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BIASES IN DYNAMIC MODELS WITH FIXED EFFECTS

BY STEPHEN NICKELL[1]

It is well known from the Monte-Carlo work of Nerlove that using the standard within-group estimator for dynamic models with fixed individual effects generates estimates which are inconsistent as the number of "individuals" tends to infinity if the number of time periods is kept fixed. In this paper we present analytical expressions for these inconsistencies for the first order autoregressive case.

INTRODUCTION

SINCE THE PIONEERING WORK of Balestra and Nerlove [2] on the demand for natural gas, economists have made extensive use of panel data in the elucidation of economic relationships. In this work it has been typically assumed that the error term corresponding to the ith individual in the tth time period, u_{it}, is made up of three components, one individual specific, one time specific, and a remainder which is both time and individual specific. Thus we have

$$(1) \qquad u_{it} = f_i + \mu_t + \epsilon_{it}$$

where the three components are often assumed to be uncorrelated with each other and, indeed, with the included variables in the equation. The fundamental question was generally considered to be, in the words of Nerlove, "whether or not to treat f_i and μ_t as parameters or as random variables."[2] This is particularly important in the case of f_i because the typical panel has vastly more individuals than time periods and treating the f_i as parameters introduces an enormous number of additional parameters into the model compared with the alternative in which the f_i are usually considered as being drawn from a distribution with but a single unknown parameter. The advantages of this latter so called random effects model over the alternative fixed effects model are thus manifest particularly when it is realized that the fixed effects model implies that one is ruling out of order all the information that may be gleaned by directly comparing one individual with another.

However, in recent years, the error components model has been looked at from a slightly different viewpoint by some researchers who view the individual effects, f_i, as relevant but unobserved characteristics which are highly likely to be correlated with the observed exogenous variables in the model. Thus, for example, in Ashenfelter's study of the effect of training programs on earnings the individual effects are talked of as capturing "such factors as ability, motivation or other previous investments in human capital"[3] which are clearly thought of as

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[2] See Nerlove [13, p. 361].

[3] See Ashenfelter [1, p. 49].

being correlated with the extent to which the individual participates in training programs. Hausman [4] takes a similar view in his brief discussion of wage equations in which he finds strong evidence that the individual effects are correlated with the observed exogenous variables and uncompromisingly rejects the uncorrelated random effects model. This is very important point because if one takes the view that, in any particular model, the individual effects are likely to be correlated with all the observed exogenous variables, then one is lead inexorably to the fixed effects model.[4] This will enable one to obtain coefficients on the exogenous variables which do not suffer from bias due to the omission of relevant individual attributes. Indeed, Mundlak [11] and Chamberlin [3] have shown, in the context of linear regression with strictly exogenous regressors, that the random effects model leads to the same estimators as the fixed effects model in situations where the individual effects are correlated with the exogenous variables and thus, in these hardly unusual circumstances, the fixed effects model assumes paramount importance.[5]

Unfortunately, as the Monte-Carlo work of Nerlove [12, 13] makes clear, the fixed effects model suffers from an important drawback. Standard methods of estimation are liable to lead to seriously biased coefficients in dynamic models. A typical set of panel data has a rather large number of individuals and a rather small number of time periods and it is in just these circumstances that the biases, which are essentially of the Hurwicz type, are most serious.[6] The fact that they will go to zero when the number of time periods becomes infinite is scant consolation. It is the purpose of this paper to investigate these biases analytically for the first-order autoregressive case. Two models will be considered. These are, omitting the time effects for simplicity of exposition,

$$y_{it} = \beta + \rho y_{it-1} + \sum_j \beta_j x_{ijt} + f_i + \epsilon_{it} \qquad (i = 1 \ldots N;\ t = 1 \ldots T), \tag{2}$$

and

$$y_{it} = \beta + \sum_j \beta_j x_{ijt} + f_i + u_{it} \qquad (i = 1 \ldots N;\ t = 1 \ldots T), \tag{3}$$

$$u_{it} = \rho u_{it-1} + \epsilon_{it}. \tag{4}$$

[4] A superior alternative to the fixed effects or within-groups estimator is available if one is prepared to assert, *a priori*, that some of the included exogenous variables are not correlated with the individual effects. This is discussed in Hausman and Taylor [5].

[5] It is, of course, always open to the investigator to take a random effects model and specify a joint distribution for the random effects and the included variables and then to integrate the former out of the likelihood function. This general procedure is discussed in Chamberlin [3] and an illustration of problems it can cause in a particular context is presented in Lancaster and Nickell [8]. The basic difficulty is, of course, that the estimates obtained often depend crucially on the distributional assumptions made about the individual effects, something on which economic theory has little to say. Unfortunately, as soon as one moves outside the framework of linear regression with strictly exogenous regressors, the fixed effects model (with its distribution-free advantages) generates inconsistent estimates for fixed T. Heckman [6] presents some Monte Carlo estimates on the size of these biases in some simple probit models.

[6] It is important to recognize that the Hurwicz type bias may be serious in *any* dynamic model estimated using a short time series.

f_i are fixed parameters, ϵ_{it} are $IN(0, \sigma_\epsilon^2)$, and $|\rho| < 1$. u_{it} is thus stationary and ergodic. If we let E_i represent the expectation of a random variable taken over the individuals for a fixed time period, the above ensures that $E_i \epsilon_{it} = 0$ and we assume that $E_i \epsilon_{it} f_i = 0$.

Rearranging (3) and (4) gives

$$y_{it} = \beta(1-\rho) + \rho y_{it-1} + \sum_j \beta_j (x_{ijt} - \rho x_{ijt-1}) + f_i(1-\rho) + \epsilon_{it}. \tag{5}$$

Concentrating our exposition on the lagged dependent variable model (2), the standard estimation procedure is to start by eliminating the fixed effects f_i. This may be done in any number of ways but the standard technique is to subtract the time mean of (2) from (2) itself to yield

$$y_{it} - y_{i\cdot} = \rho(y_{it-1} - y_{i\cdot-1}) + \sum_j \beta_j (x_{ijt} - x_{ij\cdot}) + (\epsilon_{it} - \epsilon_{i\cdot}) \tag{6}$$

where for any variable z_{it}, $z_{i\cdot} = (1/T)\sum_{t=1}^{T} z_{it}$ and $z_{i\cdot-1} = (1/T)\sum_{t=0}^{T-1} z_{it}$. It is clear that OLS estimates based on (6) will be biased *even* if N, the number of individuals, goes to infinity. This arises because in these circumstances the correlation between y_{it-1} and $\epsilon_{i\cdot}$, for example, does not go to zero. The remainder of the paper is devoted to an analysis of these biases and is set out as follows. In the next section we shall compute the bias as $N \to \infty$ in the model with no exogenous variables and we shall then look at the effect of including exogenous variables on these results. In subsequent sections we compare our analytical computations with some of the extensive Monte-Carlo results presented in Nerlove [**12**, **13**] and Maddala [**9**].

1. BIASES IN AUTOREGRESSIVE MODEL WITH FIXED EFFECTS

If we remove the exogenous variables from (6), we have

$$y_{it} - y_{i\cdot} = \rho(y_{it-1} - y_{i\cdot-1}) + (\epsilon_{it} - \epsilon_{i\cdot}) \qquad (i = 1, \ldots, n;\ t = 1 \ldots T). \tag{7}$$

Note that this equation follows from both the lagged dependent variable and the residual autoregression model. In order to estimate ρ we have a number of options. The standard method is to use OLS on (7) pooling all the cross-sections. It is perfectly legitimate, however, to use directly only one cross-section in estimating (7) although the data on all the others have, of course, already been used to compute the time means. If we use the tth cross-section we may define an OLS estimate

$$\hat{\rho}_t = \sum_{i=1}^{N} (y_{it-1} - y_{i\cdot-1})(y_{it} - y_{i\cdot}) \Big/ \sum_{i=1}^{N} (y_{it-1} - y_{i\cdot-1})^2. \tag{8}$$

We shall now compute the asymptotic bias or inconsistency by taking probability limits as $N \to \infty$. Thus we have, using (7),

$$\text{(9)} \qquad \operatorname*{plim}_{N\to\infty} \hat{\rho}_t = \rho + \frac{\text{plim}_{N\to\infty} 1/N \sum_{i=1}^{N} (y_{it-1} - y_{i\cdot -1})(\epsilon_{it} - \epsilon_{i\cdot})}{\text{plim}_{N\to\infty} 1/N \sum_{i=1}^{N} (y_{it-1} - y_{i\cdot -1})^2}$$

or

$$\text{(10)} \qquad \operatorname*{plim}_{N\to\infty} (\hat{\rho}_t - \rho) = A_t / B_t, \qquad \text{say.}$$

To avoid continual use of more complicated terminology we shall, in future, generally refer to anything of the form $\text{plim}_{N\to\infty}(\hat{\rho}_t - \rho)$ as a bias. Since ϵ_{it} are random drawings from a normal distribution when t is fixed we can replace[7] plims as $N \to \infty$ by expectations across i, E_i, and thus obtain

$$A_t = E_i (y_{it-1} - y_{i\cdot -1})(\epsilon_{it} - \epsilon_{i\cdot})$$

or

$$\text{(11)} \qquad A_t = -E_i y_{it-1}\epsilon_{i\cdot} - E_i y_{i\cdot -1}\epsilon_{it} + E_i y_{i\cdot -1}\epsilon_{i\cdot},$$

noting that $E_i y_{it-1}\epsilon_{it} = 0$. Before proceeding it is worth pointing out that stationarity implies the following result. Removing the exogenous variables and time effects, (2) implies

$$\text{(12)} \qquad y_{it} = (\beta + f_i)/(1-\rho) + \sum_{j=0}^{\infty} \rho^j \epsilon_{it-j}.$$

Then (11) and (12) imply

$$A_t = -E_i \left\{ \left(\sum_{j=0}^{\infty} \epsilon_{it-j-1}\rho^j \right) \left(\frac{1}{T} \sum_{s=1}^{T} \epsilon_{is} \right) \right\} - E_i \left\{ \frac{\epsilon_{it}}{T} \sum_{s=1}^{T} \sum_{j=0}^{\infty} \epsilon_{is-j-1}\rho^j \right\}$$
$$+ E_i \left\{ \left(\frac{1}{T} \sum_{s=1}^{T} \sum_{j=0}^{\infty} \epsilon_{is-j-1}\rho^j \right) \left(\frac{1}{T} \sum_{s=1}^{T} \epsilon_{is} \right) \right\}$$

where we have used the fact that $E_i \epsilon_{it} = E_i \epsilon_{it} f_i = 0$. Noting that $E_i \epsilon_{it}^2 = \sigma_\epsilon^2$ independent of t, we have after some manipulation

$$A_t = -\frac{\sigma_\epsilon^2}{T} \frac{(1-\rho^{t-1})}{1-\rho} - \frac{\sigma_\epsilon^2}{T} \frac{(1-\rho^{T-t})}{1-\rho} + \frac{\sigma_\epsilon^2}{T} \left\{ \frac{1}{1-\rho} - \frac{1}{T} \frac{(1-\rho^T)}{(1-\rho)^2} \right\}$$

where the three terms correspond to those in the previous section. Collecting terms then yields

$$\text{(13)} \qquad A_t = -\frac{\sigma_\epsilon^2}{T(1-\rho)} \left\{ 1 - \rho^{t-1} - \rho^{T-t} + \frac{1}{T} \frac{(1-\rho^T)}{(1-\rho)} \right\}.$$

[7]Our analysis of the bias does not depend on the normality of ϵ_{it}. So long as the ϵ_{it} are identically and independently distributed for each t, then the results will go through.

Proceeding to the denominator of the bias, B_t, we have, using (12),

$$
\begin{aligned}
B_t &= E_i\left(\sum_{j=0} \rho^j \epsilon_{it-j-1} - \frac{1}{T} \sum_{s=1}^{T} \sum_{j=0}^{\infty} \rho^j \epsilon_{is-j-1} \right)^2 \\
&= E_i\left(\sum_{j=0}^{\infty} \rho^j \epsilon_{it-j-1} \right)^2 - \frac{2}{T} E_i\left(\sum_{j=0}^{\infty} \rho^j \epsilon_{it-j-1} \right)\left(\sum_{s=1}^{T} \sum_{j=0}^{\infty} \rho^j \epsilon_{is-j-1} \right) \\
&\quad + \frac{1}{T^2} E_i\left(\sum_{s=1}^{T} \sum_{j=0}^{\infty} \rho^j \epsilon_{is-j-1} \right)^2 \\
&= \frac{\sigma_\epsilon^2}{1-\rho^2} - \frac{2\sigma_\epsilon^2}{T(1-\rho^2)} \left\{ \frac{1-\rho^t}{1-\rho} + \rho \frac{(1-\rho^{T-t})}{1-\rho} \right\} \\
&\quad + \frac{\sigma_\epsilon^2}{T(1-\rho)^2} \left\{ 1 - \frac{2\rho(1-\rho^T)}{T(1-\rho^2)} \right\}
\end{aligned}
$$

where again the three terms in the final expression correspond to those in the previous line. Collecting terms and using (13) we obtain

$$
B_t = \frac{\sigma_\epsilon^2}{1-\rho^2}\left(1 - \frac{1}{T}\right) + \frac{2\rho}{1-\rho^2} A_t. \tag{14}
$$

The final value for the bias is thus given by

$$
\operatorname*{plim}_{N\to\infty} (\hat{\rho}_t - \rho) = \left\{ \frac{2\rho}{1-\rho^2} - \left[\frac{1+\rho}{T-1} \left(1 - \rho^{t-1} - \rho^{T-t} + \frac{1}{T} \frac{(1-\rho^T)}{(1-\rho)} \right) \right]^{-1} \right\}^{-1} \tag{15}
$$

or

$$
\begin{aligned}
\operatorname*{plim}_{N\to\infty} (\hat{\rho}_t - \rho) &= \frac{-(1+\rho)}{T-1} \left\{ 1 - \rho^{t-1} - \rho^{T-t} + \frac{1}{T} \frac{(1-\rho^T)}{1-\rho} \right\} \\
&\quad \times \left\{ 1 - \frac{2\rho}{(T-1)(1-\rho)} \left[1 - \rho^{t-1} - \rho^{T-t} + \frac{1}{T} \frac{(1-\rho^T)}{1-\rho} \right] \right\}^{-1}.
\end{aligned} \tag{16}
$$

The first of these expansions is computationally simpler but the second reveals clearly that the inconsistency is $O(1/T)$. There are a number of interesting points about this bias. First, if $\rho > 0$, it is invariably negative since $A_t < 0$. Second, the bias depends on t and hence varies with the cross-section which is used to generate the estimate. Indeed, (15) clearly reveals that the bias will be smaller if we use cross-sections at the ends of the sample period and it will be largest if we use the middle one. More relevant for practical purposes, however, is the bias if we generate our estimate, $\hat{\rho}$, using the whole sample. Thus we have

$$\hat{\rho} = \sum_{t=1}^{T} \sum_{i=1}^{N} (y_{it} - y_{i\cdot -1})(y_{it} - y_{i\cdot}) / \sum_{t=1}^{T} \sum_{i=1}^{N} (y_{it-1} - y_{i\cdot -1})^2$$

and in our standard notation the bias is given by

$$\operatorname*{plim}_{N\to\infty} (\hat{\rho} - \rho) = \sum_{t=1}^{T} A_t / \sum_{t=1}^{T} B_t$$

which yields

$$\text{(17)} \qquad \operatorname*{plim}_{N\to\infty} (\hat{\rho} - \rho) = \left\{ \frac{2\rho}{1-\rho^2} - \left[\frac{1+\rho}{T-1} \left(1 - \frac{1}{T} \frac{(1-\rho^T)}{(1-\rho)} \right) \right]^{-1} \right\}^{-1}$$

or

$$\text{(18)} \qquad \operatorname*{plim}_{N\to\infty} (\hat{\rho} - \rho) = \frac{-(1+\rho)}{T-1} \left\{ 1 - \frac{1}{T} \frac{(1-\rho^T)}{1-\rho} \right\} \times \left\{ 1 - \frac{2\rho}{(1-\rho)(T-1)} \left[1 - \frac{1}{T} \frac{(1-\rho^T)}{1-\rho} \right] \right\}^{-1}.$$

Furthermore, *for reasonably large values* of T we have the simple approximation

$$\text{(19)} \qquad \operatorname*{plim}_{N\to\infty} (\hat{\rho} - \rho) \simeq \frac{-(1+\rho)}{T-1}.$$

On the other hand for small values of T we have

$$\operatorname*{plim}_{N\to\infty} (\hat{\rho} - \rho) = \frac{-(1+\rho)}{2} \quad \text{for } T = 2,$$

$$= \frac{-(2+\rho)(1+\rho)}{2} \quad \text{for } T = 3,$$

with the latter confirming the result in Chamberlin [3, p. 228]. These results are of considerable interest. Apart from the fact that the bias is always negative if $\rho > 0$, we can see how large it is if T is small. Even with $T = 10$, which is the order of magnitude of most sets of panel data, if $\rho = 0.5$ then the bias is -0.167

which can hardly be ignored. Furthermore, the bias does not go to zero as ρ goes to zero. However, these biases are not as severe as the standard Hurwicz biases associated with first-order autoregressive processes with a constant term. In this case, at least to order $1/T$, the bias is $-(1+3\rho)/T-1$ which is larger than those considered above. This approximation and a large number of related results may be found in Mariott and Pope [10] and Kendall [7]. The standard Hurwicz bias differs from that given in (17) basically because if we let $A = \sum_t (y_{it-1} - y_{i.-1})(\epsilon_{it} - \epsilon_{i.})$ and $B = \sum_t (y_{it-1} - y_{i.-1})^2$, the Hurwicz bias in the standard regression (with one value of i) is given by $E(A/B)$. We, of course, are computing the bias as $N \to \infty$ and are thus considering $E(A)/E(B)$ where expectations are all taken across i. These expressions are related in the sense that the approximation to the standard Hurwicz bias to order T^{-1} is given by

$$E(A/B) = \frac{E(A)}{E(B)}\left\{1 - \frac{\operatorname{cov}(AB)}{E(A)E(B)} + \frac{\operatorname{var}(B)}{E^2(B)}\right\}$$

which yields the formula cited above. It is, of course, the second and third terms which make the standard bias bigger. Nevertheless it usually troubles us less because the typical time series is very much longer than typical panel. Furthermore, when we introduce exogenous variables the situation gets worse as we shall see in the next section.

2. THE INCLUSION OF EXOGENOUS VARIABLES

In this section we shall concentrate on model (2) with the lagged endogenous variable. If we define the following matrices

$$\tilde{y}_t = [\,y_{it} - y_{i.}\,], \quad N \times 1 \text{ vector},$$

$$\tilde{y}_{t-1} = [\,y_{it-1} - y_{i.-1}\,], \quad N \times 1 \text{ vector},$$

$$\tilde{X}_t = [\,x_{ijt} - x_{ij.}\,], \quad N \times J \text{ matrix},$$

$$\tilde{\epsilon}_t = [\,\epsilon_{it} - \epsilon_{i.}\,], \quad N \times 1 \text{ vector},$$

$$b = [\,\beta_j\,], \quad J \times 1 \text{ vector},$$

(6) may then be written in deviation form as

$$\tilde{y}_t = \rho\tilde{y}_{t-1} + \tilde{X}_t b + \tilde{\epsilon}_t \tag{20}$$

where we have introduced J exogenous variables. To obtain the most efficient estimates we may now stack these equations over the T time periods to obtain

$$\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_T \end{bmatrix} = \rho \begin{bmatrix} \tilde{y}_0 \\ \tilde{y}_1 \\ \vdots \\ \tilde{y}_{T-1} \end{bmatrix} + \begin{bmatrix} \tilde{X}_1 \\ \tilde{X}_2 \\ \vdots \\ \tilde{X}_T \end{bmatrix} b + \begin{bmatrix} \tilde{\epsilon}_1 \\ \tilde{\epsilon}_2 \\ \vdots \\ \tilde{e}_T \end{bmatrix} \tag{21}$$

or

(22) $$\tilde{y} = \rho\tilde{y}_{-1} + \tilde{X}b + \tilde{\epsilon}$$

again in obvious notation. If $\hat{\rho}$, $\hat{b}$ are the OLS estimates, using standard procedures we obtain

(23) $$\hat{\rho} - \rho = (\tilde{y}'_{-1}M\tilde{y}_{-1})^{-1}\tilde{y}'_{-1}M\tilde{\epsilon},$$

(24) $$\hat{b} - b = -(\tilde{X}'\tilde{X})^{-1}\tilde{X}'\tilde{y}_{-1}(\hat{\rho} - \rho) + (\tilde{X}'\tilde{X})^{-1}\tilde{X}\tilde{\epsilon},$$

where $M = I - \tilde{X}(\tilde{X}'\tilde{X})^{-1}\tilde{X}'$. Taking plims as $N \to \infty$ and noting that

$$\operatorname*{plim}_{N\to\infty} \frac{1}{NT}\tilde{y}'_{-1}M\tilde{\epsilon} = \operatorname*{plim}_{N\to\infty} \frac{1}{NT}\tilde{y}'_{-1}\tilde{\epsilon}$$

since $\tilde{X}$ is exogenous, we have

(25) $$\operatorname*{plim}_{N\to\infty}(\hat{\rho} - \rho) = \left(\operatorname*{plim}_{N\to\infty}\frac{1}{NT}\tilde{y}'_{-1}M\tilde{y}_{-1}\right)^{-1}\operatorname*{plim}_{N\to\infty}\frac{1}{NT}\tilde{y}'_{-1}\tilde{\epsilon}$$

and

(26) $$\operatorname*{plim}_{N\to\infty}(\hat{b} - b) = -\operatorname*{plim}_{N\to\infty}\left[(\tilde{X}'\tilde{X})^{-1}\tilde{X}'\tilde{y}_{-1}\right]\operatorname*{plim}_{N\to\infty}(\hat{\rho} - \rho).$$

Note first from our previous analysis that we have already calculated $\operatorname{plim}_{N\to\infty}(1/NT)\tilde{y}'_{-1}\tilde{\epsilon}$ and this is given by

(27) $$\operatorname*{plim}_{N\to\infty}\frac{1}{NT}\tilde{y}'_{-1}\tilde{\epsilon} = \frac{1}{T}\sum A_t = \frac{-\sigma_\epsilon^2}{T(1-\rho)}\left(1 - \frac{1}{T}\frac{(1-\rho^T)}{1-\rho}\right).$$

This result remains unaltered by the introduction of exogenous variables since their incorporation into equation (12) will have no effect given they are uncorrelated with the error term. When ρ is positive we may derive the direction of the biases. Since A_t is negative, $\operatorname{plim}(\hat{\rho} - \rho)$ must be negative and will indeed be larger (in absolute value) than if the exogenous variables are omitted since the denominator of the expression in (25) is reduced by the inclusion of M. The bias on $\hat{b}$ depends on the relationship between the exogenous variables and $\tilde{y}_{-1}$. If an exogenous variable is positively related (in the regression sense) to $\tilde{y}_{-1}$, then (26) indicates that its coefficient will be upward biased and vice-versa.

Having derived all these explicit results for the inconsistencies generated in the dynamic fixed effects model it is obviously worth comparing them with what we know from Monte Carlo experiments. Nerlove **[12]** obtains a number of results on fixed effect first-order autoregressive models with no exogenous variables. He concludes on page 58 that the bias in ρ is uniformly negative (he only considers $\rho > 0$) as we would expect. Furthermore, his actual estimates provide most compelling confirmation of the usefulness of our results. Using a sample size of $T = 10$, $N = 25$, he computes a large number of estimates of $\hat{\rho}$ corresponding to true values of ρ equal to 0.0, 0.1, 0.5, and 0.9. He does this for a large number of different values of the variance of f_i relative to that of ϵ_{it}, a number which does not influence the asymptotic bias and indeed, as he himself points out, does not affect his $\hat{\rho}$ estimates except when $\rho = 0.9$. For $\rho = 0.0$, 0.1, 0.5 the average Nerlove Monte Carlo estimates of $\hat{\rho}$ reported in Table C1 are -0.10083, -0.01115, and 0.33354 respectively.[8] The corresponding $\hat{\rho}$'s given by equation (17) are -0.10000, -0.01108, and 0.33779. They are thus more or less exact even though they are only asymptotic in N and Nerlove has $N = 25$. The results for $\rho = 0.9$ are not quite so clear cut. Equation (17) gives $\hat{\rho} = 0.65677$ whereas the Monte Carlo results only yield this value if the variance of f_i is not large relative to the total error variance. Thus the average Monte Carlo estimate of $\hat{\rho}$ is 0.657 if we only consider those experiments where the variance of f_i is less than one third of the total—otherwise the Monte Carlo estimates become considerably higher and our asymptotic result is no longer accurate.

Turning to the introduction of exogenous variables we may note an experiment by Maddala for $\rho = 0.7$ where he again has $N = 25$ and although he claims to have $T = 10$ it appears that he only generates ten values of the dependent variable for each i, which would imply $T = 9$ in our notation given the lagged value on the right-hand side. When there is no exogenous variable he generates $\hat{\rho} = 0.475$ (equation (17) yields $\rho = 0.4805$) whereas the introduction of an exogenous variable with a true coefficient equal to 0.5 reduces $\hat{\rho}$ to 0.3178 and generates an estimate of β which is strongly upward biased as we might expect from (26).

SUMMARY

We have presented analytical expressions for the asymptotic biases in first-order autoregressive models estimated by OLS using panel data and including individual fixed effects. These asymptotic biases are shown to be both large and to coincide almost exactly with the estimates provided by the Monte Carlo studies of Nerlove (1967) and Maddala (1971).

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[8] When $\rho = 0.0$, Nerlove reports a sequence of $\hat{\rho}$'s for various different values of var(f_i)/(total error variance). All the numbers in this sequence bar one lie between -0.086 and -0.115. The odd man out is -0.010 and this has been omitted in computing the average presented in the text on the grounds that it is probably a typographical error.

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