

Economics 508: Applied Econometrics  
Problem Set 1

The budget data presented below are taken from two studies of the standard of living of English rural laborers conducted during the period 1787-1795 by the Reverend David Davies and Frederick Morton Eden. To my knowledge, these were the first examples of studies in that long and honorable liberal tradition of econometrically snooping into the private lives of the poor. By the mid 19th century such studies were being conducted all over Europe by such notables as Ernst Engel, Frederick Engels, Frederick LePlay and others. See George Stigler's essay "The Early History of Empirical Studies of Consumer Behavior" which has been reprinted in his splendid volume of *Essays in the History of Economics*. A somewhat more detailed description of the data may be found in my ancient paper "Was Bread Giffen?" in *REStat*, May, 1977. This was the first empirical paper I wrote as a graduate student so you shouldn't expect a very high level of sophistication from it.

More conveniently, the data are available from the 472 class webpage. The objective of the original paper was to document an example of that Loch Ness Monster of economics -- the Giffen good. Unfortunately, as you will see as you do the problem, bread does not seem to be Giffen among this group of households in the late 18th century. Nonetheless, I believe that the problem serves as a good review of some basic demand theory and some basic ideas of hypothesis testing. And I have a sentimental attachment to it.

All expenditure variables are in old pence per week. The price of bread is in old pence per half peck loaf, the price of meat is in old pence per lb of bacon. In cases where consumption of meat was not in the form of bacon an equivalent quantity (in money terms) of bacon was computed. Similarly, in cases where the household purchased flour rather than bread an equivalent quantity of bread was computed.

1. Estimate the following two bread equations:

$$Q_B = \mu_B + \alpha_B Y + u$$

$$Q_B = \mu_B + \alpha_B Y + \gamma_B S + \beta_{BB} P_B + \beta_{BM} P_M + u.$$

In the first only-income-matters model, test the hypothesis that the Engel curve for bread is homogeneous, i.e.,  $\mu_B = 0$ , against the alternative hypotheses that families have some "committed quantity" of bread which they will purchase regardless of family income. In the second model, test the following hypotheses:

- (i) Family size, and the prices of bread and meat are not "significant" influences on bread consumption.
  - (ii) Bread is a "normal" good.<sup>1</sup>
  - (iii) The price of meat is not a "significant" factor in determining bread consumption.
  - (iv) Compare the plots of the Engel curves for bread for the "short" and "long" versions of the model using the partial residual plot method for the latter model. Verify for this example that the least squares fit to the partial residual scatter plot yields the same estimate of  $\alpha_B$  as does the full least squares regression.
2. One difficulty with linear models is that the interpretation of the estimated parameters is intimately connected with the units of measurement of the included variables. When weekly income rises by 10 pence, average family bread consumption rises by  $10\alpha$  half peck loaves per week. (A half-peck loaf is the amount of bread which can be made with a bit less than a half-peck of (wheaten) flour -- about 8lbs. 11 oz of bread.) For those not attuned to such esoterica it is often convenient to present estimates of elasticities of demand with respect to income, prices, or whatever.<sup>2</sup> However, since here the estimated relationship is linear, elasticities will depend upon where they are evaluated. The usual practice in such circumstances is to discuss elasticities evaluated at the point of sample means. Compute the following based on your estimates of model (ii):
- (a) The income elasticity of bread
  - (b) The uncompensated own price elasticity of bread
  - (c) The compensated own price elasticity of bread
  - (d) The compensated cross (meat) price elasticity of bread

Interpret these estimates -- that is, explain what they mean in language that an historian, for example, might understand.<sup>3</sup> One way to explore this might be to compare the estimates you obtain in (a-d) with those you would get if you had specified the original demand equations in *log-linear form*. A better way to explore this would be to estimate Box-Cox forms of the bread equation, or to use the Andrews test introduced in class. The linearity assumption is particularly suspect for the family size variable, one might think.

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1. Such "composite" hypotheses are typically tested using a one-tailed test against the relevant simple hypothesis -- in this case  $\alpha_B = 0$ . Why is this? Because, in classical testing situations you would like control the probability of Type 1 error to be less than or equal to some fixed  $\alpha$  and choosing this "sharp null" assures that if the size of the test is  $\alpha$  for this version of the test then the probability of a Type 1 error is strictly less than  $\alpha$  for any other value of the parameter consistent with the composite version of the hypothesis. Why?

2. Keynes says in his *Essays in Biography*, "In the provision of terminology and apparatus to aid thought I do not think that Marshall did economists any greater service than by the explicit introduction of the idea of 'elasticity'." (p.187) He goes on to remark in a footnote, "Mrs. Marshall tells me that he hit on the notion of elasticity as he sat on the roof at Palermo shaded by the bath-cover in 1881 and was highly delighted with it."

3. On the language of economics and econometrics I can't resist recommending Donald N. McCloskey's *The Rhetoric of Economics*, (Wisconsin U. Press, 1985). It is an excellent treatise on, and model of, economic style.

Recall that the distinction between compensated and uncompensated price elasticities is the following: "uncompensated" means "*income* held constant", i.e., the Marshallian demand elasticity, "compensated" means "*utility* held constant", i.e., Hicksian demand elasticity. Of course, the following (Slutsky) relationship holds

$$\eta_{ij}^h = \eta_{ij}^g + \theta_j \eta_{iy}$$

where  $\eta_{ij}^h$  is the *compensated* (Hicksian) elasticity of demand for good  $i$ ; with respect to the price of good  $j$ .

$\eta_{ij}^g$  is the corresponding *uncompensated* (Marshallian) elasticity

$\theta_j$  is the budget share of good  $j$ , i.e.,  $\theta_j = p_j q_j / y$

$\eta_{iy}$  is the income elasticity of demand for good  $i$ .

3. Estimate the demand for meat equation:

$$Q_M = \mu_M + \alpha_M Y + \gamma_M S + \beta_{MB} P_B + \beta_{MM} P_M + u.$$

Test the following:

- (a) Meat is a luxury, recall that this implies that it has an income elasticity greater than one.
- (b) Meat and bread are substitutes, recall that this implies that the Slutsky effect is positive.

The formal definition of substitutes and complements used in economics is usually based on the *Hicksian* demand derivative, sometimes called the Slutsky effect, which in this case is, at sample means,  $\beta_{MB} + \bar{Q}_B \alpha_M$ . Recall that this *should be* identical to  $\beta_{BM} + \bar{Q}_M \alpha_B$ . Of course this Slutsky symmetry should hold at all price-income configurations try to investigate whether it holds (approximately) away from the point of means. Consider whether departures from symmetry should be blamed on the irrationality of the English workers, bad data collection, poor specification of the model, etc. Again, it might be useful to compare your estimates with what is obtained by estimating the model in logarithms

SOME BUDGETS OF ENGLISH RURAL LABORERS							
County	Date	Bread Exp	Meat Exp	Family Size	Total Exp	Price Bread	Price Meat
Berks	1787	79	8	7	99	11.5	8
		68.5	16	7	96.5	11.5	8
		68.5	8	6	88.5	11.5	8
		32	21	5	75	11.5	3.3
		53	12	4	75	11.5	8
		48	20	5	78	11.5	8
		56	18	5	90.5	13.5	8
		98	12	7	124	13.5	8
		50	12	3	81.25	13.5	8
		95.5	0	8	120.25	13.5	8
Dorset	1789	49.5	0	7	60.5	13	7.5
		61	12	6	85.5	13	7.5
		37.5	8	5	58	13	7.5
		37	8	4	65.5	13	7.5
		43	12	5	68.5	13	7.5
		43	8	4	61	13	7.5
		59	30	4	106.25	13	7.5
		59	30	7	108.5	13	7.5
		50	10.5	4	70.75	13	7.5
		41	22.5	4	80.25	13	7.5
		58.5	15	6	88.5	13	7.5
Derby	1788	54	18	6	104	12	7.5
Dorset	1789	74	8	6	99	11.5	8
		40	0	4	69.75	11.5	8
		58	8	5	84	11.5	8
		95	4	9	113	11.5	8
		75	0	8	107.75	11.5	8
		79	4	5	89	11.5	8
		98	8	9	115.25	14	8
		84	24	8	162	14	8
		48	12	5	87	14	8
Oxford	1795	59.5	18	4	109	22	10
		87	12	6	113	22	10
		117	36	8	183.5	22	10
		78	18	4	114	22	10