Instrumental Variable Models for Discrete Outcomes

Department Seminar: UIUC Economics Department

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CeMMAP & UCL

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Single equation IV model for discrete data

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h weakly **monotonic** in U, **non-decreasing**.

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  - Extensions/applications.

Threshold crossing representation

- $Y \in \{0, 1, \ldots, M\}$ determined by X and $U \sim Unif(0, 1)$:

$$Y = h(X, U) \qquad h \uparrow U \qquad U \perp\!\!\!\perp Z$$

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- Threshold crossing representation. Consider some h_0.

$$h_0(x,u) = \begin{cases} 0 & , \quad 0 < u \le p_0^0(x) \\ 1 & , \quad p_0^0(x) < u \le p_1^0(x) \\ \vdots & \vdots \quad \vdots \quad \vdots \quad \vdots \\ M & , \quad p_{M-1}^0(x) < u \le 1 \end{cases}$$

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- Consider a structure $S_0 \equiv \{h_0, F^0_{UX|Z}\}$ with

$$F^0_{UX|Z}(u, x|z) \equiv \Pr[U \leq u \cap X \leq x | Z = z]$$

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- Consider a structure $S_0 \equiv \{h_0, F_{UX|Z}^0\}$ with

$$F_{UX|Z}^0(u, x|z) \equiv \Pr[U \leq u \cap X \leq x | Z = z]$$

- It determines a distribution function of Y and X given Z

$$F_{YX|Z}^0(m, x|z) = F_{UX|Z}^0(p_m^0(x), x|z)$$

Observational equivalence

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- The model admits **observationally equivalent** $S_* \neq S_0$ with:

$$F^0_{YX|Z}(m,x|z) = F^0_{UX|Z}(p_m^0(x),x|z) = F^*_{UX|Z}(p_m^*(x),x|z)$$

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- $U \perp\!\!\!\perp Z$ limits adjustment of the U and X arguments of admissible $F_{UX|Z}$ because for all τ, z

$$F_{UX|Z}(\tau, \infty|z) \equiv F_{U|Z}(\tau|z) = F_U(\tau) = \tau$$

Some related results:

- **Continuous outcomes**: Chernozhukov and Hansen (2005) and related papers.

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- **Triangular models**: structural equation for (continuous) X:

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Chesher (2003, 2005), Imbens &Newey (2003).

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- **Simultaneous models**: "single equation" analysis of Tamer's (2003) entry game.

$$Y_1^* = \alpha_1 Y_2 + Z\beta_1 + \varepsilon_1 \qquad Y_2^* = \alpha_2 Y_1 + Z\beta_2 + \varepsilon_2$$

$$Y_1 = 1[Y_1^* \geq 0] \qquad Y_2 = 1[Y_2^* \geq 0] \qquad (\varepsilon_1, \varepsilon_2) \perp\!\!\!\perp Z$$

The single equation IV model: inequalities

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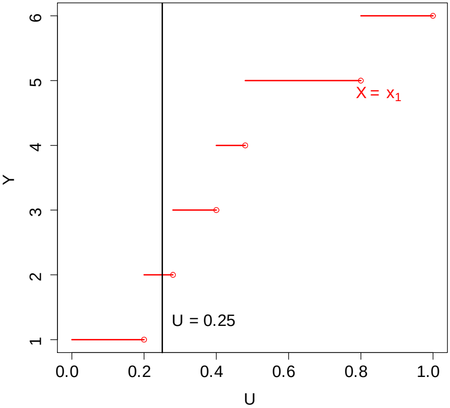
- These characterise the identified set.

For all x, $P[h(X,U) \leq h(X,0.25)|x,z] \geq P[U \leq 0.25|x,z]$



Averaging over X: $P[Y \leq h(X,0.25)|z] \geq 0.25$

Results concerning the identified set

- $\mathcal{H}_0$ is the set of structural functions, h, in admissible structures **observationally equivalent** to $S_0 \equiv \{h_0, F^0_{UX|Z}\}$:

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- **(A)**: If $h_* \in \mathcal{H}_0$ then for all $\tau \in (0,1)$ and $z \in \Omega$:

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- **Proof**:
  - If h_* is in an admissible structure delivering $F^*_{YX|Z}$ then for all $\tau \in (0,1)$ and $z \in \Omega$

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  - If S_* and S_0 are observationally equivalent $F^*_{YX|Z} = F^0_{YX|Z}$.

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fails to hold then $h_* \notin \mathcal{H}_0$.

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- **Proof**: by contradiction.

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- **(C)**. Sharpness. If for all $\tau \in (0,1)$ and $z \in \Omega$:

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then there exists a distribution function $F^*_{UX|Z}$ such that $S_* \equiv \{h_*, F^*_{UX|Z}\}$ is admissible and generates $F^*_{YX|Z} = F^0_{YX|Z}$ for all $z \in \Omega$.

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- **Proof**: constructive - see Annex of the paper.

Binary Y and discrete X

- Binary Y delivered by:

$$Y = \begin{cases} 0 \quad , \quad 0 < U \leq p(X) \\ 1 \quad , \quad p(X) < U < 1 \end{cases} \qquad U \perp\!\!\!\perp Z \qquad X \in \{x_1, \ldots, x_K\}$$

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- For $k \in \{1, \ldots, K\}$ data are informative about:

$$\alpha_k(z) \equiv P[Y = 0 | X = x_k, Z = z] \qquad \beta_k(z) \equiv P[X = x_k | Z = z]$$

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- What is the set defined by

$$\left\{ h : \begin{pmatrix} P[Y \leq h(X,\tau) | Z = z] \geq \tau \\ P[Y < h(X,\tau) | Z = z] < \tau \end{pmatrix} \qquad \forall \quad \tau \in (0,1), \quad z \in \Omega \right\}$$

in this case?

The identified set

- The (proposed) order of $\theta_1, \ldots, \theta_K$ is important. There are $K!$ orderings. Suppose

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- For j such that $\theta_j < \tau \leq \theta_{j+1}$, the event $\{p(X) < \tau\}$ occurs iff $X \in \{x_1, \ldots, x_j\}$ so

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$$P[Y < h(X, \tau)|Z = z] = \sum_{k=1}^{j} \alpha_k(z)\beta_k(z)$$
- This is less than τ for **all** $\tau \in (\theta_j, \theta_{j+1}]$ only if
$$\sum_{k=1}^{j} \alpha_k(z)\beta_k(z) \leq \theta_j.$$

The identified set

- A similar argument for the event $\{Y \leq h(X,\tau)\}$ delivers

$$P[Y \leq h(X,\tau)|Z=z] = \sum_{k=1}^{j} \beta_k(z) + \sum_{k=j+1}^{K} \alpha_k(z)\beta_k(z) \geq \tau$$

for $\tau \in (\theta_j, \theta_{j+1}]$ and so

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$$P[Y \leq h(X,\tau)|Z=z] = \sum_{k=1}^{j} \beta_k(z) + \sum_{k=j+1}^{K} \alpha_k(z)\beta_k(z) \geq \tau$$

for $\tau \in (\theta_j, \theta_{j+1}]$ and so

$$\sum_{k=1}^{j} \beta_k(z) + \sum_{k=j+1}^{K} \alpha_k(z)\beta_k(z) \geq \theta_{j+1}$$

- Combining, for $j \in \{1,\ldots,K\}$

$$\sum_{k=1}^{j} \alpha_k(z)\beta_k(z) \leq \theta_j \leq \sum_{k=1}^{j-1} \beta_k(z) + \sum_{k=j}^{K} \alpha_k(z)\beta_k(z)$$

The identified set

- These must hold for all $z \in \Omega$, so for $j \in \{1, \ldots, K\}$

$$\max_{z\in\Omega}\left(\sum_{k=1}^{j}\alpha_k(z)\beta_k(z)\right) \leq \theta_j \leq \min_{z\in\Omega}\left(\sum_{k=1}^{j-1}\beta_k(z) + \sum_{k=j}^{K}\alpha_k(z)\beta_k(z)\right)$$

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- Exchange of indices gives the result for other orderings.

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- The identified set is up to $K!$ boxes in the K dimensional unit cube.

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- Monotonicity (smoothness) restriction on effect of X is **very** informative.

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- With $K = 2$ for each $z \in \Omega$, (but drop z from notation)

$$\begin{aligned} \alpha_1\beta_1 &\leq \theta_1 \leq \alpha_1\beta_1 + \alpha_2\beta_2 \\ \alpha_1\beta_1 + \alpha_2\beta_2 &\leq \theta_2 \leq \beta_1 + \alpha_2\beta_2 \end{aligned}$$

The identified set

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- Swapping indexes.

$$\begin{aligned} \alpha_2\beta_2 &\leq \theta_2 \leq \alpha_2\beta_2 + \alpha_1\beta_1 \\ \alpha_2\beta_2 + \alpha_1\beta_1 &\leq \theta_1 \leq \beta_2 + \alpha_1\beta_1 \end{aligned}$$

Calculations for a binary Y binary X example

- Here is a process for $Y \in \{0, 1\}$ and $X \in \{0, 1\}$

$$\begin{aligned} Y^* &= \alpha_0 + \alpha_1 X + \varepsilon \\ X^* &= \beta_0 + \beta_1 Z + \eta \end{aligned}$$

$$Y = 1[Y^* > 0] \qquad X = 1[X^* > 0] \qquad \begin{bmatrix} \varepsilon \\ \eta \end{bmatrix} | Z \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \right)$$

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- Here is an **IV model**

$$Y = \begin{cases} 0 \quad , & 0 < U \le p(X) \\ 1 \quad , & p(X) < U \le 1 \end{cases} \qquad Z \perp\!\!\!\perp U \sim Unif(0,1)$$

Consider identification of $p(0)$, and $p(1)$.

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Consider identification of $p(0)$, and $p(1)$.

- The IV model is correctly specified:

$$Z \perp\!\!\!\perp U \equiv \Phi(\varepsilon) \sim Unif(0,1)$$

$$Y = \begin{cases} 0 \quad , & 0 < U \leq \Phi(-\alpha_0 - \alpha_1 X) \\ 1 \quad , & \Phi(-\alpha_0 - \alpha_1 X) < U \leq 1 \end{cases}$$

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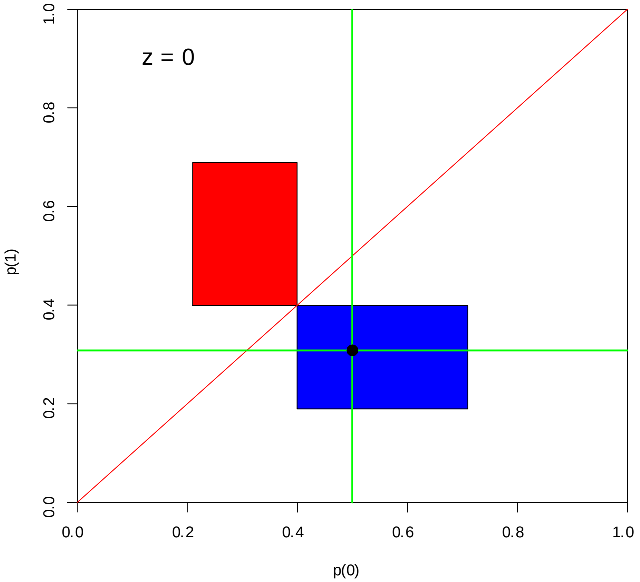
- Consider the case with $\rho = -0.25$ and

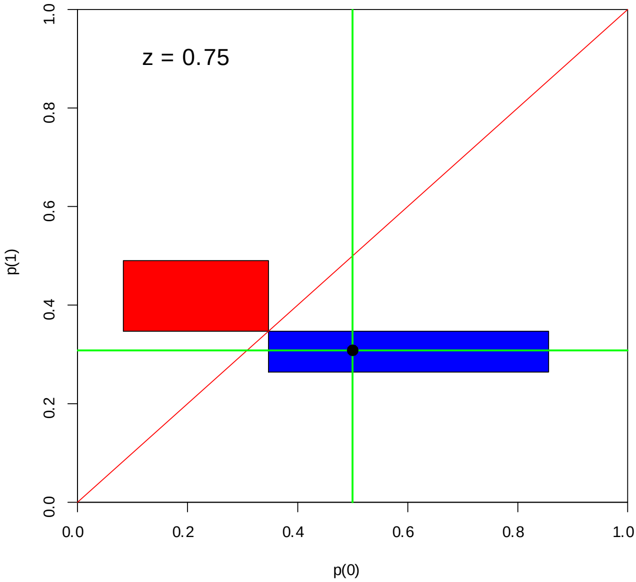
$$\alpha_0 = 0 \qquad \alpha_1 = 0.5$$

$$\beta_0 = 0 \qquad \beta_1 = 1$$

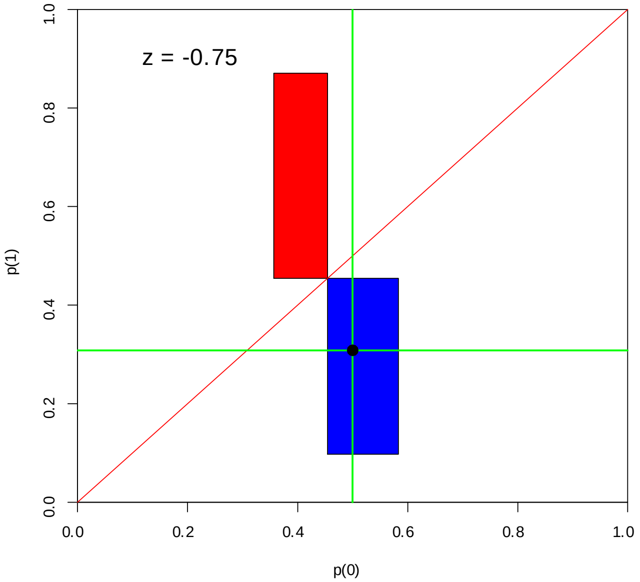
for which

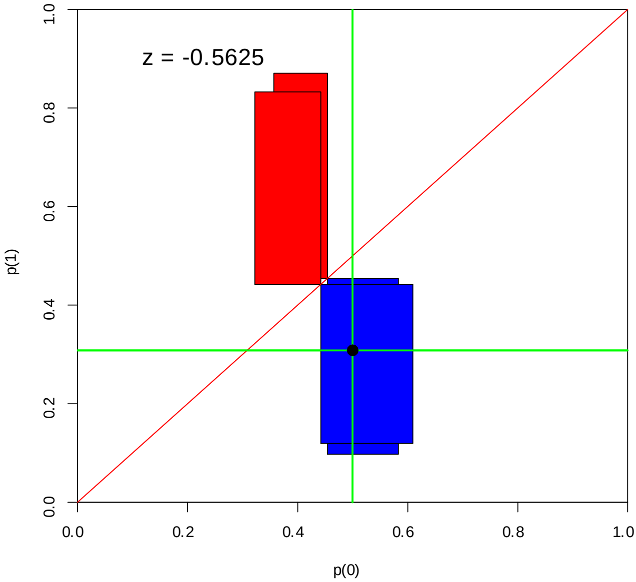
$$\begin{aligned} p(0) &= \Phi(-\alpha_0) = 0.5 \\ p(1) &= \Phi(-\alpha_0 - \alpha_1) = 0.308 \end{aligned}$$


z = 0
p(1)
p(0)
0.0
0.2
0.4
0.6
0.8
1.0

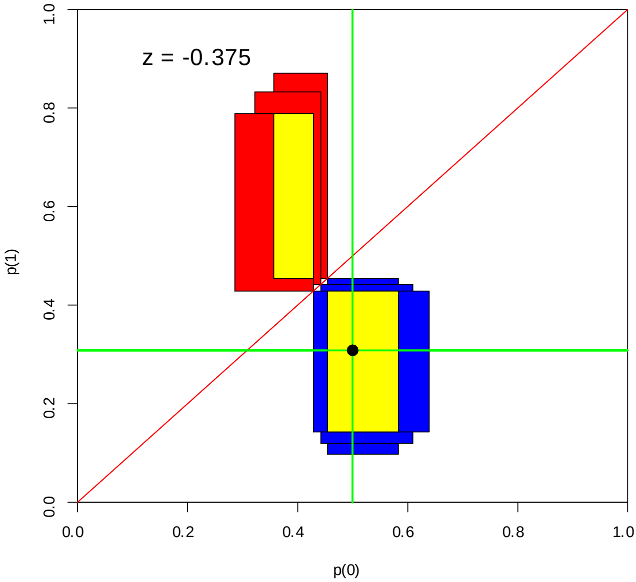


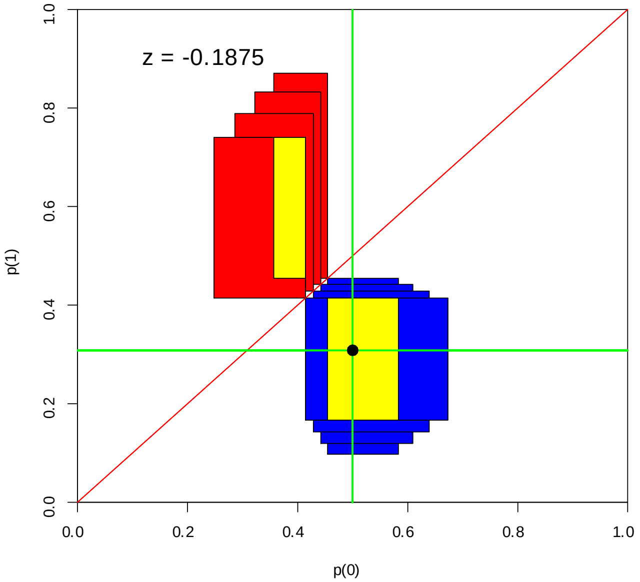
z = 0.75
p(1)
p(0)
0.0
0.2
0.4
0.6
0.8
1.0

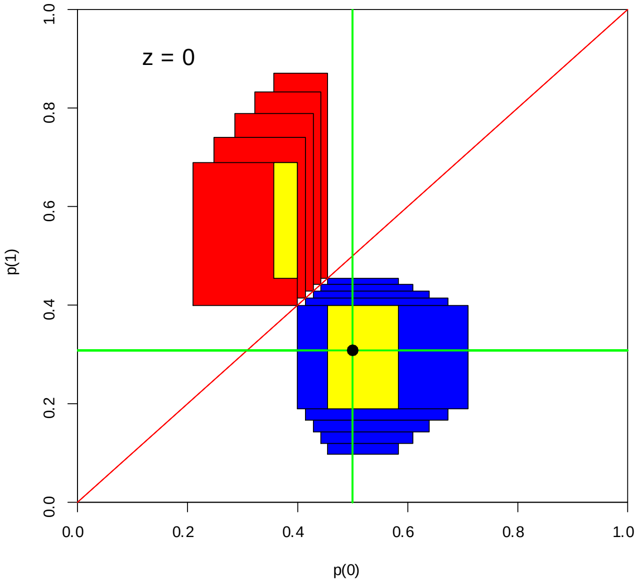

z = -0.75
p(1)
p(0)
0.0
0.2
0.4
0.6
0.8
1.0

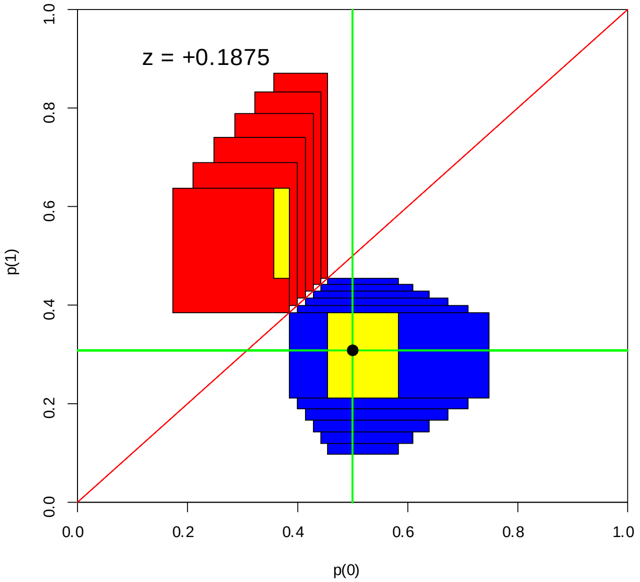


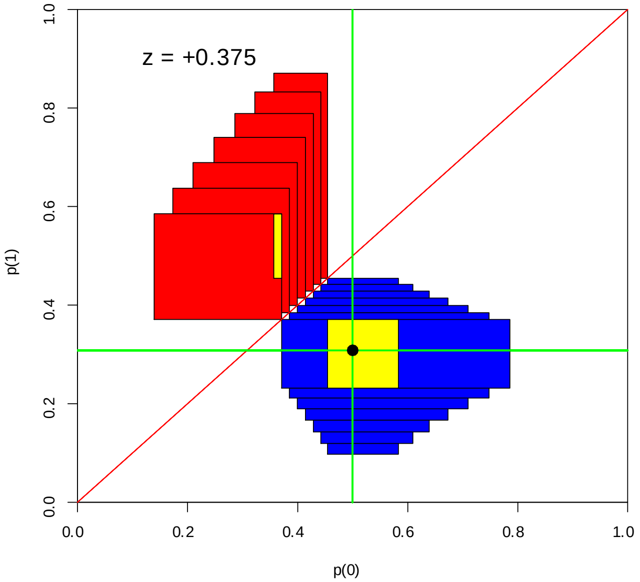
z = -0.5625
p(1)
p(0)
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.2
0.4
0.6
0.8
1.0

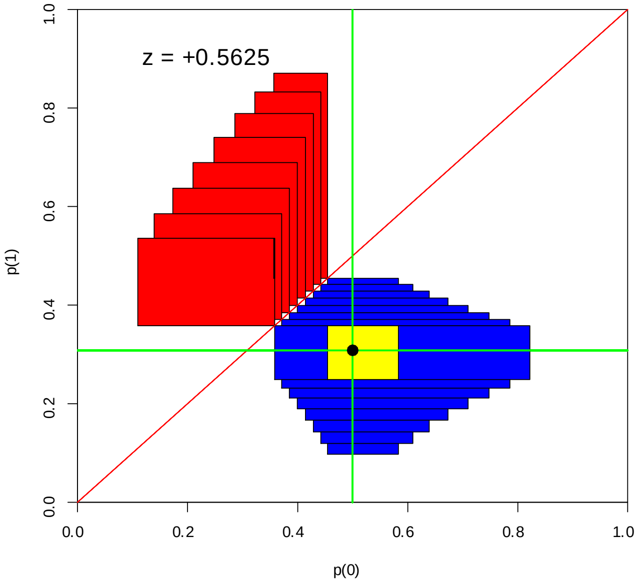

z = -0.375
p(1)
p(0)
0.0
0.2
0.4
0.6
0.8
1.0

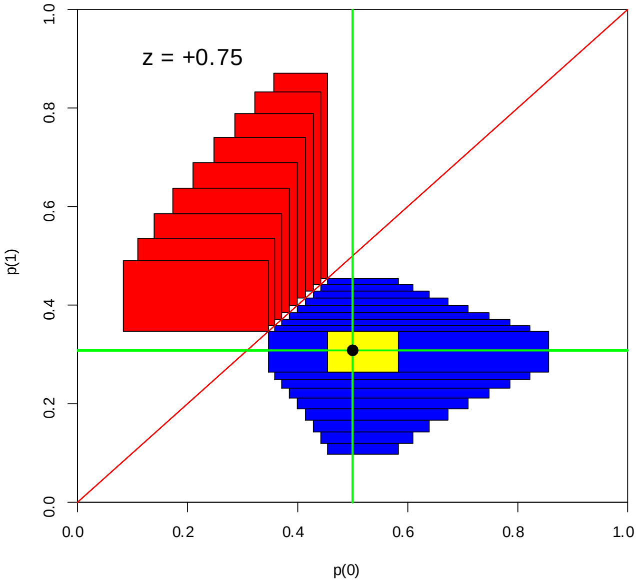

z = -0.1875
p(1)
p(0)
0.0
0.2
0.4
0.6
0.8
1.0

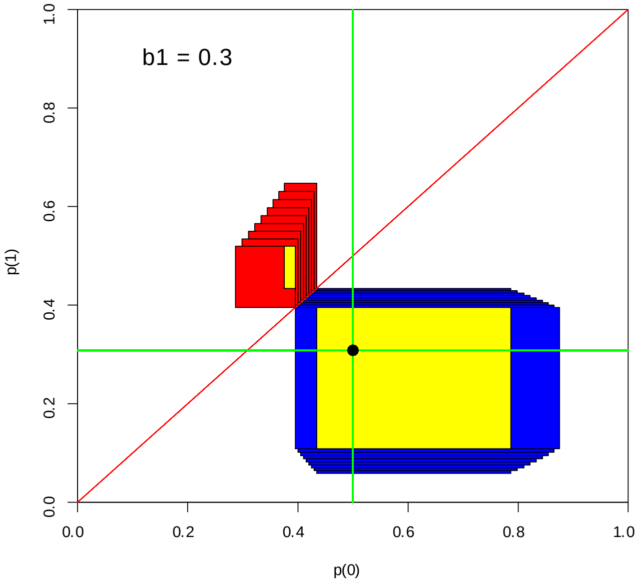

z = 0
p(1)
p(0)
0.0
0.2
0.4
0.6
0.8
1.0

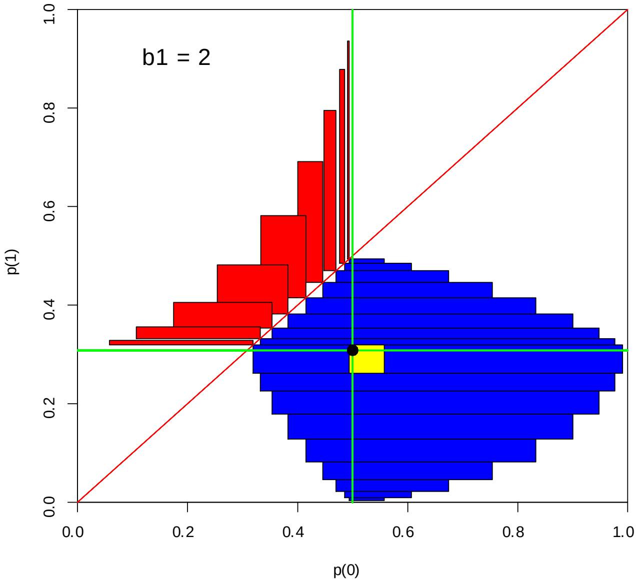

z = +0.1875
p(1)
p(0)
0.0
0.2
0.4
0.6
0.8
1.0


z = +0.375
p(1)
p(0)
0.0
0.2
0.4
0.6
0.8
1.0


z = +0.5625
p(1)
p(0)
0.0
0.2
0.4
0.6
0.8
1.0


z = +0.75
p(1)
p(0)
0.0
0.2
0.4
0.6
0.8
1.0


b1 = 0.3
p(1)
p(0)
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.2
0.4
0.6
0.8
1.0


b1 = 2
p(1)
p(0)
0.0
0.2
0.4
0.6
0.8
1.0

A parametric example: an ordered probit IV model

- Known thresholds $c_1, \ldots, c_{M-1}$ and independence: $Z \perp\!\!\!\perp U \sim \mathit{Unif}(0,1)$

$$Y = \begin{cases} 1 \quad , & 0 < U \leq \Phi(c_1 - \alpha_0 - \alpha_1 X) \\ 2 \quad , & \Phi(c_1 - \alpha_0 - \alpha_1 X) < U \leq \Phi(c_2 - \alpha_0 - \alpha_1 X) \\ \vdots & \vdots \quad \vdots \quad \vdots \\ M & \Phi(c_{M-1} - \alpha_0 - \alpha_1 X) < U \leq 1 \end{cases}$$

A parametric example: an ordered probit IV model

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- Consider the set of values of a_0 and a_1 identified by this model when:

$$Y^* = a_0 + a_1 X + \varepsilon \qquad X = b_0 + b_1 Z + \eta$$

$$Y = m, \qquad c_{m-1} < Y^* \le c_m \qquad \begin{bmatrix} \varepsilon \\ \eta \end{bmatrix} |Z \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & s_{\varepsilon\eta} \\ s_{\varepsilon\eta} & s_{\eta\eta} \end{bmatrix} \right)$$

with: $\Omega = [-1, 1]$, $a_0 = 0$, $a_1 = 1$, $b_0 = 0$, $s_{\varepsilon\eta} = 0.6$, $s_{\eta\eta} = 1$.

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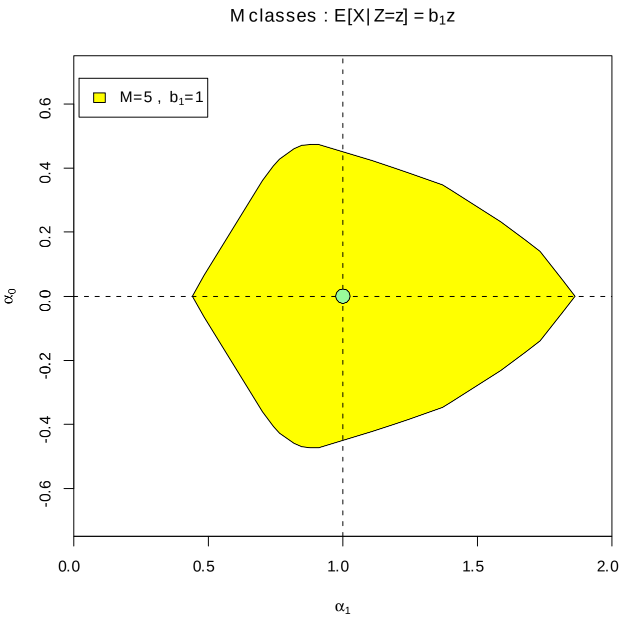
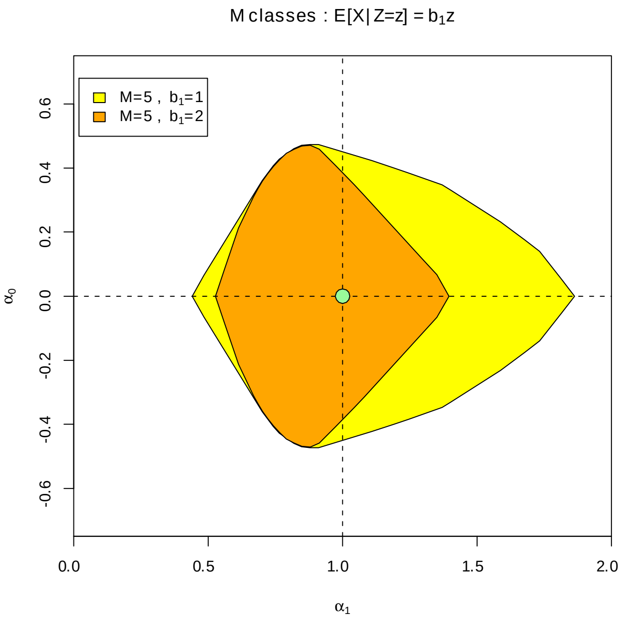
with: $\Omega = [-1, 1]$, $a_0 = 0$, $a_1 = 1$, $b_0 = 0$, $s_{\varepsilon\eta} = 0.6$, $s_{\eta\eta} = 1$.

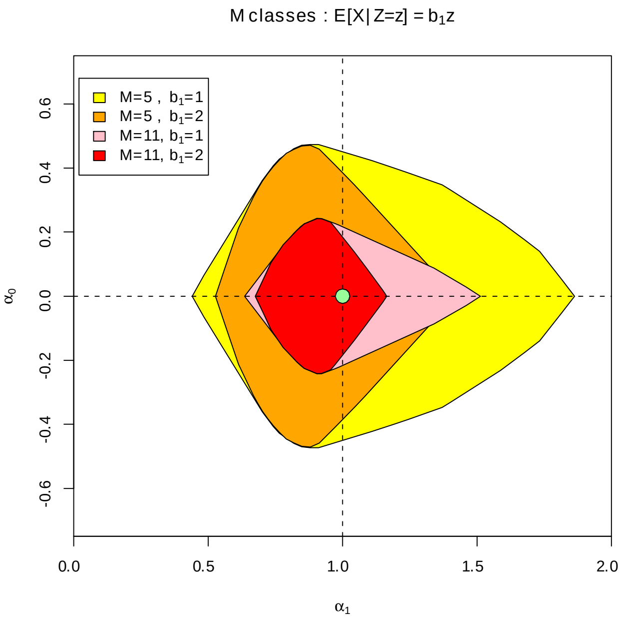
- We:

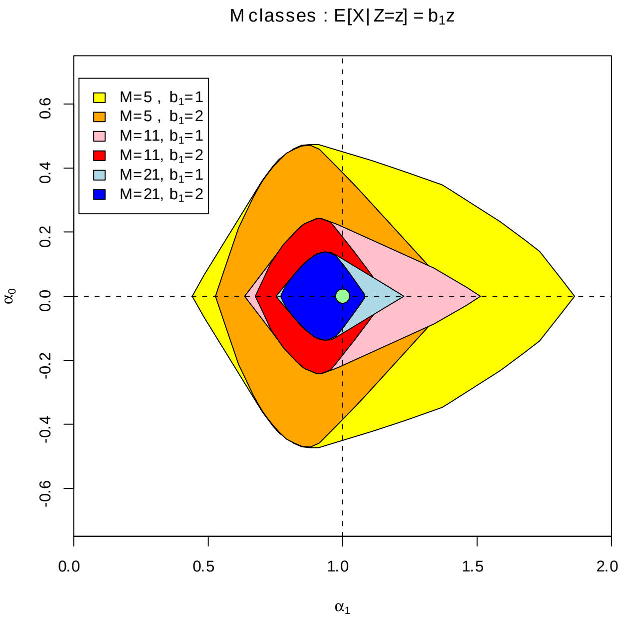
vary discreteness: $M \in \{5, 11, 21\}$

vary strength/support of instrument: $b_1 \in \{1, 2\}$


M classes : E[X| Z=z] = b1z
M=5 , b1=1
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
α0
0.0
0.5
1.0
1.5
2.0
α1


M classes : E[X| Z=z] = b1z
M=5 , b1=1
M=5 , b1=2
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
α0
0.0
0.5
1.0
1.5
2.0
α1


M classes : E[X| Z=z] = b1z
M=5 , b1=1
M=5 , b1=2
M=11, b1=1
M=11, b1=2
α0
α1
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
0.0
0.5
1.0
1.5
2.0



M classes : E[X| Z=z] = b1z
M=5 , b1=1
M=5 , b1=2
M=11, b1=1
M=11, b1=2
M=21, b1=1
M=21, b1=2
α0
α1
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
0.0
0.5
1.0
1.5
2.0

Estimation

- **Intersection bounds**: for each distribution $F^0_{YX|Z}$ the identified set of structural functions $\mathcal{H}_0$ is all h such that

$$\left.\begin{array}{l} \min_{z\in\Omega} P_0[Y \leq h(X,\tau)|Z=z] \geq \tau \\ \max_{z\in\Omega} P_0[Y < h(X,\tau)|Z=z] < \tau \end{array}\right\} \quad \text{for all } \tau \in [0,1]$$

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- Enumerate the set defined using an estimate, $\hat{F}^0_{YX|Z}$ - Chernozhukov, Lee & Rosen (2008).

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- **Moment inequalities**: for any $w(z) > 0$ and all $\tau \in (0,1)$

$$E_{YXZ}[(1[Y \leq h(X,\tau)] - \tau) \times w(Z)] \quad \geq \quad 0$$

$$E_{YXZ}[(1[Y < h(X,\tau)] - \tau) \times w(Z)] < 0$$

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  - Andrews, Berry, Jia (2004), Rosen (2006), Pakes, Porter, Ho, Ishii (2006).

Multivariate discrete outcomes

- $Y = (Y_1, \ldots, Y_T)$ with
$$Y_t = h_t(X, U_t)$$
each h_t non-decreasing in $U_t \sim Unif(0,1)$ and $U \equiv (U_1, \ldots, U_T) \perp\!\!\!\perp Z$.

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- Consider $S_0 \equiv \{h_1^0, \ldots h_T^*, F_{UX|Z}^0\}$ with copula $F_{U|Z}^0 = F_U^0$ delivering $F_{YX|Z}^0$.

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- Consider $S_0 \equiv \{h_1^0, \ldots h_T^*, F_{UX|Z}^0\}$ with copula $F_{U|Z}^0 = F_U^0$ delivering $F_{YX|Z}^0$.
- **Identified set**: consists of admissible
$$\{h_1^*, \ldots, h_T^*, F_U^*\}$$
such that for all $\tau \in (0,1)^T$, $z \in \Omega$
$$P_0[\bigcap_{t=1}^{T}(Y_t \underset{(<)}{\leq} h_t^*(X, \tau_t))|Z = z] \underset{(<)}{\geq} F_U^*(\tau)$$

Binary Y, measurement error

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- Define

$$V \equiv C(b^{-1}(U) + W'\beta) \sim Unif(0,1) \perp\!\!\!\perp Z$$

then

$$Y = \begin{cases} 0 & , \quad 0 \leq \quad V \quad \leq C(X'\beta) \\ 1 & , \quad C(X'\beta) < \quad V \quad \leq 0 \end{cases} \qquad Z \perp\!\!\!\perp V \sim Unif(0,1)$$

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  - identification catalogues: identified sets for $S = \{h, F_{UX|Z}\}$ and from this for functionals $\theta(S)$.