Fall 2001
Midterm Exam

Please answer all 3 questions. Even if you are unsure about some aspect of the questions, try to write something sensible – partial credit will be given. The exam is closed book, closed notes and will last 2 hours.

1. [20 points] The partial residual plot is a useful device for visualizing the bivariate relationship between two variables, $y$ and $z$, in the multivariate regression model

$$y = X\beta + z\gamma + u.$$  

(1)

It involves “controlling for $X$” by plotting $\hat{y} = M_x y$ versus $\tilde{z} = M_x z$. A dubious alternative to the partial residual plot is the so-called “Barro plot”, which in its simplest form computes

$$\hat{y}_i = \hat{u}_i + z_i \hat{\gamma}$$

and then plots $\hat{y}_i$ versus $z_i$. (Here the $\{\hat{u}_i\}$ are the OLS residuals from (1) and $\hat{\gamma}$ is the OLS estimate of $\gamma$ from (1)). Suppose a straight line through the origin

$$\hat{y}_i = a z_i + v_i$$

is fit by minimizing $\sum v_i^2$ with respect to $a$. Show that the least squares estimator, $\hat{a} = \hat{\gamma}$, and briefly explain under what conditions, and why, this plot gives a misleading impression of the precision of the estimate $\hat{\gamma}$.

2. [40 points] Consider the model

$$y = X\beta + u$$

and the following expressions for the asymptotic covariance matrix of candidate estimators, $\hat{\beta}$:

\[
\begin{align*}
(i) & \quad \sigma^2(X'X)^{-1} \\
(ii) & \quad (X'\Omega^{-1}X)^{-1}
\end{align*}
\]
\[(iii) \quad (X'X)^{-1}(X'\Omega X)(X'X)^{-1}\]
\[(iv) \quad \sigma^2(X'P_Z X)^{-1}\]
\[(v) \quad (X'P_Z X)^{-1}(X'P_Z \Omega P_Z X)(X'P_Z X)^{-1}\]
\[(vi) \quad (X'Z(Z\Omega Z)^{-1}Z'X)^{-1}\]

(a) For each of the cases (i-vi) give an explicit expression for the estimator, \(\hat{\beta}\), and the state assumptions on \(u\) and \(Z\) that would justify the covariance expression.

(b) In the cases that the \(u\) assumptions are compatible for several different estimators, explain briefly which estimators are preferred.

3. **[40 points]** Suppose you have estimated the model for U.S. coffee demand,

\[y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \beta_0 + \beta_1 p_t + \beta_2 p_{t-1} + \beta_3 z_t + u_t\]

based on quarterly observations 1970-2000, where \(y_t = \log\text{ per capita annual demand for coffee (in metric tons)}, p_t = \log\text{ price per ton}, z_t = \log\text{ per capita income.}

(a) For your report you need to present an estimate of the long-run (steady-state) elasticity of demand with respect to price, how would you do this?

(b) How would you verify that the estimated model had stable, i.e. non-explosive, dynamic behavior?

(c) Explain briefly how you would use the estimated model to make a forecast of demand for the four quarters of 2001.

(d) Suppose you want a revised version of your steady state elasticity in part a.) assuming that an increase in price would be compensated by a commensurate increase in income that would leave consumers as well off as before the price increase. How would you estimate the new elasticity?

(e) It is suggested that you should have included the price of tea in the model, supposing that over the sample period tea and coffee prices were strongly positively correlated, explain how this omission would be expected to affect the long-run price elasticity computed in part (a.).

(f) At the end of your conference presentation of these results someone in the audience asks: How do you know that what you have estimated is a demand equation, and not a supply equation? How would you respond?