University of Illinois
Fall 2008

Department of Economics
Professor R. Koenker

ECON 508
Final Exam Review Questions

These review questions constitute a sample of questions from prior exams. They represent in style and substance a fair indication of the kind of questions that will appear on the exam, but you should *not* expect that exam questions will be drawn directly from these examples.

1. Consider a fictional, balanced panel data model for household demand for gasoline,

$$y_{it} = \alpha_i + x_{it}\beta + z_i'\gamma + u_{it}$$

where $i = 1, \ldots, n$ and $t = 1, \ldots, T$, and

y_{it} = log of household demand for gasoline in gallons/month
x_{it} = log of average price of gasoline paid by household i in month t
z_i = a vector of time invariant household characteristics including income, family structure, etc.

A serious potential problem with the model is that the price variable, x_{it}, may be correlated with the individual specific effect α_i. This problem arises because the price paid by household i must be inferred by dividing recorded expenditure on gasoline by the number of gallons purchased – since some households may search more intensively than others for a lower price, some of the observed price variation may be due to this endogenous "shopping-effect" rather than purely exogenous market variability. In particular one might expect that this endogeneity would be correlated with the household specific demand effect α_i.

(a) Explain briefly why OLS is an unsatisfactory method of estimating this model.

(b) Since the primary objective in estimating this model is to recover an accurate estimate of the price elasticity, β, suggest a way to accomplish this which avoids the problems alluded to above regarding the endogeneity of the price variable.

(c) Suppose you now estimate the model by the technique recommended in part (b.) and obtain $\hat{\beta} = -.70$ with a standard error of .08. Then, ignoring the endogeneity effect, you also estimate the model using the random effects estimator, i.e., treating the α_i's as a random sample with mean α_0 and variance σ_α^2. From this you obtain $\hat{\beta} = -.40$ with a standard error of .06. Someone suggests using these results to test for bias due to the endogeneity. Explain the test briefly and carry it out.

(d) How do the conclusions drawn in part (c.) affect your ability to estimate the parameter vector γ. Explain briefly how the availability of a new time varying covariate, say, household income, would affect your estimation strategy.

2. You have estimated the logit model

$$\text{logit}(p_i) = -4.5 + 1.7x_i - .25x_i^2$$

where p_i is the probability that a paper submitted to the *Phuzics Review* is accepted for publication and x_i is the natural logarithm of the length of the paper in pages.

(a) If you would like to maximize the probability of acceptance, how long should your paper be?

(b) By how much do you change the probability of acceptance when you cut the length of a 50 page paper to 40 pages, assuming the content is undamaged?

3. "Value at risk" is a common finance concept for evaluating portfolio risk. Value at risk is defined to be the loss incurred on the portfolio if the realized return happens to take a specified low quantile value. For a portfolio with initial value V, the value at risk at τ is thus given by:

$$\text{VAR}(\tau) = -V \cdot Q_R(\tau)$$

where $Q_R(\tau)$, denotes the τth quantile of the random return, R, of the portfolio. For example, the daily return of the S&P 500 index over the 1000 trading days between Nov 1, 1993 and Mar 4, 2003 was below $\hat{Q}_R(\tau) = -0.0227$ on 5 percent of these days. So on a portfolio with initial value $V = 20,000$ the estimated $VAR(.05) = \$454$. For more extreme values of τ, that is values closer to zero, it becomes less reliable to estimate VAR(τ) based on the ordinary sample quantiles of returns. For this purpose, analysts frequently resort to the Pareto model.

Suppose that the random (daily) return, R, on a portfolio has a Pareto left tail,

$$P(R < -x) = Kx^{-\alpha}$$

for $x > 0$. Let $R_{(1)} < \ldots < R_{(m)}$ denote the $m \ll n$ lowest realized returns and consider the regression model,

$$\log(k/n) = \log(K) - \alpha \log(-R_{(k)}) + u_k, \quad k = 1, \ldots, m.$$

(a) Explain briefly the rationale for this model given the Pareto assumption and how you would use it to estimate α.

(b) Suppose you have estimated $\hat{\alpha} \approx 3$ in the above model and you wish to now use it to estimate VAR(τ) for some $\tau < \tau_0 = .05$. Show that under the Pareto assumption

$$\text{VAR}(\tau) = \text{VAR}(\tau_0) \left(\frac{\tau_0}{\tau} \right)^{1/\alpha}$$

for τ sufficiently near τ_0 that the Pareto assumption holds. Estimate VAR(.01) for the S&P 500 example.

(c) Interpret the parameter α: If, for another portfolio, $\alpha = 1$, is it more or less "risky" than the S&P 500 with $\alpha = 3$?

4. A standard model for ranking sports teams is the Bradley-Terry, or paired comparison logit model. Under this model the probability, π_{ij}, of team i beating team j is given by

$$\text{logit}(\pi_{ij}) = \alpha_i - \alpha_j$$

(a) Given data on the binary outcomes of n games involving p teams with $n \gg p$, explain briefly how you would estimate the vector of parameters $\alpha \in \mathcal{R}^p$.

(b) Given estimates of the parameters α_k and α_l representing the strengths of teams k and l how would you estimate the probability that team k beats team l.

(c) Suppose you wanted to include a "home field advantage" in the model, how would you do this?

5. In a randomized experiment involving 400 new applicants for unemployment insurance half of the participants were assigned to attend a 2 hour class on "how to apply for a job" and the other half of the participants were given a ticket to the nearby movie theater.

(a) For *all* participants we observe the duration of their unemployment spell and estimate the Cox proportional hazard model

$$\lambda(t|x) = \lambda_0(t) \exp\{x\beta\}$$

where x is the binary indicator of class attendance. Suppose the median duration in unemployment for the control (movie) group is 17.4 weeks, and $\hat{\beta} = -1$, describe how to estimate the probability that a class attendee would stay unemployed more than 17.4 weeks?

(b) Now consider an alternative analysis in which we adopt the model,

$$\text{median}\{\log(T_i)\} = \alpha + \beta x_i$$

where T_i denotes the observed unemployment duration for the i^{th} participant. Explain how to estimate this model and how to interpret the parameters.

(c) Show that a special case of this quantile regression model is the accelerated failure time model

$$\log T_i = x_i^\top \beta + u_i$$

where u_i is iid and e^{u_i} has distribution function F. In particular, show that for this model $P(T_i > t) = 1 - F(te^{-x^\top \beta})$.

(d) Explain why randomization is important to the validity of the interpretation of these models, why voluntary choice of a movie or class participation would be problematic.

6. Consider a life cycle productivity model like the one estimated for phuzicists in which you decide to adopt a more flexible model for the "experience" effect. You estimate the model

$$y_{it} = \alpha_1 y_{it-1} + \alpha_2 y_{it-2} + \sum_{k=0}^{p} \varphi_k(x_{it})\beta_k + z_i^\top \gamma + u_{it}$$

where y_{it} is productivity of individual i in year t, x_{it} is experience of i in year t, the vector z_i consists of time invariant characteristics of individual i, and the α_i are individual specific effects.

(a) Suppose you adopt a linear spline model with

$$\varphi_k(x) = \max\{0, x - \tau_k\}$$

where $p = 3, \tau_0 = 0, \tau_1 = 6, \tau_2 = 12, \tau_3 = 24$. Given an estimated version of this model explain how you would test the hypothesis that productivity declines after 24 years of experience.

(b) Given the same estimated model, how would you test the hypothesis that productivity increases linearly with experience up until $x_{it} = 24$ but may, or may not, decrease after that.

(c) Suppose that you now would like to introduce individual fixed effects, say, α_i's into the model. Supposing that the u_{it} were iid, explain why ordinary least squares estimation of the model in first differences is ill-advised and suggest an alternative.