1. A Sample of 20 observations corresponding to the model

$$y_i = \alpha + \beta x_i + u_i$$

gave the following data:

$$\begin{aligned} \sum y_i &= 21.9 \quad \sum (y_i - \bar{y}) = 86.9 \quad \sum (x_i - \bar{x})(y_i - \bar{y}) = 106.4 \\ \sum x_i &= 186.2 \quad \sum (x_i - \bar{x})^2 = 215.4 \end{aligned}$$

Estimate α and β by OLS.

2. Consider the following regression

$$\begin{aligned} col\hat{G}PA &= 1.39 + 0.412 hsGPA + 0.015 ACT - 0.083 skipped \\ & \quad\ (0.33) \quad (0.094) \qquad\quad (0.011) \qquad\quad (0.026) \\ n &= 141, \ R^2 = 0.234 \end{aligned}$$

a) Using the standard normal approximation, find the 95% confidence interval for β_{hsGPA}.
b) Can you reject the hypothesis H_0:$\beta_{hsGPA} = 0.4$ against H_1:$\beta_{hsGPA} \neq 0.4$.
c) Can you reject the hypothesis H_0:$\beta_{hsGPA} = 1$ against H_1:$\beta_{hsGPA} \neq 1$.

3. The following equation is estimated as a production function for Q:

$$\begin{aligned} \ln Q &= 1.37 + 0.632 \ln K + 0.452 \ln L \\ & \qquad\qquad (0.257) \qquad\quad (0.219) \\ R^2 &= 0.98 \qquad\quad cov(b_k, b_l) = 0.055 \end{aligned}$$

where the standard errors for b_k and b_l are given in parentheses. Test the following hypotheses:
a) The capital and labor elasticities of output are identical.
b) There are constant returns of scale.

4. The following estimated equation was obtained by ordinary least squares using quarterly data for 1971 to 1976 inclusive

$$\begin{aligned} Y_t &= 1.10 - 0.0096 X_{1t} - 4.56 X_{2t} + 0.0345 X_{3t} \\ & \quad\ (2.12) \quad (0.0034) \qquad (3.35) \qquad (0.007) \end{aligned}$$

The numbers inside the brackets are estimated standard errors.
Explained sum of squares = 109.24 and Residual sum of squares = 20.22.
a) Test the significance of each of the slope coefficients.
b) Calculate the coefficient of determination R^2.
c) Test the overall significance of the regression.

5. A firm has a production function:

$$Y = AK^\alpha L^\beta M^\gamma e^u$$

where Y is the output, K is capital, L is labour, M is managerial input and u is a random disturbance. At fixed levels of K and L, and given fixed prices, p_y and p_m, for output and managerial services, show that if the firm maximizes profits (i.e. chooses M such that p_m is equal to the value of the marginal product of M), the following relationship holds:

$$lnM = constant + \frac{1}{1-\gamma} \ln \frac{p_y}{p_m} + \frac{\alpha}{1-\gamma} \ln K + \frac{\beta}{1-\gamma} \ln L + error.$$

Suppose now that an econometrician estimates the following misspecified production function:

$$\ln Y_i = a_0 + a_1 \ln K_i + a_2 \ln L_i + v_i$$

by least squares for a cross section sample of such firms, and estimates the degree of returns-to-scale in the industry by

$$\hat{r} = \hat{a}_1 + \hat{a}_2.$$

What would you expect about the estimator $\hat{r}$?