

**Econ 508**  
**Applied Econometrics**  
**Problem Set 2**

Since the “oil shock” of 1973 there has been a continuing controversy about tax policy for gasoline and other petroleum distillates. A crucial component of any such debate is a reliable model for demand. In this problem set we will analyze U.S. postwar demand for gasoline and some implications for tax policy.

A general dynamic model for the demand for gasoline is (see Harvey 8.4.1)

$$y_t = \alpha_0 + (\alpha_1 y_{t-1} + \sum_{j=1}^{r-1} \delta_j \Delta y_{t-j}) + x_t \beta + \sum_{j=0}^{s-1} \gamma_j \Delta x_{t-j} + u_t \quad (1)$$

where *all variables are in natural logarithms*,  $\Delta y_t = y_t - y_{t-1}$ , and

$y_t$  = per capita personal consumption on gasoline in thousands of gallons (at annual rates)

$x'_t = (z_t, p_t)$

$z_t$  = per capita personal income (in 1000's of 1982 \$ at annual rates)

$p_t$  = real price/gallon of gasoline in 1982 \$ (1 gallon = \$ at 1982 prices)

Data on these variables has been extracted from DRI tapes. There are quarterly observations, from 1947.1 to 1997.1, available from the class website as `gasq.data`.

1. Estimate model (1) with  $r = 2, s = 2$ , and use Schwarz's BIC criterion to simplify the model.
2. Compute the long-run income and price elasticities corresponding to your final model. Compare with results you would get from the simple static model with  $\alpha_1 = 0$ , and  $\delta_j = \gamma_j = 0$  for all  $j$ . Try to interpret the differences. What revenue implication do these long run elasticities have for contemplated increases in the gasoline tax. If the current tax rate is 14 cents per gallon, what would be the net per-capita revenue gain expected from the imposition of an additional 10, 20 and 50 cent per gallon tax?
3. Plot the impulse response functions for your final model for both income and price changes. Interpret. Put the model in error-correction form (Harvey §8.5) and reinterpret.
4. The constant elasticity equilibrium model has some implausible features from a policy analysis standpoint. An alternative somewhat more appealing equilibrium model is the following:

$$y_t = \beta_0 + \beta_1 z_t + \beta_2 p_t + \beta_3 (p_t)^2 + \beta_4 p_t z_t + u_t \quad (2)$$

In model (2) the price elasticity of demand is

$$\eta = \frac{\partial y}{\partial p} = \beta_2 + 2\beta_3 p_t + \beta_4 z_t$$

If, as seems to be the case in US postwar data,  $\beta_2 < 0$ ,  $\beta_3 < 0$ , and  $\beta_4 > 0$ , the model implies that gasoline demand is (i) more elastic as price increases, and (ii) less elastic as income increases.

- (a) Do these implications seem intuitively plausible? Why, or why not?
  - (b) Recalling that revenue is maximized when  $\eta = -1$ , suppose per capita income is  $x_0$  and give a formula for computing the price which maximizes gasoline revenue assuming model (2) is correct.
  - (c) Use the partial residual plot to visually evaluate whether the quadratic term is justified. Then, formally test for the significance of the quadratic effect.
  - (d) Estimate model (2) and compute the revenue maximizing price level assuming per capita income is \$15,000 per year. Use either the  $\delta$ -method or bootstrap to compute a confidence interval for this estimate. Recall that the data as distributed has per capita income in 1000's of 1982 dollars.
5. A serious problem with model (2) is that it assumes that demand adjusts instantaneously to changes in price and income. A more plausible model is

$$y_t = \alpha y_{t-1} + \delta \Delta y_{t-1} + \beta_0 + \beta_1 z_t + \beta_2 p_t + \beta_3 (p_t)^2 + \beta_4 p_t z_t + u_t \quad (3)$$

where  $\Delta y_{t-1} = y_{t-1} - y_{t-2}$ . The parameters  $\alpha$  and  $\delta$  determine the short run dynamics of the model. Put model (3) in equilibrium form and interpret, then estimate model (3) and compare your results with the equilibrium model (2) results. Evaluate this final specification of the model in the light of diagnostics for autocorrelation and other possible departures from classical Gaussian linear model conditions.

6. Monthly data on gasoline demand is also available from the class website as `gasm.data` from January 1959 to 2002. The variables are defined in roughly equivalent manner, except that price is only available as a price index so neither price nor quantity data have a simple per gallon interpretation. Again, data is in logarithms. As a sanity check of your prior results estimate model (1) with  $r = s = 6$ , and then compute point estimates of the long run price and income elasticities and compare with your prior results.