

## Lecture 5 Some Welfare Econometrics of Empirical Demand Analysis

The second problem set investigates U.S. demand for gasoline. There are at least 3 rationales for this:

1. The gasoline tax and therefore the economics of gasoline demand continues to be an important policy issue in the US.
2. It is important to explore the connection between demand *theory* as it is developed in micro courses and its empirical analogues.
3. Gasoline demand provides an interesting context to explore methods of estimation and testing in dynamic econometric models.

I won't say much about point (1.) at this stage, we will talk more about it on the day we discuss the results of the problem set. I might only say that the US is rather unusual in having quite low taxes on gasoline and therefore has quite low gasoline prices and relatively high per-capita gasoline demand. See the chart reproduced from the *Economist* below.

We have several basic formulations of the theory of demand which are interrelated. Consider first the most natural empirical formulation, the Marshallian demand function,

$$(1) \quad x = g(p, y)$$

where  $y, p$  and  $x$  denote, respectively, income, price and the quantity demanded. We will focus on the simplest case in which only a single commodity is considered and all else is regarded as fixed.

The simplest empirical implementation of this model of demand is the constant elasticity or log-linear formulation,

$$(2) \quad \log x = \alpha + \beta \log p + \gamma \log y$$

where  $\beta$  and  $\gamma$  may be interpreted as price and income elasticity respectively, and are assumed in this formulation to be constants.

This model can be easily estimated using time series data, however, in most applications one would need to be careful to consider possible dynamic elaborations of the model. We will explore some of these a bit later. For the moment we might consider (2) to be an equilibrium relationship.

Why do we care about estimating demand equations like this one? A typical reason is that we would like to do revenue forecasting for tax changes, or perhaps in a slightly more sophisticated vein we might like to investigate the burden, or "dead weight loss", associated with an increase in the tax. I will briefly discuss both of these exercises.

### *Revenue Forecasting*

Suppose there is a per gallon tax on gasoline, so the price is given by

$$p = p_0 + \tau$$

and thus, revenue from the tax is given by

$$R = \tau Q(p_0 + \tau)$$

where  $Q(\cdot)$  is the aggregate Marshallian demand function, Differentiating we have,

$$(3) \quad \frac{dR}{d\tau} = Q + \tau \frac{dQ}{dp}$$

or, as illustrated in Figure 1,

$$\Delta R \approx \Delta\tau \cdot Q + \tau \cdot \Delta Q$$

Such approximations only work well for small changes in the tax rate, for moderate changes one would be better off using

$$\Delta R = \tau_1 Q(p_0 + \tau) - \tau_0 Q(p_0 + \tau_0).$$

A question which arises in the problem set and one that was made infamous in income taxation by the (laughable) Laffer-effect is: Is there a point, i.e., a tax rate, which maximizes revenue?

We may explore this question first in the constant elasticity model. We can reformulate the question as can we find  $\tau$  to make the right hand side of (3.) zero? Try multiplying through by the positive quantity  $p/Q$  to obtain

$$\frac{dR}{d\tau} = p + \tau\beta = p_0 + (1 + \beta)\tau$$

where  $\beta$  denotes the elasticity of demand with respect to price. Solving for  $\tau$  to make this zero yields

$$\tau = -\frac{p_0}{1 + \beta}$$

but note that if  $-1 \leq \beta \leq 0$  as we might expect, i.e., gasoline is *inelastic*, then there is no positive  $\tau$  which accomplishes this. What does this imply about tax policy?

### *Deadweight Loss of a Tax*

The simplest analysis of the efficiency loss due to taxation is the Harberger triangle which we may interpret in terms of consumer's surplus. If we raise the price (via taxation) from  $p_0$  to  $p_1$  in Figure 1, demand is restricted from  $q_0$  to  $q_1$ . There is a revenue gain of  $R^+$ , a revenue loss of  $R^-$ , but viewed in terms of consumers' willingness to pay there is a loss of the shaded triangle which Harberger called the dead weight loss of the tax. This is simply the change in the area above the demand curve usually called consumers' surplus.

Obviously, the simplest way to compute the DWL is Harberger's formula

$$DWL \cong \Delta p \Delta Q / 2$$

but for nonlinear demand specifications we might prefer the formula,

$$DWL = \int_{p_0}^{p_1} x^M(p, y) dp - \Delta p Q_1$$

which is "exact" at least according to the consumer surplus measure generated by the Marshallian demand function. A moments recollection of the basic welfare economics of the theory of demand however, suggests that one would really prefer to compute DWL using the Hicksian demand function. How would we go about doing this?

## Marshallian Gasoline Demand

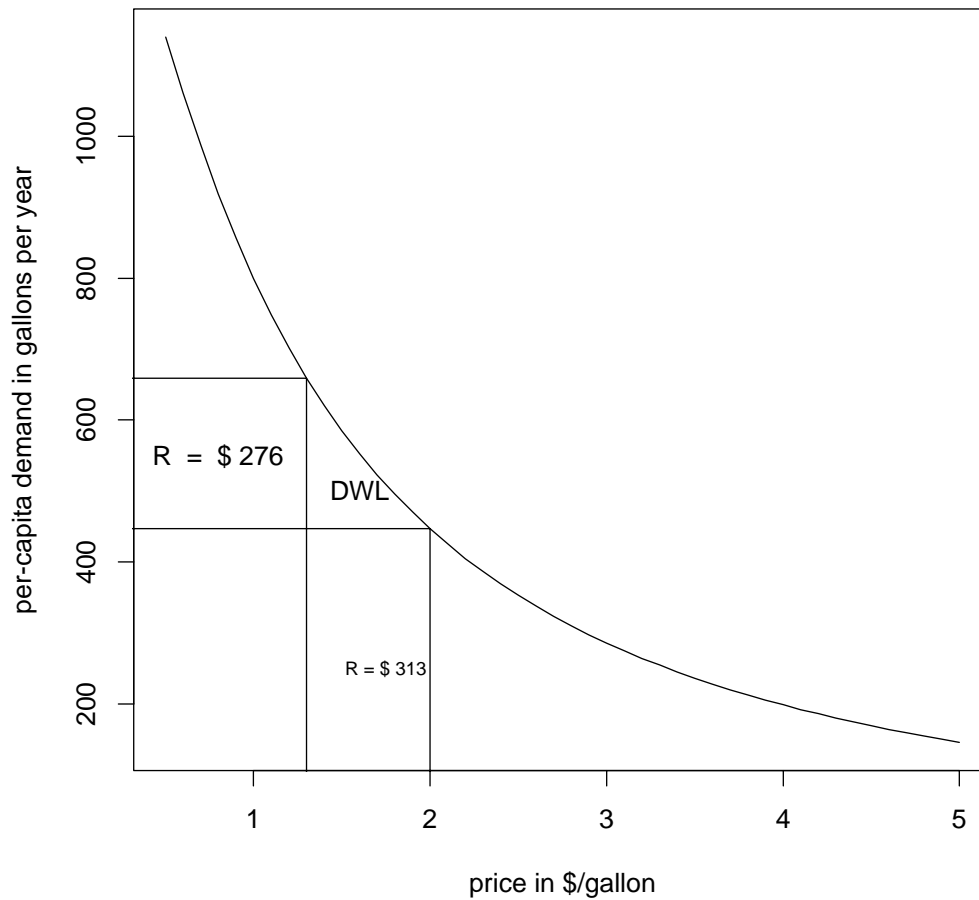


FIGURE 1. This figure illustrates an empirical demand function for percapita U.S. gasoline consumption estimated from postwar quarterly data. The figure also illustrates a policy experiment increasing the price from \$1.30 per gallon to \$2.00 per gallon, which for this demand model has the effect of reducing expenditure on gasoline.

Recall that the Hicksian, or compensated demand function is

$$x^H(p, u) = x^M(p, y(p, u))$$

where  $y(p, u)$  denotes the *expenditure function*, i.e., the income required to achieve utility  $u$  at prices  $p$ . From Shephard's lemma we know that

$$y'(p) = x(p, y(p, u_0))$$

## Marshallian and Hicksian Gasoline Demands

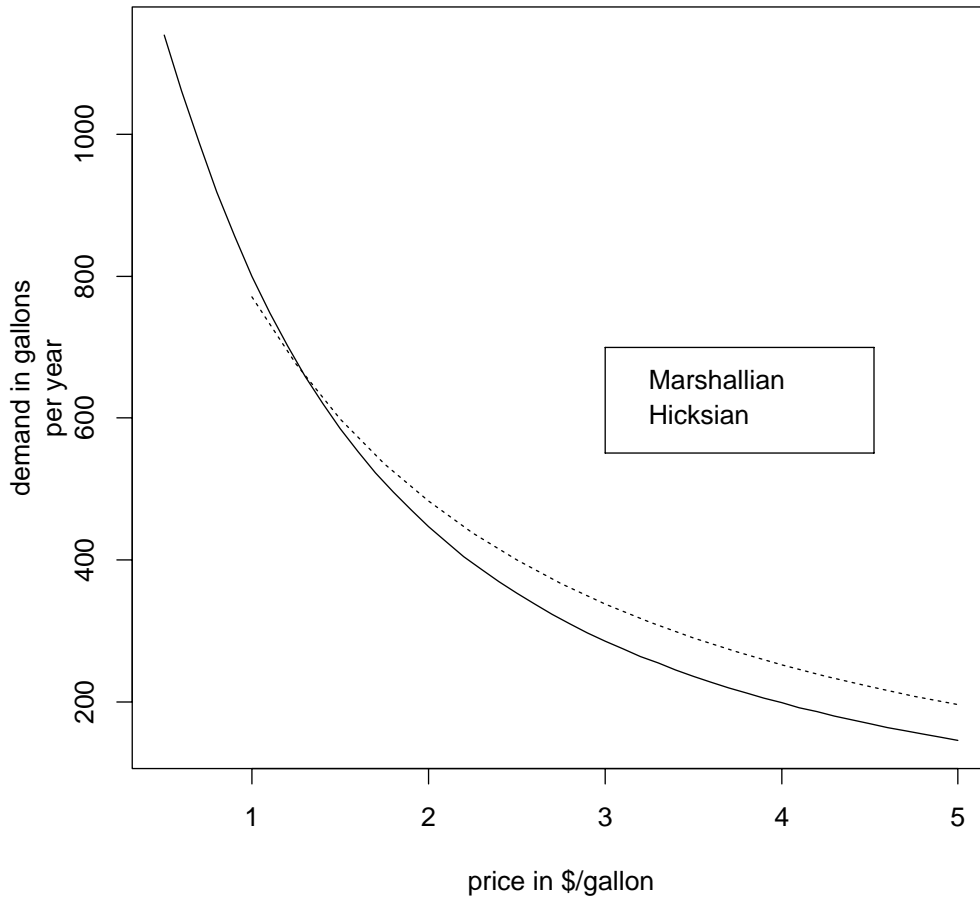


FIGURE 2. This figure illustrates an two empirical demand functions for per-capita U.S. gasoline consumption estimated from postwar quarterly data. The solid line is the Marshallian demand function also depicted in Figure 1, while the dotted line is the Hicksian demand function obtained by solving the differential equation given in the text at the reference price of \$1.30 per gallon. Obviously, there is a Hicksian demand curve corresponding to any chosen initial point on the Marshallian demand curve, at which the two curves intersect.

where the reference level of utility is chosen to satisfy the initial condition

$$y(p_0, u_0) = y_0.$$

The Hicksian demand function holds utility constant as price changes, or to put it slightly differently compensates the consumer for changes in prices by altering his income along the path of a price change. This differential equation is often rather difficult to solve analytically but can

be solved numerically in most cases as the attached *Mathematica* notebook pages illustrate. In the Figure 2, I illustrate the difference between the Marshallian and Hicksian demand equations as estimated for the US gasoline demand data in Problem Set 3. The *Mathematica* notebook pages are included as an attempt to provide a small tutorial on the use of *Mathematica* in empirical economics.

## References

Hausman, J. (1981), "Exact Consumer Surplus and Dead Weight Loss," *Am. Econ. Rev.*