University of Illinois	**Econ 476**	Department of Economics
Spring 2002	**Final Exam**	Professor Roger Koenker

This is a take home final exam. It will be available on the web at 9am Monday May 6 and it is due on Friday, May 10 by 5pm. You can put it in my mailbox in 484 Wohlers Hall. I hope that you find the questions challenging, but not too challenging! Feel free to send email to clarify the questions, but since I'll be away there may not be immediate response. I should in the office on thursday and friday.

1. Consider the "errors in variables" model

$$\begin{aligned} y_i &= \alpha + \beta x_i + u_i \qquad\qquad i = 1, \ldots, n \\ z_i &= x_i + v_i \end{aligned}$$

with

$$\begin{pmatrix} x_i \\ u_i \\ v_i \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_x^2 & 0 & 0 \\ 0 & \sigma_u^2 & 0 \\ 0 & 0 & \sigma_v^2 \end{pmatrix} \right)$$

(a) Find the limiting distribution of Wald's grouped IV estimator

$$\hat{\beta}_{IV} = \frac{\sum y_i \text{ sgn } (z_i - \hat{z}_i)}{\sum z_i \text{ sgn } (z_i - \hat{z}_i)}$$

where $\hat{z}_i =$ median $\{z_i\}$.

(b) Find the asymptotic relative efficiency of the Wald estimator relative to the ordinary least squares estimator when $\sigma_v^2 = 0$.

(c) Comment on the effectiveness of the Wald approach in dealing with the bias of the OLS estimator, when $\sigma_v^2 > 0$.

2. Consider the problem of estimating the location parameter, θ, from a random sample of Cauchy random variables with density

$$f(x, \theta) = \frac{1}{\pi} \left(\frac{1}{1 + (x - \theta)^2} \right).$$

(a) Describe briefly how to compute "one-step version" of the mle, assuming that you use the sample median as an initial estimator.

(b) Extend the result in (a) to describe how to compute the mle.

(c) Write an R function to compute the estimators in (a) and (b) and conduct a *small** monte-carlo experiment to compare the performance of the mean, median, one step mle, full mle. Hints: (i) see $nlm(\quad)$ to automate the mle computation, (ii) Cauchy rv's can be generated as $rt(n, 1)$.

*Small means something like the following design: 3 sample sizes say 20, 50 and 100 with 1000 replications.

(d) Based on the monte-carlo, make a conjecture about the asymptotic variance of the Cauchy mle, and then find the limiting distribution of the mle and compare with your conjecture.

3. Suppose $\{y_1, \dots, y_n\}$ are iid random variables each normally distributed with mean μ, and variance μ^2. Find the mle of μ and argue its consistency. Compare the asymptotic efficiency of the mle in this problem with that of the sample mean as an estimator of μ. This problem is closely related to problems of estimating models of heteroscedasticity in linear regression models that have common parameters in the location and scale components. Elaborate *very* briefly on this extension.