodes. Nevertheless, these equilibria suggest an alternative explanation for high-inflation episodes. They might be optimal penalty phases in an
economy with private government information. APS [2], in a different context, provide conditions under which optimal recursive equilibria in private information games necessarily jump between the extreme points of an equilibrium payoff set. APS refer to such equilibria as “bang–bang.” It remains to be determined whether there are conditions under which optimal credible equilibria in private information macroeconomic policy games are necessarily bang-bang as well.

As developed, the model investigates the relationship between the parameters that define the preferences and technologies of agents and the set of credible, incentive compatible monetary policies. The focus has been on the relationship between the preference parameter \( \phi \) and this set. These relationships are suggestive of positive policy choices that might lead to welfare improvements. For example, any policy that increased the competitiveness of the goods market would tend to reduce the extent of the incentive compatibility problem thus permitting equilibria that allow the government to make better use of its private information. Any policy that reduced the rate at which the government discounted the future would also tend to have beneficial implications. It is well known that in full information environments patient governments can obtain a larger set of credible outcomes. The same result holds in private information environments of the sort analyzed here. This has implications for the design of policy making institutions; e.g., it might rationalize the lengthy terms enjoyed by Federal Reserve governors.\(^{20}\) Further investigation of the relationship between the design of policy making institutions and the nature of credible, incentive compatible policy is left for future work.

APPENDIX: PROOFS

Proof of Lemma 4.1. Let \((c, l, x)\) be an allocation–monetary policy pair in \(\mathcal{E}'(\chi)\) and let \((c, l, x)(\hat{h}_t^x)\) be as defined in the lemma. By definition \((c, l, x)\) is supported by some competitive equilibrium with an associated multiplier sequence. The zeroth period elements of \((c, l, x)\) and the first pair of elements of the multiplier sequence satisfy the conditions in (1) of the lemma. It is straightforward to verify that \((c, l, x)(\hat{h}_t^x)\) is a competitive allocation-monetary policy pair. Equations (11) and (16) evaluated at \(\hat{h}_t^x\) then imply that \((c, l, x)(\hat{h}_t^x)\in\mathcal{E}'(\hat{\lambda}_0/\beta \psi_0, \hat{h}_t^x)\) each \(\hat{h}_t^x\). Let \(x'\) denote a monetary policy publicly indistinguishable from \(x\). Suppose that after some \(\hat{h}_t^x\)

\[
E\left( \sum_{t=1}^{\infty} \beta^{t-1} u(c_t, l_t) \bigg| (c, l, x)(\hat{h}_t^x) \right) < E\left( \sum_{t=1}^{\infty} \beta^{t-1} u(c_t, l_t) \bigg| (c, l, x')(\hat{h}_t^x) \right).
\]

(36)

\(^{20}\) I thank an anonymous referee for this point.
But then the monetary policy \( x'' \), defined as follows:

\[
x''(h'^*_t) = \begin{cases} 
    x'_t(h'^*_t) & \text{if } h'^*_t = (h^*_{t}, \ldots, h^*_{t-1}, x_{t-1}, x_t) \\
    x_t(h'^*_t) & \text{otherwise},
\end{cases}
\]

gives a higher payoff to the government than \( x \) and is publicly indistinguishable from \( x \). This violates the incentive compatibility condition (21), leading to a contradiction. Hence, \((c, l, x)(h^*_{t}) \in \mathcal{E}(\lambda_0, \mu_0(h^*_{t}))\). Finally, if after some \( s_0 \),

\[
\sum_{\theta} u(c_0, x_0(s_0), \theta), l_0(x_0(s_0), \theta)) + \beta v_0(x_0(s_0), \theta) f(\theta | s_0)
\]

\[
< \sum_{\theta} u(c_0, x'_0(s'), \theta), l_0(x'_0(s'), \theta)) + \beta v_0(x'_0(s'), \theta) f(\theta | s_0)
\]

(37)

for some \( x'_0 \) indistinguishable from \( x_0 \), then the monetary policy:

\[
x''(h'^*_t) = \begin{cases} 
    x'_t(h'^*_t) & \text{if } h'^*_t = (s_0) \\
    x_t(h'^*_t) & \text{otherwise}
\end{cases}
\]

is publicly indistinguishable from and gives a higher payoff than \( x \). This also contradicts the incentive compatibility of \((c, l, x)\).

The converse is now proved. The definition of the functions \( \lambda_0 \) and \( \mu_0 \), the conditions in part 1 of the Lemma and the fact that

\[
(c, l, x)(h^*_{t}) \in \mathcal{E}(\lambda_0, \mu_0(h^*_{t}))
\]

all \( h^*_{t} \) implies that there exists a multiplier sequence consistent with \((c, l, x)\), such that (16) holds for all \( t \) and such that \( E\Psi_0 = \zeta \). The other elements of a competitive equilibrium that supports \((c, l, x)\) can be recovered from this sequence as described in the text. Hence, \((c, l, x) \in \mathcal{E}(\zeta)\). It remains to establish that \((c, l, x) \) is incentive compatible. Suppose it was not, then for some \( x' \) publicly indistinguishable from \( x \):

\[
E\left( \sum_{i=0}^{\infty} \beta^{i-1} u(c_{i}, l_{i}) \bigg| c, l, x \right)
= E(u(c_0, l_0) | c_0, l_0, x_0)
\]

\[
+ \beta E\left( E\left( \sum_{i=1}^{\infty} \beta^{i-1} u(c_{i}, l_{i}) \bigg| (c, l, x)(h^*_{t}) \right) \bigg| c_0, l_0, x_0 \right)
\]

\[
< E(u(c_0, l_0) | c_0, l_0, x'_0)
\]

\[
+ \beta E\left( E\left( \sum_{i=1}^{\infty} \beta^{i-1} u(c_{i}, l_{i}) \bigg| (c, l, x')(h^*_{t}) \right) \bigg| c_0, l_0, x'_0 \right).
\]
But since \((c, l, x)(h_t^\ast)\) is incentive compatible for each \(h_t^\ast\),
\[
E\left(\sum_{t=1}^\omega \beta^{-1}u(c_t, l_t) \right | (c, l, x)(h_t^\ast)) \geq E\left(\sum_{t=1}^\omega \beta^{-1}u(c_t, l_t) \right | (c, l, x')(h_t^\ast)).
\]
But this fact coupled with the temporary incentive compatibility constraint at 0 implies that \((c, l, x)\) is incentive compatible, which is a contradiction.

**Proof of Proposition 4.3.** The proof is obtained by making a few small alterations to arguments in Abreu et al. \[2\].

**Proof of Lemma 5.2.** It is straightforward to extend the proof of Lemma 4.1 to the case of the altered monetary policy bounds considered in Section 5. By Lemma 4.1, if \((v, \xi)\) is in \(\mathcal{Y}'\), then there is some incentive compatible allocation–policy pair with initial payoff \(v\) and initial multiplier \(\xi\) and continuation payoff and multiplier functions \(v'\) and \(\xi'\) such that for every \(h_t^\ast\) \((v', \xi') \in \mathcal{Y}'\). Consider the optimization
\[
\sup_{i, j} \sum_i \{ [\ln c_j - c_j/\theta_i] + \beta v'_{ij} \} f_q i,
\]
subject to: \((\xi_{i, j}', v_{i, j}') \in \mathcal{Y}'\), \(\sum c_i q_i, \sum (1/c_i) q_i \leq Q\), and for all \((j, k)\)
\[
\sum_i \{ [\ln c_j - c_j/\theta_i] + \beta v'_{ij} \} f_q i \geq \sum_i \{ [\ln c_k - c_k/\theta_i] + \beta v'_{ik} \} f_q i,
\]
(38)
and \(c_i \geq 0\) all \(i\). It may be shown (see the working paper version of this paper) that a solution exists to this problem.

Denote such a solution by \(\{c_i, \{v_{i, j}', \xi_{i, j}'\}\}\) and its payoff by \(w^*\). Note that \(w^* \geq v^* = \max\{v: (v, \xi) \in \mathcal{Y}\}\). This follows from the fact that, by Lemma 4.1, any competitive equilibrium that attains \(v^*\) must satisfy the constraints of the above optimization. Select \(\xi \in \mathcal{E}\) arbitrarily, we now show that there exists a \((c, l, x) \in \mathcal{E}'(\xi')\) that attains the payoff \(w^*\). Take the solution to the previous optimization and set \(p\) as follows:
\[
p = \frac{1}{\xi} \sum_q \left( \frac{\psi_{q}}{c_q} \right) q_i.
\]
Set \(l_{i, j} = c_i/\theta_i, 1 + x_i = p c_i, \psi_j = 1/(1 + x_i), \lambda_{i, j} = \beta \psi_j \xi(i, j)\). Note that \(E(1/(1 + x)) = \xi\) and so satisfies the bounds on \(E(1/(1 + x))\) assumed in the discussion of monetary policies. It is easy to check that \(1 + x_i \geq 0\) each \(i\). Also \(E(1 + x_i) E(1/1 + x_i) = Ec, E(1/c_i) \leq Q\). Since \(v_{i, j}' \in \mathcal{E}'(\xi_{i, j}'\)) there exists
a continuation allocation-policy pair supported by some equilibrium that gives a payoff of \( v_{l,j} \) and is consistent with \( \xi_{l,j} \). Thus, by Lemma 4.1 \( \{c_i, l_i, j, s_i\}_{l,i} \) form the initial choices of an allocation-policy pair.

**Proof of Lemma 5.3.** The first order conditions for \( v(\theta_i, s_i) \) are: \( f_{i1} q_1 - \xi_{i,1} + \eta f_{i1} = 0 \) where \( \xi_{i,1} \) is the multiplier on the relevant bound (27) and \( \eta \) is the multiplier on the incentive compatibility constraint (30). The non-negativity of \( \eta \) implies \( \xi_{i,1} > 0 \) so \( v(\theta_i, s_i) = v^* \). The first order conditions for \( v(\theta_i, s_2) \) are \( f_{i2} q_2 - \xi_{i,2} - \eta f_{i1} = 0 \). The result then follows from the fact that \( f_{i2}/f_{i1} > f_{i2}/f_{i1} \).

**Proof of Lemma 5.4.** Suppose the constraint (28) does not bind. Then the first order conditions for \( c_1 \) and \( c_2 \) imply that \( c_1 > c_2 \) where \( c_1^* = 1/\alpha_1 \). This, however, violates (28). The first order condition for \( u_i \) is: \( (q_1 + \eta) - \theta_i = 0 \), where \( \eta \) is the Lagrange multiplier on constraint (30) and \( \theta_i \) is the multiplier on constraint (29). Since \( q_1 > 0 \), it follows that \( \theta_i > 0 \) and constraint (29) binds. Let \( \theta_2 \) be the multiplier on constraint (29). If this constraint does not bind, the first order condition for \( u_2 \) implies that \( q_2 - \eta = 0 \). Hence, \( \eta > 0 \) and the incentive compatibility constraint must bind. The first order conditions for \( v(\theta_i, s_2) \), \( i = 1, 2 \) are \( f_{i2} q_2 - \xi_{i,2} - \eta f_{i1} = 0 \) where \( \xi_{i,2} \), \( i = 1, 2 \) are the multipliers on the constraints (27). But since \( \eta = q_j > (f_{i2}/f_{i1}) q_2 \), this implies that the problem has no solution and that the value of the supremum is \( -\infty \). This is a contradiction.

Suppose that \( c_1 > c_2 \). By the previous result, \( u_i = \ln c_i - \alpha_i c_i \). Then consider altering the solution by replacing \( c_1 \) and \( c_2 \) with \( c = \sum_{i=1,2} q_i c_i \), setting \( v(\theta_i, s_i) = v^* \) and \( u_i = \ln c - \alpha_i c_i \). Since \( c_1 > c_2 \) and \( c_1 > c_2 \) this alteration satisfies constraint (28). It is also incentive compatible. Additionally, \( \sum_i q_i (\ln c_i - \alpha_i c_i) < \ln c - \sum_{i=1,2} q_i \alpha_i c_i \). So this alternative is feasible and delivers a higher payoff to the government. It follows that \( c_1 < c_2 \).

It is easy to verify that if the upward constraint does not bind the solution automatically satisfies the downward incentive compatibility constraint. Suppose the upward constraint does bind. Then, by Lemma 5.3, and the fact that (29) binds

\[
\ln c_1 - \alpha_1 c_1 + \beta v^* = \ln c_2 - \alpha_2 c_2 + \beta v^* f_{i1} + \beta v(\theta_i, s_2) f_{i1}, \tag{39}
\]

Combining (29), (39), and (32) and rearranging gives

\[
[1 - \frac{f_{i2}}{f_{i1}}] (\ln c_2 - \alpha_2 c_2) + \frac{f_{i2}}{f_{i1}} (\alpha_1 - \alpha_2) c_2
\]

\[\geq [1 - \frac{f_{i2}}{f_{i1}}] (\ln c_1 - \alpha_2 c_1) + \frac{f_{i2}}{f_{i1}} (\alpha_1 - \alpha_2) c_1.\]
Suppose this constraint does not hold at the solution. Then since $f_{12}/f_{11} \leq 1$ and, by an argument given above, $c_2 \geq c_1$, so the failure of the previous inequality implies $\ln c_1 - c_2 c_1 > \ln c_2 - c_2 c_2$. But then consider setting $c_2$ equal to $c_1$ and $v'(\theta, s) = v^*$ in each state. This is feasible and delivers a higher payoff to the government contradicting the optimality of the original solution.

**Proof of Lemma 6.6.** Consider the following policy functions for the government and the private sector players:

1. The government sets $x_t(h_t) = \bar{x} \forall t, h_t$.
2. Firms set prices equal to $\bar{p} = \beta \bar{x} + E(1 + \bar{x})^2$.
3. Households choose consumption equal to $(1 + \bar{x})/\bar{p}$ and labor supply equal to $(1 + \bar{x})/(\partial \bar{p})$.

These choices are consistent with a competitive equilibrium after all histories. This competitive equilibrium is characterized by a multiplier sequence in which: $\psi_t = 1/1 + \bar{x}$ and $\lambda_t = \beta/(1 + \bar{x})^2$. They are also consistent with a CIC equilibrium. After a realization of the signal $s$, the money growth rate that maximizes the current payoff of the government is

$$1 + x^*(s) = \frac{\bar{p}}{E(1/\bar{p})} = \beta \frac{\phi}{\phi - 1} \frac{E(1/\bar{p})}{E(1/\bar{p})} (1 + \bar{x})^2 > 1 + \bar{x},$$

where the last inequality stems from the assumption in the proposition. Given the assumption and given that firms set a price equal to $\bar{p}$, the government’s current payoff is strictly increasing in $x < \bar{x}$ for all realizations of $s$. Thus the government cannot raise its current payoff by lowering the rate of monetary growth below $\bar{x}$. Nor can it raise its payoffs in the future by lowering its current rate of monetary growth. Thus no one-shot deviation is optimal. Hence (Abreu [1]) it is optimal for the government to set $x(h_t) = \bar{x}, \forall t, h_t$. Define $g$ as follows:

$$g(p, x) = \max_{s | (x(s))} \sum_{\theta, s} \left[ \ln \left( \frac{1 + x(s)}{p} \right) - \frac{1 + x(s)}{\partial p} \right] f(\theta, s).$$

Let $P$ be the set of prices possible in the CIC equilibrium. Note that $\bar{p}$ is its largest element. Also $g^*(p) = \max_{x(s) \in [\bar{x} - 1, \bar{x}]} g(p, x)$ is non-increasing in $p$ so that the solution to $\min_{p \in P} \max_{x(s) \in [\bar{x} - 1, \bar{x}]} g(p, x)$ entails $p = \bar{p}$. 

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and $x(s) = \bar{x}, \forall s$. So that the equilibrium described previously attains this minmax value. But the worst CIC equilibrium payoff, $w$, satisfies:

$$w \geq \min_{p \in [p, \bar{p}]} \max_{x(s) \in \{x_{-1}, x_s\}} g(p, x) + \beta w.$$  \hspace{1cm} (53)

Hence, $w = \min_{p \in [p, \bar{p}]} \max_{x(s) \in \{x_{-1}, x_s\}} g(p, x)/(1 - \beta)$.

REFERENCES

25. C. Sleet, On credible monetary policy with private information, mimeo, University of Texas at Austin, 1999.