1. A Sample of 20 observations corresponding to the model

\[ y_i = \alpha + \beta x_i + u_i \]

gave the following data:

\[
\begin{align*}
\sum y_i &= 21.9 & \sum (y_i - \bar{y}) &= 86.9 & \sum (x_i - \bar{x})(y_i - \bar{y}) &= 106.4 \\
\sum x_i &= 186.2 & \sum (x_i - \bar{x})^2 &= 215.4
\end{align*}
\]

Estimate \( \alpha \) and \( \beta \) by OLS.

2. Consider the following regression

\[
\text{colGPA} = 1.39 + 0.412 \text{hsGPA} + 0.015 \text{ACT} - 0.083 \text{skipped}
\]

\begin{align*}
(0.33) & \quad (0.094) & \quad (0.011) & \quad (0.026) \\
n &= 141, \quad R^2 = 0.234
\end{align*}

a) Using the standard normal approximation, find the 95% confidence interval for \( \beta_{\text{hsGPA}} \).
b) Can you reject the hypothesis \( H_0: \beta_{\text{hsGPA}} = 0.4 \) against \( H_1: \beta_{\text{hsGPA}} \neq 0.4 \).
c) Can you reject the hypothesis \( H_0: \beta_{\text{hsGPA}} = 1 \) against \( H_1: \beta_{\text{hsGPA}} \neq 1 \).

3. The following equation is estimated as a production function for \( Q \):

\[
\ln Q = 1.37 + 0.632 \ln K + 0.452 \ln L
\]

\begin{align*}
(0.257) & \quad (0.219) \\
R^2 &= 0.98 & \text{cov}(b_k, b_l) &= 0.055
\end{align*}

where the standard errors for \( b_k \) and \( b_l \) are given in parentheses. Test the following hypotheses:

a) The capital and labor elasticities of output are identical.
b) There are constant returns of scale.

4. The following estimated equation was obtained by ordinary least squares using quarterly data for 1971 to 1976 inclusive

\[
Y_t = 1.10 - 0.0096X_{1t} - 4.56X_{2t} + 0.0345X_{3t}
\]

\begin{align*}
(2.12) & \quad (0.0034) & \quad (3.35) & \quad (0.007)
\end{align*}

The numbers inside the brackets are estimated standard errors. Explained sum of squares = 109.24 and Residual sum of squares = 20.22.

a) Test the significance of each of the slope coefficients.
b) Calculate the coefficient of determination \( R^2 \).
c) Test the overall significance of the regression.

5. A firm has a production function:

\[ Y = AK^\alpha L^\beta M^\gamma e^u \]
where $Y$ is the output, $K$ is capital, $L$ is labour, $M$ is managerial input and $u$ is a random disturbance. At fixed levels of $K$ and $L$, and given fixed prices, $p_y$ and $p_m$, for output and managerial services, show that if the firm maximizes profits (i.e. chooses $M$ such that $p_m$ is equal to the value of the marginal product of $M$), the following relationship holds:

$$lnM = constant + \frac{1}{1 - \gamma} \ln \frac{p_y}{p_m} + \frac{\alpha}{1 - \gamma} \ln K + \frac{\beta}{1 - \gamma} \ln L + error.$$ 

Suppose now that an econometrician estimates the following misspecified production function:

$$\ln Y_i = a_0 + a_1 \ln K_i + a_2 \ln L_i + v_i$$

by least squares for a cross section sample of such firms, and estimates the degree of returns-to-scale in the industry by

$$\hat{r} = \hat{a}_1 + \hat{a}_2.$$ 

What would you expect about the estimator $\hat{r}$?