THE HUBER ROUND ROBIN PROBLEM

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Abstract. A relatively obscure paper by Peter Huber reveals some surprising features of round robin tournament play.

1. Introduction

Huber (1963) in a brief remark on Trawinski and David (1963) considers a balanced round robin tournament among t teams $T_0, T_1, \ldots, T_t$. Each team plays every other team n times. It is assumed that there is one superior team, T_0, that is capable of beating every other team with probability $p > \frac{1}{2}$, while all other team play is a "toss-up." At the end of the tournament each team receives a score, $a_i = \sum_{j \neq i}\{T_i \succ T_j\}$, and the winner is the team with the largest score. Huber focuses on $n = 1$, and denotes by $P_{p,t} = P_{p,t,1}$ the probability that T_0 wins.

Figure 1, taken from Trawinski and David (1963), plots $P_{p,t,n}$ for $n = 1$ and $n = 10$. Huber comments that the left panel for $n = 1$ "seems to indicate" that while $P_{n,t,1}$ tends to 1 for p near 1, $P_{n,t,1}$ tends to 0 for p near $\frac{1}{2}$. On the contrary, Huber demonstrates that $\lim_{t\to\infty} P_{p,t,1} = 1$ for any $p > \frac{1}{2}$.

As a warm up exercise, Huber notes, citing Feller, that when $p = \frac{1}{2}$ all the a_i are binomial so $\tilde{a}_i = (a_i - t/2)/\sqrt{4t} \rightsquigarrow \mathcal{N}(0, 1)$ and provided that $t \to \infty$, $x_t \to \infty$ and $x_t^2/\sqrt{4t} \to 0$,

$$\mathbb{P}(\tilde{a}_i > x_t) \sim \frac{1}{x_t\sqrt{2\pi}} e^{-x_t^2/2},$$

where the symbol $\sim$ indicates that the ratio of the two expressions tends to 1.

When $p > \frac{1}{2}$, he then shows that $\max_{1\leq i\leq t} a_i/t$ concentrates around $m_t = \frac{1}{2} + (\log t/2t)^{1/2}$ with dispersion $o(1/\sqrt{t})$, so the best of the similar teams has $a_i^*/t \to \frac{1}{2}$, while $a_0/t \to p > \frac{1}{2}$. So T_0 eventually triumphs. But how big does t need to be? Big enough so that $m_t - \frac{1}{2}$ becomes comparable to $p - \frac{1}{2}$, which means that if $p = 0.56$ we need $t \approx 950$, and for $p = 0.51$ we need $t \approx 54,000$.

Careful examination of Figure 1 reveals the plausibility of all this, but it is yet another lesson that asymptotic approximations sometime require large samples. The most interesting aspect of the Huber paper may be the proof of his main lemma showing majorization of the pairwise outcome distribution by an independent version.

References

Huber, P. J. (1963), 'A Remark on a Paper of Trawinski and David Entitled: "Selection of the Best Treatment in a paired-Comparison Experiment"', *The Annals of Mathematical Statistics* **34**, 92 – 94.

Trawinski, B. J. and David, H. A. (1963), 'Selection of the Best Treatment in a Paired-Comparison Experiment', *The Annals of Mathematical Statistics* **34**, 75 – 91.

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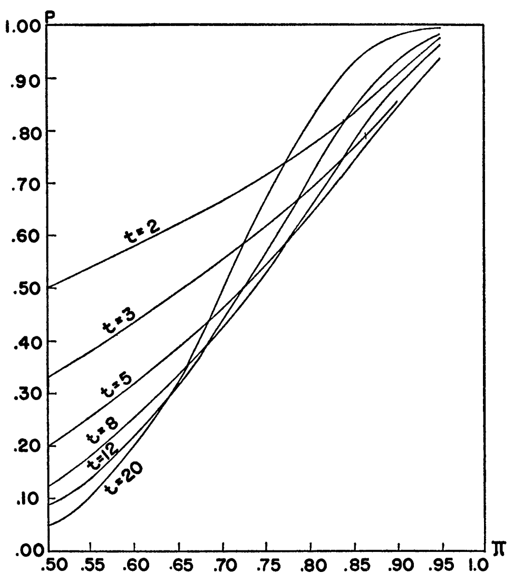

Fig. 1A. $n = 1$

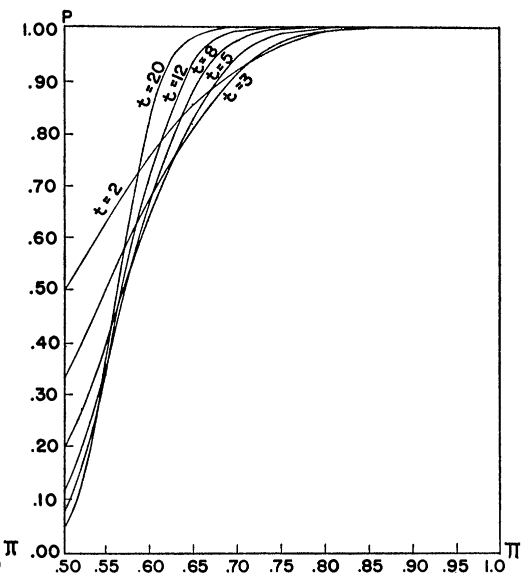

Fig. 1B. $n = 10$

Figure 1.