Appendix A

Test problem 1 (Wormersley, 1986)

The results of temperature accelerated life tests on electrical insulation in 40 motorettes are recorded in Table A.1. This data is originally from Schmee and Hahn (1979). Ten motorettes were tested at each of four temperatures. Testing was terminated at different times at each temperature. The model used to fit the data is

$$log_{10}H = x_1 + \frac{1000x_2}{(T + 273.2)} + \epsilon,$$

where H is the failure time and T is the temperature.

Table A.1 : Data for motorettes example

	Test temperature T^oC			
	150	170	190	200
Failure times H in hours		1764	408	408
		2772*	408	408
		3444	1344	504
		3542	1344	504
		3780	1440	504
		4860		
		5196		
Termination time $\bar{H}$	8064	5448	1680	528
	10 units	3 units	5 units	5 units

$*$ Wormersley gives the second failure time at 170^o as 2722, but his results are consistent with the value recorded here from Schmee and Hahn.

At each temperature there is an upper bound $\bar{H}$ (the time at which testing was stopped) on the observed failure times, so the logarithms of the observed failure times are given by

$$min\left(log_{10}\bar{H},\ x_1 + \frac{1000x_2}{(T+273.2)} + \epsilon\right).$$

Test problem 2 (Bard, 1970)

$$f_i(x) = y_i - \left(x_1 + \frac{u_i}{v_i x_2 + w_i x_3}\right)$$

where $i = 1, 2, \cdots, 15$, $u_i = i$, $v_i = 16 - i$, $w_i = min(u_i, v_i)$, and

i	y_i	i	y_i	i	y_i
1	0.14	6	0.32	11	0.73
2	0.18	7	0.35	12	0.96
3	0.22	8	0.39	13	1.34
4	0.25	9	0.37	14	2.10
5	0.29	10	0.58	15	4.39

Test problem 3 (Beale, 1958)

$$f_i(x) = y_i - x_1(1 - x_2^i),$$

where $i = 1, 2, 3$, $y_1 = 1.5$, $y_2 = 2.25$ and $y_3 = 2.625$.

Test problem 4 (Biggs, 1971)

$$f_i(x) = x_3 exp(-t_i x_1) - x_4 exp(-t_i x_2) + x_6 exp(-t_i x_5) - y_i,$$

where $i = 1, \cdots, 13$, $t_i = i/10$ and

$$y_i = exp(-t_i) - 5exp(-10t_i) + 3exp(-4t_i).$$

Test problem 5 (Brown and Dennis, 1971)

$$f_i(x) = (x_1 + t_i x_2 - exp(t_i))^2 + (x_3 + x_4 sin(t_i) - cos(t_i))^2,$$

where $i = 1, \cdots, 20$, and $t_i = i/5$.

Test problem 6 (El-Attar 5.1, 1979)

$$\begin{aligned} f_1(x) &= x_1^2 + x_2 - 10 \\ f_2(x) &= x_1 + x_2^2 - 7 \\ f_3(x) &= x_1^2 - x_2^3 - 1 \end{aligned}$$

Test problem 7 (El-Attar 5.2)

$$\begin{aligned} f_1(x) &= x_1^2 + x_2^2 + x_3^2 - 1 \\ f_2(x) &= x_1^2 + x_2^2 + (x_3 - 2)^2 \\ f_3(x) &= x_1 + x_2 + x_3 - 1 \\ f_4(x) &= x_1 + x_2 - x_3 + 1 \\ f_5(x) &= 2x_1^3 + 6x_2^2 + 2(5x_3 - x_1 + 1)^2 \\ f_6(x) &= x_1^2 - 9x_3 \end{aligned}$$

Test problem 8 (Madsen, see Overton and Murray, 1981)

$$f_1(x) = x_1^2 + x_2^2 + x_1 x_2$$
$$f_2(x) = sin(x_1)$$
$$f_3(x) = cos(x_2)$$

Test problem 9 (Osborne 1, 1972)

$$f_i(x) = y_i - (x_1 + x_2 exp(-t_i x_4) + x_3 exp(-t_i x_5))$$

where $i = 1, 2, \cdots, 33$, $t_i = 10(i-1)$, and

i	y_i	i	y_i	i	y_i
1	0.844	12	0.718	23	0.478
2	0.908	13	0.685	24	0.467
3	0.932	14	0.658	25	0.457
4	0.936	15	0.628	26	0.448
5	0.925	16	0.603	27	0.438
6	0.908	17	0.580	28	0.431
7	0.881	18	0.558	29	0.424
8	0.850	19	0.538	30	0.420
9	0.818	20	0.522	31	0.414
10	0.784	21	0.506	32	0.411
11	0.751	22	0.490	33	0.406

Test problem 10 (Osborne 2)

$$\begin{aligned} f_i(x) =& y_i - \big(x_1 exp(-t_i x_5) + x_2 exp(-(t_i - x_9)^2 x_6) \\ & + x_3 exp(-(t_i - x_{10})^2 x_7) + x_4 exp(-(t_i - x_{11})^2 x_8)\big) \end{aligned}$$

where $i = 1, 2, \cdots, 65$, $t_i = (i-1)/10$, and

i	y_i	i	y_i	i	y_i
1	1.366	23	0.694	45	0.672
2	1.191	24	0.644	46	0.708
3	1.112	25	0.624	47	0.633
4	1.013	26	0.661	48	0.668
5	0.991	27	0.612	49	0.645
6	0.885	28	0.558	50	0.632
7	0.831	29	0.533	51	0.591
8	0.847	30	0.495	52	0.559
9	0.786	31	0.500	53	0.597
10	0.725	32	0.423	54	0.625
11	0.746	33	0.395	55	0.739
12	0.679	34	0.375	56	0.710
13	0.608	35	0.372	57	0.729
14	0.655	36	0.391	58	0.720
15	0.616	37	0.396	59	0.636
16	0.606	38	0.405	60	0.581
17	0.602	39	0.428	61	0.428
18	0.626	40	0.429	62	0.292
19	0.651	41	0.523	63	0.162
20	0.724	42	0.562	64	0.098
21	0.649	43	0.607	65	0.054
22	0.649	44	0.653		

Test problem 11 (Powell, 1962)

$$f_1(x) = x_1 + 10x_2$$
$$f_2(x) = 5^{1/2}(x_3 - x_4)$$
$$f_3(x) = (x_2 - 2x_3)^2$$
$$f_4(x) = 10^{1/2}(x_1 - x_4)^2$$

Test problem 12 (Rosenbrock, 1960)

$$f_1(x) = 10(x_2 - x_1^2)$$
$$f_2(x) = 1 - x_1$$

Test problem 13 (Watson, see Kowalik and Osborne, 1968)

$$f_i(x) = \sum_{j=2}^{n}(j-1)x_j t_i^{j-2} - \left(\sum_{j=1}^{n} x_j t_i^{j-1}\right)^2 - 1,$$

where $i = 1, \cdots, 29$, $t_i = i/29$, $f_{30}(x) = x_1$ and $f_{31}(x) = x_2 - x_1^2 - 1$.

Test problem 14 (Wood, see Colville, 1968)

$$f_1(x) = 10(x_2 - x_1^2)$$
$$f_2(x) = 1 - x_1$$
$$f_3(x) = 90^{1/2}(x_4 - x_3^2)$$
$$f_4(x) = 1 - x_3$$
$$f_5(x) = 10^{1/2}(x_2 + x_4 - 2)$$
$$f_6(x) = 10^{-1/2}(x_2 - x_4)$$