University of Illinois **Econ 574** **Department of Economics**
Spring 2005 **Final Exam** **Roger Koenker**

1. A typical model for the distribution of income is the Pareto model with density

$$f(y, \gamma, \lambda) = \gamma \lambda^{\gamma} y^{-(\gamma+1)} \qquad y > \lambda, \;\; \gamma > 0, \;\; \lambda > 0.$$

   (a) Suppose that $\lambda = \lambda_0$ is known, and that you have a random (iid) sample $\{y_1, \ldots, y_n\}$ from the Pareto density. Find the maximum likelihood estimator, $\hat{\gamma}$, of γ.

   (b) What is Fisher's information number for the parameter γ in the Pareto model assuming $\lambda = \lambda_0$ is known?

   (c) Explain how to construct a Wald $100(1-\alpha)\%$ confidence interval for γ.

   (d) Explain how to construct a Likelihood ratio $100(1-\alpha)\%$ confidence interval for γ.

   (e) Show that the median income from the Pareto density is $\lambda 2^{1/\gamma}$.

   (f) Explain how to construct a $100(1-\alpha)\%$ confidence interval for median income based on the mle.

   (g) Now consider simply estimating median income by the sample median of the y_i's, explain how to construct a $100(1-\alpha)\%$ confidence interval for median income based on this estimate.

   (h) Which interval f.) or g.) would you expect to be more "accurate" and what do you mean by accurate in this context?

2. In the 2002 Exam there was a question intended to compare various estimators of the location parameter of the Cauchy distribution. Generalize the setting to the bivariate regression setting, that is, assume that the the conditional distribution of Y is Cauchy with location parameter $\alpha + \beta X$, and unit scale parameter. For purposes of the Monte-Carlo you may take X to be standard normal. Analyse the behavior of both the intercept and slope parameter estimates. Hint: Note that the 2002 question *and answer* are available in the course's web exam directory 2002.