University of Illinois | **Econ 476** | Department of Economics
Spring 2004 | **Final Exam** | Professor Roger Koenker

This is a take home final exam. It will be available on the web at 9am Friday May 7 and it is due on Friday, May 14 by 5pm. You can put it in my mailbox in 484 Wohlers Hall.

1. (Bayesian Credulity) When our prior beliefs conflict drastically with new evidence reconciliation of the conflict is usually difficult, even painful. In psychology this experience is sometimes called cognitive dissonance. In statistics it is exemplified by situations in which prior beliefs and new evidence are reconciled by Bayesian updating. To consider a very elementary example suppose we have a prior belief that a location parameter μ lies in the interval $[6, 8]$ with high probability. Now we see a sample $\{.2, .5, 2.5\}$ which we believe constitute a random sample from a Gaussian distribution with mean μ and standard deviation .8.
   (a) Suppose that we begin by formulating the prior as Gaussian with mean 7 and standard deviation of .25. Find the posterior density. Interpret the posterior as the solution to a weighted least squares (regression) problem. Do you find this updating credible?
   (b) As an alternative, suppose that we adopt the view that the prior is Cauchy with mean 7 and scale parameter .2, i.e., that $Z = 5(\mu - 7)$ is standard Cauchy. Again, find the posterior and compare to the result of a.) commenting on whether the result seems more reasonable. Compare this Cauchy prior with the normal prior used in part (a).
   (c) Do some sensitivity checking to see how changes in the precision of the prior affects the posterior in parts a.) and b.).
   (d) Suppose we maintain the Cauchy prior but alter our view that the observations were Gaussian and assume instead that they too came from a standard Cauchy distribution. Again, plot the posterior and compare with the previous results. Suggest a strategy for making a confidence or credibility region based on this posterior.
   (e) Conclude by making some comment on the consequences of the normality assumption in the reconciliation of this kind of statistical cognitive dissonance.
2. In the 2001 exam for Econ 476 there was an elaborate question about U-statistics. It was intended as an auto-tutorial introduction to U-statistics, which seem to be playing an increasing important role in semiparametric econometrics. The original exam questions and my rather sketchy answers to the questions are available on the 476 website.
   (a) Read over the 2001 exam and the answers, and then write a short commentary on part g.) that fills in some mathematical details, explains a bit better the basic structure of the argument, and discusses the rationale for the estimator.

(b) In the 2002 exam for 476 there was a question about comparing estimators of the location parameter of the Cauchy distribution. Again the exam and my sketchy answers are available on the web. Using the R code provided there adapt the simulation experiment to include the Hodges-Lehmann estimator, and adapt the reporting of the results so that it is convenient to compare with the asymptotic behavior of the estimators. Explain the estimators and their asymptotic behavior briefly, and provide some commentary on the monte-carlo code that explains some of the obscure aspects. Hints: Rescale the mse's by $\sqrt{n}$. The Fisher information for the location parameter of the standard Cauchy is .5. A reasonably efficient R function for the Hodges-Lehmann estimator is `HL <- function(x){A <- outer(x,x,"+"); median(A[upper.tri(A)])/2}`