## Econ 536 <br> Applied Econometrics <br> Problem Set 2

Since the first "oil shock' of 1973 there has been a continuing controversy in the U.S. about tax policy for gasoline and other petroleum distillates. A crucial component of any such debate is a reliable model for demand. In this problem set we will analyze U.S. demand for gasoline and some implications for tax policy.

A general dynamic model for the demand for gasoline is (see Harvey 8.4.1)

$$
\begin{equation*}
y_{t}=\alpha_{0}+\left(\alpha_{1} y_{t-1}+\sum_{j=1}^{r-1} \delta_{j} \Delta y_{t-j}\right)+x_{t} \beta+\sum_{j=0}^{s-1} \gamma_{j} \Delta x_{t-j}+u_{t} \tag{1}
\end{equation*}
$$

where all variables are in natural logarithms, $\Delta y_{t}=y_{t}-y_{t-1}$, and
$y_{t}=$ per capita personal consumption of gasoline in gallons (at annual rates)
$x_{t}^{\prime}=\left(z_{t}, p_{t}\right)$
$z_{t}=$ per capita personal income (in 1000's of $1982 \$$ at annual rates)
$p_{t}=$ real price $/$ gallon of gasoline in $1982 \$(1$ gallon $=\$ 1$ at 1982 prices $)$
Data on these variables has been extracted from the national income and product accounts. There are quarterly observations, from 1947.Q1 to 1997.Q1, available from the class website as GasOld.Rda, as an $R$ save file, and as GasOld.csv, as ascii csv file. I've made a valiant effort to update this data, and there are corresponding files GasNew.Rda, and GasNew.csv, but unfortunately definitions are difficult to match and results for the new data are not as enlightening as with the earlier data, so I recommend that you use the old data, and then, if you want to be adventurous try out the new. Note that one dollar in 1982, is roughly equivalent to $\$ 2.50$ at current, 2016, prices according to the consumer price index, so if you would like to compare your computations below with current prices "at the pump" you need to make this adjustment. Although our demand estimates are based on aggregate market data we are naturally tempted to interpret them as reflecting behavior of a representative consumer. Thus, for such a mythical average consumer with consumption of, say, 300 gallons per year would imply annual miles driven of about 9,000 assuming a car averaging 30 mpg .

1. Estimate model (1) with $r=2, s=2$, and use Schwarz's BIC criterion to simplify the model.
2. Compute the long-run income and price elasticities corresponding to your final model. Compare with results you would get from the simple static model with $\alpha_{1}=0$, and $\delta_{j}=\gamma_{j}=0$ for all $j$. Try to interpret the differences.
3. What revenue implication do these long run elasticities have for contemplated increases in the gasoline tax? If the current tax rate is 18 cents per gallon, what would be the net per-capita revenue gain expected from the imposition of an additional 50 cent, $\$ 1$, and $\$ 2$ per gallon tax? Note that the Dutch gasoline tax is roughly $\$ 3.50$ per gallon, so these values are not so unreasonable by international standards.

There are two ways to do these computations: starting from the long-run form of the model, one can simply predict tax revenue from various levels of tax, or one can try to extrapolate using the long-run elasticity. It is instructive to compare these two approaches; in the first case we have $R(\tau)=\tau q\left(p_{0}+\tau\right)$ and we simply evaluate revenue for various choices of the tax rate $\tau$. In the second approach we approximate the change in demand by effectively linearizing the demand curve and then multiplying by the appropriate change in price. For small changes in price these two approaches should produce very similar answers, but it is instructive to compare their results in the current context when larger changes are being contemplated.
4. Plot the impulse response functions for your final model for both income and price changes. Interpret. Put the model in error-correction form (Harvey $\S 8.5$ ) and reinterpret.
5. The constant elasticity equilibrium model has some implausible features from a policy analysis standpoint. An alternative somewhat more appealing equilibrium model is the following:

$$
\begin{equation*}
y_{t}=\beta_{0}+\beta_{1} z_{t}+\beta_{2} p_{t}+\beta_{3}\left(p_{t}\right)^{2}+\beta_{4} p_{t} z_{t}+u_{t} \tag{2}
\end{equation*}
$$

In model (2) the price elasticity of demand is

$$
\eta=\frac{\partial y}{\partial p}=\beta_{2}+2 \beta_{3} p_{t}+\beta_{4} z_{t}
$$

If, as seems to be the case in US data, $\beta_{2}<0, \beta_{3}<0$, and $\beta_{4}>0$, the model implies that gasoline demand is (i) more elastic as price increases, and (ii) less elastic as income increases.
(a) Do these implications seem intuitively plausible? Why, or why not?
(b) Recalling that revenue is maximized when $\eta=-1$, suppose per capita income is $\$ 30,000$ and give a formula for computing the price which maximizes gasoline revenue assuming model (2) is correct.
(c) Use the partial residual plot to visually evaluate whether the quadratic term is justified. Then, formally test for the significance of the quadratic effect.
(d) Estimate model (2) and compute the revenue maximizing price level assuming per capita income is $\$ 30,000$ per year. Use either the $\delta$-method or bootstrap to compute a confidence interval for this estimate. Recall that the data as distributed has per capita income in thousands of 1982 dollars, so you should adjust by the factor 2.50 to get current (2016) dollars.
6. A serious problem with model (2) is that it assumes that demand adjusts instantaneously to changes in price and income. A more plausible model is

$$
\begin{equation*}
y_{t}=\alpha y_{t-1}+\delta \Delta y_{t-1}+\beta_{0}+\beta_{1} z_{t}+\beta_{2} p_{t}+\beta_{3}\left(p_{t}\right)^{2}+\beta_{4} p_{t} z_{t}+u_{t} \tag{3}
\end{equation*}
$$

where $\Delta y_{t-1}=y_{t-1}-y_{t-2}$. The parameters $\alpha$ and $\delta$ determine the short run dynamics of the model. Put model (3) in equilibrium form and interpret, then estimate model
(3) and compare your results with the equilibrium model (2) results. Evaluate this final specification of the model in the light of diagnostics for autocorrelation and other possible departures from classical Gaussian linear model conditions. Note that such nonlinear (in variables) dynamic models can have somewhat bizarre behavior, so it is worthwhile exploring the behavior of this one via simulation.

