

The Impact of Entrepreneur Characteristics and Bankruptcy Rules on Firm Performance

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Abstract

How important for firm performance are differences in owner's personal characteristics (risk tolerance or optimism) versus the institutional environment in which the firm operates (bankruptcy institutions, access to credit or return distributions)? To answer this question we examine SSBF data on small incorporated firms and construct a dynamic, computable model with heterogeneous agents in which entrepreneurs weigh the firm's current return against expected future returns. The model accounts for three puzzles found in the data: the risk/return trade-off from operating a small firm seems unattractive; owners' personal investments are poorly diversified; and many owners "bail out" their firm rather than declare bankruptcy. We show that entrepreneurs need not have personal characteristics that are significantly different from standard values, but the environment in which they operate matters greatly. The option to declare bankruptcy insures an entrepreneur against extreme current loss and the ability to bail out the firm with personal funds preserves the potential for high future gains. Welfare gains from bankruptcy reform or improved access to credit are equivalent to increasing net-worth by 35% for some agents.

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1 Introduction

How important for firm performance are differences in owner personal characteristics versus the institutional environment in which a firm operates? This question is important because bankruptcy institutions, access to credit and a firm's capital structure can be affected by policy, but innate characteristics such as risk tolerance or optimism cannot. We derive facts and theory to assess the impact of these personal characteristics and institutions on firms. We first analyze data from the Survey of Small Business Finance (SSBF) and find three puzzles: entrepreneurs face a seemingly unattractive risk/return trade-off, have poorly diversified personal investments, and many have negative equity. Second, we construct a dynamic, computable model with three features: (i) Forward looking entrepreneurs weigh current gains/losses against expected future returns. (ii) The distribution of firm returns from SSBF data is peaked in the middle, with "fat" asymmetric tails. (iii) Bankruptcy protection insures entrepreneurs against poor returns, but permits upside gain. Finally, we show that institutions such as bankruptcy are important and interact with differences in personal characteristics to reconcile the puzzles found in the data.¹

Small firms are a significant and vital part of the macroeconomy of most economies. The U.S. Small Business Administration defines a firm as small if it has less than 500 employees, but the median number of employees in the 1993 and 1998 SSBF samples is 7, with median assets of about \$270,000. Small firms account for 99.7 percent of all U.S. employer firms (89.3 percent for firms with less than 20 employees), produce more than 50 percent of non-farm private U.S. GDP, employ half of all private sector employees and pay 45 percent of total private payroll. They are a source of "good jobs," generating 60 to 80 percent of net new jobs annually over the last decade, employing 41 percent of high tech workers (scientists, engineers, and computer workers) and producing 13 to 14 times more patents per employee than large patenting firms.²

We study these firms in a model economy with many long lived agents, that differ in their willingness to bear risk, and a representative investor (e.g., bank). Entrepreneurs are agents that choose to operate a risky production technology. The risk neutral investor (bank), with an elastic supply of funds, makes risky short term loans to entrepreneurs. Each period, the heterogeneous entrepreneurs decide the scale of their firm and the composition of firm

¹The analysis is challenging because risk aversion is central to the debate on entrepreneurship, but individual coefficients and the distribution of risk aversion are not directly observable. The data also document substantial heterogeneity in entrepreneur behavior, hence we cannot use a representative agent model.

²See <http://www.sba.gov/advo/stats/sbfaq.pdf>.

finance: the mix of personal funds and bank loans. At the end of the period entrepreneurs observe their firm's return and decide whether to repay the loan or default. If default occurs, the bank recovers only a fraction of the loan and the firm cannot operate for several periods. We solve an individual agent's problem for consumption, the amount of personal net-worth to invest, firm scale, and firm debt-equity structure.³ We next derive cumulative probability distribution functions (cdfs) to account for heterogeneous risk aversion and uncertain firm returns. We then use the model to predict the cdfs of firms' capital structure, assets, and the percentage of personal net-worth entrepreneurs invest in their firms. The discipline imposed by the check for consistency between the distributions predicted by the model and those observed in the data is the analog of matching moments predicted by models with summary statistics from data in quantitative macroeconomic models (cf., Prescott (2006)).

There is a large literature on how institutions affect firms' ability to raise finance. For example, Beim and Calomiris (2001) note that legal systems define property rights, specify procedures to enforce contracts, establish firms as legal entities (e.g., corporations have limited liability but debt of an unincorporated firm is a personal liability), and specify laws to resolve firm insolvencies, all of which affect entrepreneur decisions. There is also a recent literature on the quantitative effects of consumer bankruptcy rules in dynamic models with limited commitment and incomplete markets. Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) and Livshits, MacGee, and Tertilt (2007) analyze quantitatively U.S. consumer bankruptcy, which provides consumers with partial insurance against bad luck due to health, job, divorce or family shocks, but also drives up interest rates, which impedes intertemporal smoothing. In Livshits, MacGee, and Tertilt (2007) the insurance effect generally dominates the interest rate effect for U.S. consumers and in Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) the effect is reversed. However, in both cases the net effect on welfare is modest.

In contrast, corporate bankruptcy provides owners with insurance against bad firm returns. We find that the welfare effects for firm owners are much greater than in these consumer studies, especially for the entrepreneurs most willing to bear risk. The insurance effect of corporate bankruptcy is more important than the interest rate effect because it encourages entrepreneurs to invest more in their firms and operate at larger scales, thereby increasing output. Our model generates testable predictions for firm size, capital structure, access to credit and owner net worth, which we evaluate.

³Models with representative agents are aggregated by multiplying the optimal decision rules from the individual's problem by the number of (identical) agents. This is not possible in our setting because differences in willingness to bear risk (i.e., heterogeneous risk aversion parameters) are central to the debate on entrepreneurship. As in Krusell and Smith (1998), heterogeneity requires us to construct a distribution.

There is also a large literature on entrepreneurship. In an early paper, Kihlstrom and Laffont (1979) focus on differences in risk aversion and formalize ideas about entrepreneurship found in Knight (1921). More recently, Puri and Robinson (2007) use data from the Survey of Consumer Finance to show that the self employed are more optimistic than wage earners, but Hoelzl and Rustichini (2005) show that people are not overconfident in laboratory experiments when a task is unfamiliar and money is at stake, as is the case for entrepreneurs. There are many models of credit constraints, e.g. Evans and Jovanovic (1989). Hopenhayn and Vereshchagina (2006) examine risk taking by entrepreneurs in a model with homogeneous risk preferences, borrowing constraints, and occupational choice. They show that these constraints can give rise to a locally non-convex value function. Thus, agents who choose to become entrepreneurs may look like risk takers.

Our model differs from the previous literature on entrepreneurship because we assess the importance of risk-aversion, optimism, and credit constraints in a dynamic model with a risky return distribution, endogenous project size, endogenous capital structure, and default that occurs in equilibrium. Our results are compatible with mild entrepreneur optimism and binding credit constraints for many but not all entrepreneurs. In contrast to Hopenhayn and Vereshchagina (2006), our agents differ in their willingness to bear risk, and we estimate the distribution of risk-aversion among entrepreneurs. Surprisingly, we find that changes in bankruptcy institutions and credit constraints can have vastly different impacts on agents with only small differences in risk aversion, which indicates that agent heterogeneity is important for policy analysis.⁴

The paper proceeds as follows. Section 2 derives facts about small firms from the SSBF and Section 3 contains the model. Section 4 has theoretical results to obtain a computable problem. Section 5 constructs the distributions predicted by the model. Model parameters are specified in Section 6. Section 7 shows that our model is quantitatively plausible along a number of dimensions, including firm size, capital structure and owner characteristics. Section 8 reports policy experiments which vary the bankruptcy institution, credit constraint, optimism and the firm's return distribution; it also analyzes entrepreneur ability in the SSBF data. Section 9 concludes.

⁴We abstract from differences in ability because we found none in the SSBF data. Although initial net-worth and the return distribution are identical across firms ex-ante, net-worth and consumption evolve differently over time due to differences in risk aversion and project realizations. For models with ability heterogeneity, see for example Antunes, Cavalcanti, and Villamil (2006), Cagetti and DeNardi (2006) and Quadrini (2000).

2 Facts About Small Firms

The SSBF is a survey administered by The Board of Governors of the Federal Reserve System and the U.S. Small Business Administration. Conducted in 1987, 1993 and 1998, each survey is a cross sectional sample of about 4000 non-farm, non-financial, non-real estate small businesses that represent 5 million firms.⁵ The surveys contain information on the characteristics of small firms and their primary owner (e.g., owner age, gender, industry, type of business organization), firm income statements and balance sheets, details on the use and source of financial services, and recent firm borrowing experience (including trade credit and capital injections such as equity).

We document several facts from this survey for firms with at least \$50,000 in assets:

Fact 1: *Small firm returns are very risky.*

Table 1 provides summary statistics about return on assets for small firms in the 1993 SSBF.⁶ Solely to put the data in perspective, we compare this return to that of a typical firm in the S&P500. The median return, skewness and kurtosis are roughly similar. SSBF firms are noticeably more risky, as the standard deviation indicates, with the higher risk somewhat compensated by a higher mean. About 12% of SSBF firms lost more than 20% of assets invested (debt plus equity), 7.4% lost more than 40%, and 3.8% lost more than 100%. Returns can also be substantial: 20.7% exceeded 50%, 10.4% exceeded 100%, and 3.8% exceeded 200%. Losses and gains are less extreme for S&P500 firms, as Figure 1 shows.

Table 1: Real Firm Return Summary Statistics, 1993 SSBF and S&P500

moment	median	mean	standard dev.	skewness	kurtosis
1993 SSBF	1.094	1.30	1.57	13.2	290
95% conf.	[1.08, 1.11]	[1.22, 1.38]	[0.95, 2.13]	[2.3, 17.3]	[29, 488]
1993 S&P500	1.093	1.21	0.65	13.1	221
95% conf.	[1.07, 1.10]	[1.16, 1.28]	[0.28, 1.02]	[3.1, 14.6]	[20, 277]

⁵The 2003 survey was recently released. All surveys are available at <http://www.federalreserve.gov>.

⁶We use 1993 data because interest expenses are required to compute the return on assets (ROA); they are listed only in the 1993 data. Section 11.1 explains how to compute the ROA distributions for the SSBF and S&P500. We assume that all firms have access to the same constant returns to scale “blue print” technology. As a consequence, the return per unit of asset for a particular firm is a sample point from the distribution of the blue print technology.

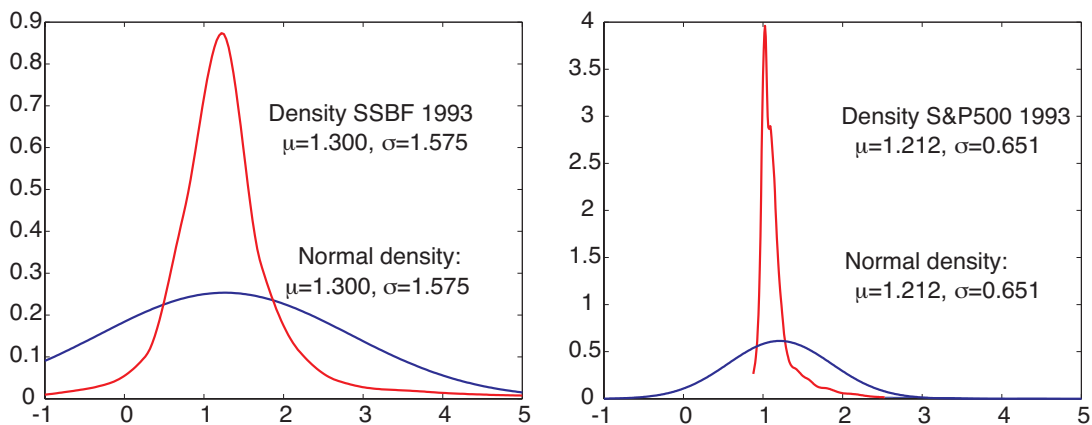


Figure 1: pdf of firm return on assets in SSBF 1993 and S&P500 (Compustat) vs. normal pdfs

Figure 1 also shows that neither distribution is normal. It compares the empirical return on asset density function to a normal density with the same mean and variance; the left panel is for SSBF data and the right is for S&P500 data. Both empirical densities are tighter around the median than a normal density because variance is generated by some firms that do exceptionally well. This, in turn, generates the high kurtosis.⁷

Fact 2: *Owners invest substantial personal net-worth in their firms.*

Table 2 reports the percentage of personal net-worth invested by entrepreneurs in their firm in the 1998 SSBF.⁸ The median amount of net-worth invested is 21%, but the data indicate a surprising lack of diversification for some entrepreneurs: 3% invest more than 80% of personal net-worth in their firm, 11% invest more than 60% and 25% invest more than 40%. This concentration of personal funds in a business is puzzling in view of the risky returns documented by table 1.

Table 2: Net-Worth Invested, 1998 SSBF

% net-worth invested	≥ 20%	≥ 40%	≥ 60%	≥ 80%	mean	median
% of entrepreneurs	52%	25%	11%	3%	27%	21%

Fact 3: *Most owners work at their firms.*

⁷In table 1 the 95% confidence bands are computed for each moment using bootstrap sampling, except for the median where the interquartile range is reported.

⁸Owner net-worth is personal net-worth plus home equity; it is only in the 1998 SSBF. We report percent net-worth invested for firms with positive net-worth outside the firm and firms with non-negative equity.

In the SSBF data for incorporated firms, the percentage of primary owners who work at their firms was 79% in 1993 and 89% in 1998. This compounds the risk return puzzle because if the firm fails, owners lose the funds invested and their jobs.⁹

Fact 4: *Negative equity for incorporated firms was 15.7% in 1993 and 21.0% in 1998.*

Negative equity means that the firm uses non-business assets to cover business losses (e.g., personal funds or unpaid bills absorbed by creditors). Entrepreneurs' willingness to use personal funds to "bail out" their firms seems puzzling since we consider only incorporated firms, which are protected by limited liability in bankruptcy. Is firm forbearance in the face of such poor performance rational? Put differently, why do these entrepreneurs not simply default on their loans?

Fact 5: *The average annual default rate on SBA loans is 3.5%.*

The low default rate documented by Glennon and Nigro (2005) on small business loans guaranteed by the Small Business Administration (SBA) deepens the puzzle.

Fact 6: *The distribution of firm capital structure is uniform.*

Figure 2 shows that the cdfs of Equity/Assets in the 1993 and 1998 SSBF are approximately uniform. By definition, total assets consist of debt plus equity, thus Equity/Assets is a measure of firm capital structure. The uniform cdfs, observed in both data sets, indicate that all capital structures are equally likely. This empirical fact for the distribution of all firms, of course, does not preclude a particular firm from having a determinate structure.¹⁰

3 The Model

Consider an economy with $t = 0, 1, \dots$ time periods. A risk-neutral competitive bank has an elastic supply of funds and makes one-period loans.¹¹ There are many infinitely lived risk averse agents who discount the future at common rate β , each with a CRRA utility function over consumption. Preferences are heterogeneous with respect to risk aversion parameter ρ ,

⁹Moskowitz and Vissing-Jorgensen (2002) find a similar risk-return puzzle for private equity investors in the Survey of Consumer Finance for large firms, but their data only allow them to determine the mean of the return distribution. We argue that more complete knowledge of the distribution, combined with bankruptcy protection and a dynamic decision problem are important for understanding the small firm puzzles.

¹⁰A uniform distribution, if individual firm capital structure is optimal, suggests agent heterogeneity.

¹¹Small firms may not have access to long-term loans because they lack payment or profit histories, audited financial statements, or verifiable contracts with workers, input suppliers or customers.

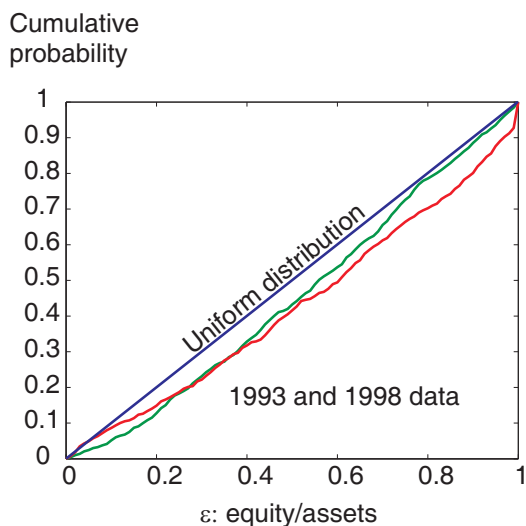


Figure 2: Equity/Assets for firms with positive equity: 1993 and 1998 SSBF Data

with $\rho \sim N(\mu, \sigma^2)$ and

$$u(c) = \frac{c^{1-\rho}}{1-\rho}.$$

Agents have a given initial endowment w_0 and access to an ex-ante identical technology. If operated, the constant returns to scale technology produces a random output. For asset investment A , the firm's return on assets is given by random variable X , with cumulative distribution function $F(x)$ and probability density function $f(x)$ which is strictly positive on support $[\underline{x}, \bar{x}]$, with $\underline{x} \leq 0$ and $\bar{x} > 0$. A negative realization means that a firm's losses in a year exceed its current level of assets, and the owner must either use personal funds to stay solvent or default. Net-worth w_t is derived from the return on investment in all periods $t \geq 1$, known at the beginning of the period, and illiquid.¹² All agents have access to an outside investment opportunity with return r .

Entrepreneurs are agents who choose to operate a firm, which means $A > 0$; agents who do not set $A = 0$. Entrepreneurs raise assets to invest in their firm at time t in two ways:

Equity: Use personal net-worth w_t to self-finance at riskless real opportunity cost r .

Debt: Take a loan, secured by business assets, which gives the bank reservation return $1+r_B$. The interest rate on the loan is determined endogenously for each entrepreneur by the model and exceeds r_B when the bankruptcy probability is positive.

¹²For example, equity in a house is illiquid but can be used to invest in a business. We assume that a loan against this equity is riskless because it is fully collateralized. Although w_0 and the risky business technology are ex-ante identical, net-worth and consumption will evolve stochastically over time.

We permit $r \neq r_B$. Equality would prevail if all net-worth were liquid (e.g., cash). Because agents are long-lived and hence can invest long-term, the opportunity cost r of using personal net-worth to fund the firm will generally be higher than bank funds, $r > r_B$, where r_B is the bank's opportunity cost of short-term funds.¹³ Given a level of business assets A in a period, an entrepreneur determines the optimal mix of firm finance by choosing the percentage of self-finance ϵ . Thus, total equity is ϵA and debt is $(1 - \epsilon)A$ at the beginning of the period. We introduce a borrowing constraint, $(1 - \epsilon)A \leq bw$, which requires a business loan to not exceed percentage b of entrepreneur net-worth. The value of b is determined endogenously.

At the end of each period assets are Ax and the entrepreneur owes $A\bar{v}$. The entrepreneur chooses whether or not to repay loan $A\bar{v}$ or default.¹⁴ If the entrepreneur does not repay, the bank can request judicial enforcement of the contract by a court described by two parameters, δ and T . The court determines the total value of firm assets and transfers $(1 - \delta)$ percent to the bank, where δ is a deadweight bankruptcy loss (e.g., firm assets are sold at a loss). The entrepreneur is protected by limited liability (only firm assets can be seized), but has the option to pay firm debt with personal funds if this is optimal. If bankruptcy occurs, the entrepreneur does not have access to the firm's returns for T periods, which has two interpretations. First, corresponding to Chapter 7 in the U.S. Bankruptcy Code, the firm may be liquidated. Because bankruptcy remains on a credit record for a period of time, creditors and customers would be unwilling to do business with the entrepreneur during this period. Second, corresponding to Chapter 11, the firm may continue to operate, but is owned by the debtholders who make investments and receive payments, or shut down the firm. After T periods, when the credit record is clean, the entrepreneur can either restart a new firm or regain control of the original firm, in Chapter 7 or 11 respectively.

The timing of events is as follows:

1. Beginning of period t (*ex-ante*) entrepreneur net-worth is w . There are two cases:

(a) *The entrepreneur has not declared bankruptcy in any of the previous T periods.*

The entrepreneur chooses consumption c , firm assets A , self-finance ϵ (debt is $1 - \epsilon$), and amount \bar{v} to repay per unit A . Bank ex-ante net return is $(1 - \epsilon)(1 + r_B)$.

¹³Consider home equity used to finance a business loan via a second mortgage. The rate on the second mortgage is typically higher than on a primary mortgage, which in turn exceeds the riskless rate. Other personal assets, such as retirement savings, have even higher penalties for early withdrawal.

¹⁴The firm may default for two reasons. It may be unable to repay loan \bar{v} if firm plus personal assets are less than $A\bar{v}$, and unwilling to repay otherwise. The entrepreneur can "bail out the firm" by using personal assets to forestall bankruptcy (but cannot be forced to do so). Default occurs in equilibrium in our model, in contrast to Kehoe and Levine (1993) and Kocherlakota (1996).

- (b) *The entrepreneur declared bankruptcy k periods ago.* The firm cannot operate for the next $T - k$ periods. Hence, only current consumption is chosen.
2. At the end of period t (*ex-post*) the firm's return on assets, x , is realized. Total end-of-period firm assets are Ax . The entrepreneur must decide whether or not to default.
- (a) *Default:* Only firm assets are seized; the entrepreneur is left with personal net-worth $(1 + r)(w - \epsilon A - c)$, personal assets invested at outside interest rate r .
- (b) *No Default:* Entrepreneur net-worth is $A(x - \bar{v}) + (1 + r)(w - \epsilon A - c)$, which includes both net-equity in the firm and the return on personal assets.

4 An Individual Agent's Problem

In this section we specify the optimization problem for an individual agent, with a given coefficient of risk aversion ρ . The objective is to determine the structure of the value function. We state the problem recursively. Let w be the entrepreneur's net-worth at the beginning of the period. If bankruptcy occurred in the previous T periods, then the state is given by (B, k, w) where k is the number of periods since default. Otherwise, the state is given by (S, w) . Denote the value functions by $V_{B,k}(w)$ and $V_S(w)$, respectively. After T periods the firm can restart, thus $V_{B,T}(w) = V_S(w)$. Let \mathfrak{B} denote the set of realizations x for which bankruptcy occurs, with complement \mathfrak{B}^c .

If the firm did not default in the previous T periods, the agent solves:

$$\begin{aligned} \mathbf{Problem\ 1} \quad V_S(w) = \max_{c,A,\epsilon,\bar{v}} u(c) + \beta \left[\int_{\mathfrak{B}} V_{B,1}((1+r)(w - \epsilon A - c)) dF(x) \right. \\ \left. + \int_{\mathfrak{B}^c} V_S(A(x - \bar{v}) + (1+r)(w - \epsilon A - c)) dF(x) \right] \end{aligned}$$

Subject to:

$$\int_{\mathfrak{B} \cap \mathbb{R}_-} x dF(x) + \int_{\mathfrak{B} \cap \mathbb{R}_+} (1 - \delta)x dF(x) + \int_{\mathfrak{B}^c} \bar{v} dF(x) \geq (1 - \epsilon)(1 + r_B) \quad (1)$$

$$x \in \mathfrak{B} \text{ if and only if } V_{B,1}((1+r)(w - \epsilon A - c)) > V_S(A(x - \bar{v}) + (1+r)(w - \epsilon A - c)) \quad (2)$$

$$(1 - \epsilon)A \leq bw \quad (3)$$

$$c \geq 0, \quad A \geq 0, \quad 0 \leq \epsilon \leq 1. \quad (4)$$

The objective is an agent's expected utility of current consumption and the expected discounted value of net-worth; the latter is the return on personal assets and net equity in the firm when it is bankrupt and solvent. Constraint (1) ensures that a representative lender (e.g., bank) is willing to supply funds. The right-hand-side indicates that the $1 - \epsilon$ percent of funds the lender invests in the firm earn at least reservation return $1 + r_B$. The left-hand side is the lender's expected return: the first term accounts for the fact that the lender may absorb some losses when the firm's return is negative,¹⁵ the second term is the net amount transferred in bankruptcy states with positive net returns (deadweight default loss δ arises only if realization x is positive and the firm has not lost more than the value of its assets during the period), and the third term is the fixed debt repayment in solvency states. Constraint (2) specifies ex-post optimality of the default decision: An entrepreneur will default if and only if the expected discounted value of future consumption after default exceeds that from solvency.¹⁶ Constraint (3) is a standard borrowing constraint, see for example Evans and Jovanovic (1989), which captures the fact that in practice there are limits on the amount of debt a firm can raise: the bank does not grant an entrepreneur a loan that exceeds b percent of net worth. Finally, (4) ensures non-negativity and that ϵ is a percentage.

Now consider the problem of a firm that defaulted $k \leq T$ periods ago. After T periods the firm can operate again, thus $V_{B,T}(\cdot) = V_S(\cdot)$. Let w' denote net-worth next period.

Problem 2 $V_{B,k}(w) = \max_{c,w'} u(c) + \beta V_{B,k+1}(w')$

Subject to:

$$c(1 + r) + w' \leq w(1 + r); \quad (5)$$

$$c, w' \geq 0. \quad (6)$$

The objective of problem 2 is the agent's expected ex-ante utility. If default occurred, a firm cannot operate for T periods and the entrepreneur can choose only consumption and saving, consistent with budget constraint (5). (6) is the non-negativity constraint.

We now use the fact that entrepreneurs have CRRA utility to determine the structure of the value function. The proof is in Appendix B.

¹⁵This can occur if the loan has an overdraft provision or the firm has trade credit. In the data, this corresponds to the case where the firm has negative equity and defaults.

¹⁶Bailing out the firm with personal funds means that the entrepreneur continues to operate the firm even if $x < \bar{v}$. In a one period model (instead of the dynamic model) both $V_{B,1}$ and V_S would be the identity mapping, and (2) would reduce to $x \in \mathfrak{B}$ if and only if $(1 + r)(w - \epsilon A - c) > A(x - \bar{v}) + (1 + r)(w - \epsilon A - c)$, which implies $x \in \mathfrak{B}$ if and only if $x < \bar{v}$ (bankruptcy only if the return is less than debt plus interest).

Proposition 1 *Suppose that the entrepreneur has constant relative risk aversion. Let $v_S = V_S(1)$ and $v_{B,k} = V_{B,k}(1)$. Then $V_S(w) = w^{1-\rho}v_S$ and $V_{B,k}(w) = w^{1-\rho}v_{B,k}$.*

Applying Proposition 1 to Problem 2 it is straightforward to compute $v_{B,k}$ as a function of v_S . Further, Lemma 1 and Lemma 2 in Appendix B prove that the investor's constraint binds and bankruptcy set \mathfrak{B} is a lower interval, with cutoff x^* . Thus, the entrepreneur's optimization problem can be rewritten as follows.

Problem 3 $v_S = \max_{c,A,\epsilon,\bar{v}} u(c) + \beta v_B \int_{\underline{x}}^{x^*} \left[(1+r)(1-\epsilon A - c) \right]^{1-\rho} dF(x)$
 $+ \beta v_S \int_{x^*}^{\bar{x}} \left[A(x - \bar{v}) + (1+r)(1-\epsilon A - c) \right]^{1-\rho} dF(x)$

Subject to:

$$\int_{\underline{x}}^0 x dF(x) + \int_0^{x^*} (1-\delta)x dF(x) + \int_{x^*}^{\bar{x}} \bar{v} dF(x) = (1-\epsilon)(1+r_B) \quad (7)$$

$$x^* = \max \left\{ \bar{v} - \left[1 - \left(\frac{v_B}{v_S} \right)^{\frac{1}{1-\rho}} \right] \frac{(1+r)(1-\epsilon A - c)}{A}, \underline{x} \right\} \quad (8)$$

$$c + \epsilon A \leq 1 \quad (9)$$

$$(1-\epsilon)A \leq b \quad (10)$$

$$c \geq 0, A \geq 0, 0 \leq \epsilon \leq 1. \quad (11)$$

The objective is to maximize the utility of current consumption and the expected discounted value of future net-worth in firm bankruptcy and solvency states. Constraint (7) corresponds to bank individual rationality constraint (1), and binds by Lemma 1 in Appendix B. Constraint (8) is the optimal default cutoff and follows from (2) by Lemma 2. When w is normalized to 1, (9) ensures feasibility and (10) is the borrowing constraint. (11) is obvious.

Problem 3 is non-convex because the timing of decisions leads to a commitment problem: c , A , ϵ , \bar{v} are chosen ex-ante, but the bankruptcy decision is made ex-post and the entrepreneur cannot commit to not declare bankruptcy. This implies that default set cutoff x^* is determined by constraint (8). Lotteries cannot be used to convexify the problem, as in Rogerson (1988), because independent randomization over A , ϵ , c , \bar{v} and x^* is not possible. See Krasa and Villamil (2000), Krasa and Villamil (2003) for an analysis of randomization and commitment.

Proposition 2 *There exist $\underline{\rho} < 1$ and $\bar{r} > \frac{1}{\beta} - 1$ such that Problem 3 has a solution for all $\rho \geq \underline{\rho}$ and for all $r \leq \bar{r}$.*

Let $\Gamma(v_S)$ be the expected utility given continuation value v_S . In general $\Gamma'(v_S) > 1$ for all v_S close to 0. Thus, Γ is not a contraction mapping because net-worth is unbounded.¹⁷ In the proof of Proposition 2 in Appendix B, we show that $\Gamma(0) \leq 0$ and there exists v_S such that $\Gamma(v_S) \geq 0$. As a consequence, continuity of Γ implies that Γ has a fixed point.

5 Heterogeneous Entrepreneurs & Model Predictions

In the SSBF data we observe the distribution of end-of-period asset values, personal net-worth invested in the firm, and the ratio of equity over assets (firm capital structure). Risk aversion ρ is not directly observable. In section 3 we assumed that ρ is normally distributed in the population of firm owners, with mean μ , standard deviation σ and pdf $g_{\mu,\sigma}(\rho)$. Given firm return pdf $f(x)$ and risk aversion pdf $g_{\mu,\sigma}(\rho)$, the cdfs predicted by the model are:

Cdf of Net-Worth: After realization x , firm assets are $A(\rho)x$ and debt is $A(\rho)\bar{v}$. Equity in the firm is $A(\rho)(x - \bar{v}(\rho))$, which is positive if $x \geq \bar{v}(\rho)$. Owner personal net-worth outside the firm is $(1+r)(1 - c(\rho) - \epsilon(\rho)A(\rho))$. The percent of total net-worth invested is

$$\mathfrak{w} = \frac{A(\rho)(x - \bar{v}(\rho))}{A(\rho)(x - \bar{v}(\rho)) + (1+r)(1 - c(\rho) - \epsilon(\rho)A(\rho))}.$$

It follows immediately that \mathfrak{w} is strictly increasing in x . We can solve this equation for $x = x(\mathfrak{w}, \rho)$. The percent of net-worth invested is less than or equal to \mathfrak{w} for all $x \leq x(\mathfrak{w}, \rho)$. For firms with positive equity, integrate to get¹⁸

$$W_{\mu,\sigma}^m(\mathfrak{w}) = \frac{\int_{-\infty}^{\underline{\rho}} \int_{\bar{v}(\rho)}^{x(\mathfrak{w},\rho)} f(x)g_{\mu,\sigma}(\rho) dx d\rho + \int_{\underline{\rho}}^{\infty} \int_{\bar{v}(\rho)}^{x(\mathfrak{w},\rho)} f(x)g_{\mu,\sigma}(\rho) dx d\rho}{\int_{\bar{v}(\rho)}^{\infty} f(x) dx}. \quad (12)$$

Cdf of Equity/Assets: The percent of equity is given by

$$\mathfrak{e} = \frac{A(\rho)(x - \bar{v}(\rho))}{A(\rho)x}.$$

Solve this equation for $x = x(\mathfrak{e}, \rho)$. For firms with positive equity, integrate to get

$$E_{\mu,\sigma}^m(\mathfrak{e}) = \frac{\int_{-\infty}^{\underline{\rho}} \int_{\bar{v}(\rho)}^{x(\mathfrak{e},\rho)} f(x)g_{\mu,\sigma}(\rho) dx d\rho + \int_{\underline{\rho}}^{\infty} \int_{\bar{v}(\rho)}^{x(\mathfrak{e},\rho)} f(x)g_{\mu,\sigma}(\rho) dx d\rho}{\int_{\bar{v}(\rho)}^{\infty} f(x) dx}. \quad (13)$$

¹⁷This precludes standard existence arguments like maximizing a continuous function over a compact set.

¹⁸The denominator is the probability that the entrepreneur has positive equity. $\underline{\rho}$ is the lowest parameter for which a model solution exists. For all $\rho < \underline{\rho}$ we assign the model solution as explained in section 7.

Cdf of End of Period Assets: The current realization of end of period assets as a percent of net-worth outside the firm is

$$\mathbf{a} = \frac{A(\rho)x}{(1+r)(1-c(\rho)-\epsilon(\rho)A(\rho))}$$

Solve this equation for $x = x(\mathbf{a}, \rho)$ and integrate to get

$$A_{\mu, \sigma}^m(\mathbf{a}) = \int_{-\infty}^{\underline{\rho}} \int_{\underline{x}}^{x(\mathbf{a}, \rho)} f(x)g_{\mu, \sigma}(\rho) dx d\rho + \int_{\underline{\rho}}^{\infty} \int_{\underline{x}}^{x(\mathbf{a}, \rho)} f(x)g_{\mu, \sigma}(\rho) dx d\rho. \quad (14)$$

6 Quantitative Analysis

In order to parameterize the model, we assign values from U.S. data to β , T , δ , r_B , r , and construct the pdf of firm returns, $f(x)$, from the SSBF 1993. We then calibrate b (borrowing constraint), μ , and σ (mean and standard deviation of risk aversion, ρ).

Table 3: Exogenous Parameters

Parameter	Interpretation	Value	Comment/ Observations
β	discount factor	0.97	determined from r and r_B
T	default exclusion period	11	U.S. credit record
δ	default deadweight loss	0.10	Boyd-Smith (1994)
r_B	bank opportunity cost	1.2%	real rate, 6 mo T-Bill, 1992-2006
r	entrepreneur opportunity cost	4.5%	real rate, 30 year mortgage, 1992-2006
$f(x)$	pdf of firm returns		SSBF 1993 (Appendix D)

Table 3 reports the parameter values taken from U.S. data. We identify r_B , the bank's opportunity cost of short-term funds, with the average real return on 6 month Treasury bills between 1992 and 2006 because this period includes the SSBF data.¹⁹ The interest rate charged by the bank will be strictly higher than r_B because of bankruptcy costs. We identify the entrepreneur's opportunity cost of funds r with the real rate on 30 year mortgages over the period; the cost of using home equity to finance a business loan will also be strictly higher. The value $\beta = 0.97$ is standard, $\beta = 1/(1 + 0.5r_B + 0.5r)$, with r and r_B weighed equally. We set the bankruptcy parameters to $T = 11$, because in the U.S. after 10 years past default is removed from a credit record, and $\delta = 0.1$ is the bankruptcy deadweight loss in Boyd and Smith (1994).²⁰ Appendix D explains how we compute $f(x)$.

¹⁹We use monthly data for T-Bill rates and deduct for each month the CPI reported by the BLS.

²⁰Section 8.1 shows the results are insensitive to tripling δ due to the low equilibrium default rate.

Parameters b, μ, σ are chosen to minimize (15) below, a measure of the difference between the cdfs of the percentage of personal net-worth invested predicted by the model and the SSBF data. We proceed as follows:

(i) The empirical cdf of the percent of net-worth invested, $W^e(\mathbf{w})$, is computed directly from the 1998 SSBF by constructing:

$$\frac{\text{owners' share * equity}}{\text{Net-worth outside the firm + owners' share * equity}}.$$

$W^e(\mathbf{w})$ is the number of observations, accounting for sample weights, at which the percent of net-worth invested is less than or equal to \mathbf{w} .

(ii) The cdf of net-worth invested implied by the model, $W_{\mu,\sigma}^m(\mathbf{w})$, is given by (12) in section 5.

(iii) The empirical distribution of end-of-period assets per unit of net-worth is $A^e(\mathbf{a})$, where

$$\frac{\text{owners' share * asset}}{\text{Net-worth outside the firm}}.$$

The model-predicted median assets are $\mathbf{a}_{\mu,\sigma}$ such that $A_{\mu,\sigma}^m(\mathbf{a}_{\mu,\sigma}) = 0.5$.

(iv) Choose parameters b, μ, σ to minimize the supnorm distance between the cdfs implied by the model and the SSBF:

$$\min_{\mu,\sigma,b \geq 0} \|W_{\mu,\sigma}^m(\mathbf{w}) - W^e(\mathbf{w})\|_{\infty} + (0.431 - \mathbf{a}_{\mu,\sigma})^+ + (\mathbf{a}_{\mu,\sigma} - 0.519)^+ \quad (15)$$

Supremum norm $\|\cdot\|_{\infty}$ is taken over all non-negative percentages of net-worth.²¹ The second and third terms impose penalties only for asset values outside the 95% confidence interval, [43.1,51.9]. Since we exclude firms with negative equity when determining W^e , net-worth invested is between 0% and 100%, but assets are unbounded.²² The lack of a well defined upper bound for assets is a problem because tail behavior would greatly impact model prediction; to solve it we require the median asset level to lie in its 95% confidence interval.

Table 4 reports the calibrated parameters b, μ, σ . The model predicts a maximal loan size of 21.5% of entrepreneur net-worth. These loans are secured by risky business assets because the firm is incorporated; the bank cannot seize personal assets in default. The median risk

²¹To compute the supremum norm we evaluate $|W_{\mu,\sigma}^m(\mathbf{w}) - W^e(\mathbf{w})|$ at 1,000 equidistant points between 0 and 1, and take the maximum. Appendix C shows the estimates are not affected by using square distance

$$\sqrt{\int (W_{\mu,\sigma}^m(\mathbf{w}) - W^e(\mathbf{w}))^2 d\mathbf{w} + ((0.431 - \mathbf{a}_{\mu,\sigma})^+)^2 + ((\mathbf{a}_{\mu,\sigma} - 0.519)^+)^2}$$

²²For example, 5% of firms had assets over ownership share that exceeded owner net-worth by 500%.

Table 4: Calibrated Parameters

Parameter	Interpretation	Est. Value
$b\%$	borrowing constraint: $\text{loan} \leq bw$	21.5
μ	median of distribution of risk aversion	1.55
σ	standard deviation of distribution of risk aversion	0.83

aversion is 1.55, with a standard deviation of 0.83. Thus, about 75% of all entrepreneurs have a coefficient of risk aversion between 1 and 3, the range in real business cycle models. Using the Consumer Expenditure Survey, Mazzocco (2006) estimates a median coefficient of risk aversion of 1.7 for men. We would expect entrepreneurs to be somewhat less risk averse than the general population; our estimate for ρ is in line with this.²³

7 Quantitative Predictions

Our model is quantitatively plausible along a number of dimensions. Figure 3 compares the cdfs predicted by the model from section 5 with the relevant SSBF data. The first panel shows the model-predicted and empirical cdfs of the percent of net-worth an owner invests in the firm. Since we fit to this empirical cdf we would expect to see a match, but the match is surprisingly good given that we use only three parameters. The next panel compares the predicted cdf of firm assets to its empirical counterpart. The match between the two asset cdfs is also good, except the model under predicts a few large firms. This occurs because model solutions do not exist below $\underline{\rho} = 0.74$, and we assign point mass of $\mu(\{\rho \leq \bar{\rho}\})$ to $\underline{\rho}$. At $\underline{\rho}$, the ex-ante level of ϵ and A are 0.720 and 0.766, respectively, and c is close to 0. Thus, end of period net-worth outside the firm, $(1 - \epsilon A - c)(1 + r)$ is 0.470. The median return in table 1 is $\bar{x} = 1.094$, and the median net-worth invested for risk aversion level $\bar{\rho}$ is $A\bar{x}/(1 - \epsilon A - c)(1 + r) = 1.786$. In the graph, this is the range where the model predicted curve moves away from the data. Note that the model predicted asset level of 47.8% is well within the 95% confidence interval of [43.1, 51.9]. Thus, the penalty term in criterion (15) is not relevant in the neighborhood of the optimal parameters.

The bottom panels of figure 3 compare the model prediction for firm capital structure to the empirical cdfs for 1993 and 1998. The left panel shows that the model somewhat over predicts equity/assets. This again occurs because no model solutions exist below $\underline{\rho}$ and (13)

²³Since Mazzocco (2006) does not estimate the distribution of risk aversion, his estimate of the standard deviation of 0.96 is close, but not directly comparable to ours. We discuss gender differences in section 8.2.

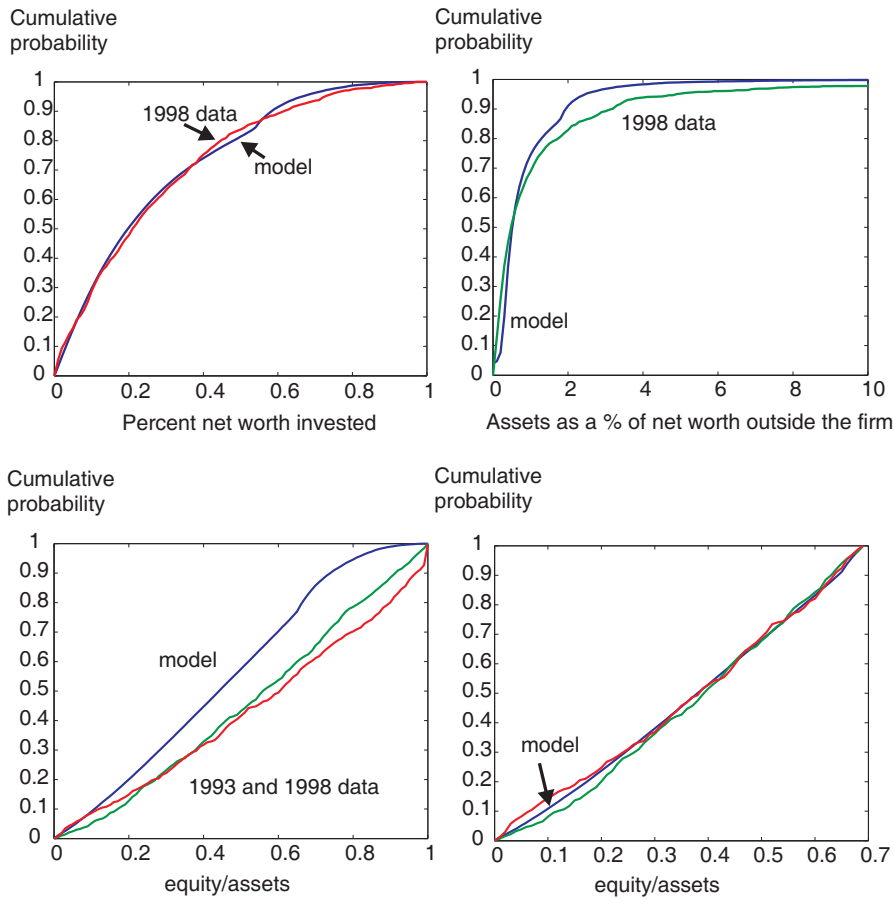


Figure 3: Model Predictions and SSBF Data: cdfs

assigns point mass to these values. At $\rho = 0.74$ the associated value of $\bar{v} = 0.335$. At median return level $x = 1.094$, this gives $(x - \bar{v})/x = 0.69$, which is about where the kink in the left panel occurs. In fact, if we compute the cdf of ϵ conditional on $\epsilon < 0.7$ then the model does an excellent job in replicating the empirical distribution of equity/assets among firms as the right panel shows.

Table 5: Model Point Estimates

Parameter	Interpretation	Value
median $A\%$	median firm assets (size)	48.1
default %	firm default rate	4.4
cons. %	consumption as a percent of net worth	3.6
neg. Eq. %	negative equity in the firm	10.6

The model predicts that 10.6% of firms have negative equity (table 5). This number is below the empirical values of 15.7% and 21.0% in 1993 and 1998. Given the high percentage

of firms with negative equity, the default rate of only 4.4% predicted by the model may seem surprising (though it is close to the empirical level of 3.5%). In a dynamic model an entrepreneur may not default, and hence continue to operate a poorly performing firm, if the firm’s expected discounted continuation value is sufficiently high. While our benchmark model’s predicted level of negative equity falls short of the values observed in the SSBF, section 8.3 shows the model can match the data if entrepreneurs are slightly optimistic. Finally, the model also predicts an average consumption level of 3.6%, which is in the range of 3-5% documented for the U.S.²⁴

Table 6: Entrepreneur’s Ex-Ante Optimal Choice and Risk Aversion

ρ	0.9	1.2	1.5	1.8	2.1	2.5	3.0	3.5	4.0
b %	21.5	21.5	21.5	21.5	21.5	18.7	15.2	12.8	11.1
A %	61.0	44.2	35.3	30.0	27.0	22.7	18.3	15.4	13.3
ϵ %	64.8	51.5	39.1	28.5	20.4	17.6	17.2	16.8	16.5
\bar{v}	0.409	0.550	0.682	0.798	0.891	0.921	0.925	0.928	0.930
default %	3.6	3.7	4.0	4.6	5.4	5.6	5.4	5.2	5.1

In order to better understand the effect of risk aversion on optimal choice, Table 6 shows how the loan limit, firm size, capital structure and default vary as risk aversion increases. The percentage of net-worth an entrepreneur can borrow, b , is constant when constraint (3) binds and falls as risk aversion increases because the borrowing constraint becomes slack.²⁵ More risk averse agents also run smaller firms, A , and use less of their own money, ϵ . As a consequence, firms become more leveraged and their debt burden rises, \bar{v} , which increases the incentive to default.

8 Policy Experiments

Overall, the model is able to account for key properties of the data. In light of this success, we now undertake a series of policy experiments to better understand the effect of bankruptcy rules, liquidity constraints, risk aversion and optimism in explaining the data. A counterfactual exercise shows the importance of the return distribution. All tables are in Section 10, Appendix A.

²⁴Point estimates for expected percent of net-worth spent on consumption and the default probability are $\int_{-\infty}^{\underline{\rho}} c(\underline{\rho})g_{\mu,\sigma}(\rho) d\rho + \int_{\underline{\rho}}^{\infty} c(\rho)g_{\mu,\sigma}(\rho) d\rho$ and $\int_{-\infty}^{\underline{\rho}} \int_{\underline{x}}^{x^*(\epsilon,\rho)} f(x)g_{\mu,\sigma}(\rho) dx d\rho + \int_{\underline{\rho}}^{\infty} \int_{\underline{x}}^{x^*(\epsilon,\rho)} f(x)g_{\mu,\sigma}(\rho) dx d\rho$.

²⁵In table 17 in Appendix A we show how welfare is affected by changes in b . Clearly, if the borrowing constraint is slack, then welfare is unaffected by changes in b , which as above occurs for sufficiently large ρ .

8.1 Bankruptcy Institution: Experiment 1

Bankruptcy Exclusion Period T : Experiment 1 in Appendix A evaluates the effect of the exclusion period on the results. Longer exclusion raises the penalty of bankruptcy. The U.S. benchmark is $T = 11$.

(i) Table 8: We re-calibrate the model for values of T from 10 to 20. The results indicate that μ and σ are very stable: as T increases, μ remains between 1.5 and 1.6 and σ between 0.74 and 0.9. Liquidity constraint b decreases a bit more significantly because the penalty increases with T ; entrepreneurs become more cautious and run smaller firms (lower A). In order to achieve the best model fit, the optimization procedure lowers b to ensure entrepreneurs use enough personal funds to finance the firm. Default decreases with T because it is more costly to the entrepreneur.

(ii) Table 9: We conduct comparative statics with respect to T (fix μ , σ , b to the benchmark case, do not re-optimize, and evaluate T from 6 to 20). The default probability decreases by 50% and firm size, measured by median asset level A , decreases as T is increased. Because b is fixed, the decrease in total investment results in a decrease in equity and an increase in debt, which raises negative equity.

(iii) Table 10: We wish to consider the effect of changes in T on welfare, but agents are heterogeneous and we cannot simply compare utilities. We use an equivalent variation: fix the base case at $T = 11$, consider changing T to T' to get the change in net-worth at $T = 11$ that would be equivalent to an actual change in the bankruptcy exclusion period to T' . Formally, let $V_S^T(w)$ be the value function in a solvency state for bankruptcy exclusion period T . The equivalent variation is given by λ such that $V_S^T(\lambda w) = V_S^{T'}(w)$. Proposition 1 implies that $V_S^T(\lambda w) = \lambda^{1-\rho} V_S^{T'}(w)$. Further, $V_S^T(w) = w^{1-\rho} V_S^T(1) = v_S^T$ and $V_S^{T'}(w) = w^{1-\rho} V_S^{T'}(1) = v_S^{T'}$. Thus, the change in welfare is

$$\lambda = \left(\frac{v_S^T}{v_S^{T'}} \right)^{\frac{1}{1-\rho}}.$$

The estimates of μ , σ and to a lesser degree b are relatively insensitive to T , but table 10 shows the bankruptcy exclusion period has a significant impact on welfare. Lowering T from 11 to 10 is equivalent to a 1.1% increase in net-worth to a person with median risk aversion. The improvement for an agent with risk aversion $\rho = 0.9$ is a more substantial increase of 6.3%. Decreasing T to 6 increases net-worth for the median agent by 7.7% and 36.9% for one with $\rho = 0.9$. If T is increased to 20, the loss of net-worth to the median person is 5.7% and 20.4% when $\rho = 0.9$.

(iv) Table 11: One of the main economic arguments in support of the recent U.S. bankruptcy reform act was that more stringent bankruptcy rules lower interest rates, and therefore help borrowers. Table 11 shows that the loan rate decreases as T increases. However, the downside of stricter bankruptcy is less insurance against bad realizations. The welfare results in table 10 indicate that this insurance effect strongly dominates the benefit from lower interest rates.

Our results indicate a tradeoff between the insurance provided by firm bankruptcy and higher interest rates induced by the increase in default. A similar tradeoff has been analyzed recently for consumer bankruptcy by Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) and Livshits, MacGee, and Tertilt (2007). In both models consumers trade off insurance against health, divorce or family shocks versus consumption smoothing; the signs of the tradeoffs differ but the magnitudes are modest.²⁶ Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) find that when the length of punishment is reduced from 10 to 5 years welfare drops by 0.05%. In other words, in their experiment the negative effect from a higher interest rate and a more binding borrowing constraint slightly dominates the insurance benefit associated with a shorter punishment period. Livshits, MacGee, and Tertilt (2007) show that the insurance effect is sometimes weakly dominant, but again in their consumer analysis the effect is modest. In contrast, table 10 reports strong welfare effects from reducing the exclusion penalty for all entrepreneurs, particularly those with low levels of risk aversion.

The main reason for the difference between our model of firm bankruptcy and the models of consumer bankruptcy is that in our model a more lenient bankruptcy rule encourages more risk taking—entrepreneurs invest more in their firms and operate at a larger scale. This leads to an increase in production, which is the main source of our large welfare effect. In this sense, even though we do not find extreme variations in ρ , risk interacts with the dynamic decision problem, the return distribution and the bankruptcy institution to have an important effect on some (heterogeneous) agents, namely those that are most invested in their firms.

Bankruptcy Costs δ : Experiment 1 also considers the effects of bankruptcy cost δ . Table 8 is the benchmark case with $\delta = 0.1$. Table 12 increases the cost to $\delta = 0.3$ and re-estimates the model and μ , σ , b are almost unaffected. This shows that the model is robust to such changes in bankruptcy costs, and that a detailed measurement of this cost is not essential within this range. Table 13 reports comparative static results in which δ varies between 0

²⁶In our model credit is secured, for example by a house, and bad luck is a poor return x rather than the health, job, divorce or family shocks in the consumer models.

and 100%, fixing b , μ and σ at the benchmark values (i.e., we do not re-estimate the model). Again, δ has almost no impact on endogenous variables — and the impact is significantly smaller than in the comparative statics with respect to T . Table 14 shows that the welfare gains or losses from δ are also minor: For an entrepreneur with the median level of risk aversion the gains/losses are less than 0.1%. The effect of δ is minor because (a) bankruptcy occurs with only a small probability, and (b) since assets Ax in bankruptcy states tend to be small, the deadweight loss δAx will also be small. Clearly, the expected costs, i.e., the product of (a) and (b) is second order.

The results indicate that changes in bankruptcy costs (e.g., court efficiency in liquidating firm assets) has a minor impact on welfare. However, if δ is very large and if, in addition, there are large fixed cost to creditors to recover payments in default, the parties will attempt to avoid costly bankruptcy, through debt forgiveness or renegotiation, and debtors will be more likely to default. The static model of Krasa, Sharma, and Villamil (2007) shows that these effects can generate substantial deadweight losses when courts are sufficiently inefficient.

8.2 Risk Aversion & Liquidity Constraints: Experiment 2

Experiment 2 in Appendix A considers the effect of changes in μ and b . Clearly, median risk aversion μ cannot be modified by policy changes, however, it helps us to understand how risk aversion of the owner affects firm decisions. Table 15 shows that owners with higher than median risk aversion run smaller firms. Because b is fixed, these smaller firms have higher levels of debt, which in turn explains why negative equity and the default rate rise with μ . Mazzocco (2006) finds that women are more risk averse than men (5 versus 1.7). In our model this parameter change would imply that (i) less women own businesses, (ii) they run smaller firms, and (iii) they have higher levels of negative equity. In fact, the SSBF data indicate that all three occur. (i) In 1993 and 1998 the percentage of women owned businesses was 16% and 24%, respectively. (ii) In 1998 median assets, normalized by net-worth outside the firm, was 39% for firms owned by women and 53% for men (the only year for which net-worth is reported). Finally, (iii) negative equity for women was 19.5% versus 14.8% for men, and 26.1% versus 19.4% in 1993 and 1998, respectively. Absent the model, the observation that more risk averse agents have higher levels of negative equity may seem counterintuitive.

In contrast to μ , credit constraint parameter b can be affected by policy. Table 16 shows that increasing b allows firms to borrow more, and hence operate at a larger scale A . The

higher levels of firm debt, however, increase the percentage of firms who default or have negative equity. Table 17 shows there are again substantial welfare effects for the least risk averse agents, but not for more risk averse agents because for sufficiently high b the credit constraint does not bind. Comparing the welfare effects of T and b shows that the median entrepreneur receives more benefit from more lenient bankruptcy rules than from reducing the borrowing constraint.

Lowering T or raising b both increase the riskiness of loans. In the U.S., regulation makes it very costly for banks to deal with creditors with high default rates. Bank CAMELS ratings establish portfolio risk, and a poor rating forces a bank to either hold more liquid assets or it raises the cost of funds.²⁷ Deposits are insured in the U.S., and these regulations are designed to prevent banks from taking excessive risk (insurance limits depositors downside loss and greater risk taking may be beneficial for the bank but inefficient for society). Thus, b cannot simply be raised by a policy maker, unless it is accompanied by other institutional changes. Similarly, making bankruptcy more lenient by lowering T could also lead banks to reduce loans, i.e., effectively reducing b . While our results clearly indicate that lowering T is beneficial, any policy that makes bankruptcy more lenient would also have to ensure that banks do not simply increase their credit constraints. In other words, in practice T and b may be linked and investigating this is an important topic for future research in a model with heterogeneity in risk aversion.

8.3 Entrepreneur Optimism: Experiment 3

How does optimism by entrepreneurs affect our results? Intuition suggests that less risk averse, less optimistic agents will behave similarly to more risk averse, more optimistic agents. This leads to an identification problem: optimistic agents may be observationally equivalent to less risk averse, non-optimistic agents. We now investigate whether the model has observable implications that are uniquely induced by optimism. We assume that an optimistic entrepreneur believes the firm's return exceeds the true return by some fixed percentage \bar{o} . Formally, this implies the entrepreneur assumes that firm returns are $X + \bar{o}$, which yields cdf $H(x - \bar{o})$ in the objective of problem 3. The bank is assumed to use the correct distribution to determine payoff (7) in problem 3.

In Experiment 3 in Appendix A we vary \bar{o} and fix all other parameters. The exclusion

²⁷The acronym CAMELS refers to the components of a bank's condition that are assessed by regulators: Capital adequacy, Asset quality, Management, Earnings, Liquidity and Sensitivity to market risk.

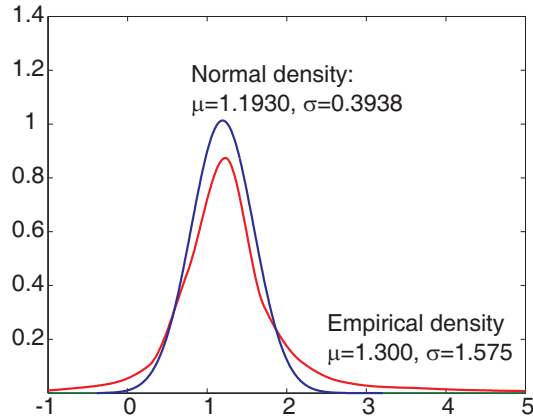


Figure 4: Empirical firm return pdf versus best-fit normal pdf, SSBF 1993

benchmark T is 11, and T is varied from 10 to 20. Table 19 shows that slight optimism (10%) improves the fit in the baseline model with $T = 11$ while keeping μ , σ and the default rate in acceptable ranges. Liquidity constraint parameter b increases slightly, as does A . Negative equity increases to a level consistent with the SSBF. The reason for the increase in negative equity is that optimistic entrepreneurs run larger firms because they expect higher future returns relative to the baseline, thereby increasing the total amount of debt \bar{v} . Equity is negative if $x < \bar{v}$. When \bar{v} is higher, $x < \bar{v}$ is more likely and this increases the percentage of projects with negative equity. Mild entrepreneur optimism can explain the level of negative equity observed in the 1993 SSBF (15.7%) and still accommodate the relatively low level of default observed in the data.

8.4 Counterfactual Exercise: Normally Distributed Firm Returns

In order to show the importance of a risky return distribution, we conduct two counterfactual experiments. We replace the empirical ROA distribution computed from SSBF data, keeping all other benchmark settings the same, with two different normal distributions: the best fit minimizes the maximum distance between the normal and empirical cdfs; the other is a normal distribution with the same mean and variance as the empirical distribution.

Best Fit Normal Distribution. Let $g_{\mu,\sigma}$ be the density of a normal distribution with mean μ and standard deviation σ and f be the density of the SSBF distribution. We solve $\min_{\mu,\sigma} \sup_x |g_{\mu,\sigma}(x) - f(x)|$ to find a normal distribution that best approximates the empirical density function. The resulting values are $\mu = 1.193$ and $\sigma = 0.394$ — both distributions are

shown in figure 4. In order to fit the “middle” this normal distribution has less mass in the tails and, as a consequence, is less risky. Thus, when re-calibrating the model, median risk aversion increases from 1.55 to 2.33 but at the same time, for given ρ , the lower project risk in the normal distribution encourages entrepreneurs to run larger firms. Default is lower, again because the normal distribution has a thinner lower tail. Finally, the thinner upper tail of the normal distribution implies that less firms will be “lucky” and have a very good realization. In order to be able to match the distribution of net-worth invested, firms must be more leveraged: Given two solvent firms with the same realization, a more leveraged firm earns a higher return because the owner receives a higher residual after making the fixed debt payment.²⁸ The somewhat higher level of debt also implies that more low realizations will result in negative equity, and the predicted percentage of firms with negative equity increases from 10.6% to 13.7%.

Normal Distribution with SSBF μ, σ . Figure 1 compares the SSBF pdf with a normal distribution with the same mean and standard deviation. Table 7 shows the results for this distribution are significantly at odds with the data, highlighting the importance of the return distribution. First, the fat tails lead to μ and σ such that all point mass is at $\underline{\rho}$ and $\bar{\rho}$, where $\bar{\rho}$ is the highest risk aversion for which we compute a solution. Generally, we can choose $\bar{\rho}$ sufficiently high that the mass above $\bar{\rho}$ is negligible; this cannot be done for this normal distribution with fat tails and $\bar{\rho}$ affects the results.²⁹ Second, the model predictions in the last column of table 7 are clearly implausible.

Table 7: Counterfactual Experiment: Normal Distributions

Parameter	Data	Empirical $f(x)$	Best Fit Normal $g(x)$ $\mu= 1.193, \sigma=0.3938$	μ, σ Normal $g(x)$ $\mu=1.300, \sigma=1.193$
μ	1-3	1.55	2.33	$4.4 * 10^8$
σ	NA	.83	1.11	$7.9 * 10^8$
b%	NA	21.5	30.0	23.4
fit	NA	0.042	0.040	.045
median A%	[43.1,51.9]	48.1	54.7	38.6
default %	3.5	4.4	1.5	61.0
cons. %	3-5	3.6	4.9	3.1
neg. Eq %	15.7	10.6	13.7	64.4

²⁸This also explains the higher value of b .

²⁹Upper bound $\bar{\rho}$ is needed for computation; it is impossible to compute solutions for a fine grid $[\underline{\rho}, \infty]$.

8.5 Entrepreneur Ability

This paper has focused on differences among agents in their willingness to bear risk because this is a central theme in discussions of entrepreneurship. There is also a large literature on differences in entrepreneurial ability. We omit this type of heterogeneity from our analysis because we examined the SSBF data for evidence of differences in entrepreneur ability and found none. We assume that all agents have similar ability to be consistent with this data. In the Lucas (1978) “span of control” model, the foundation of entrepreneur ability models, a firm with a more able manager has better return realizations than a firm with a less able manager. Our model implies that a more able entrepreneur (with a better return distribution) would run a firm that is larger in relation to the entrepreneur’s net-worth w than a less able one. Thus, if ability is relevant in our data, then A/w should be positively correlated with realizations x . We use the 1998 SSBF data to test whether a positive correlation exists.³⁰

The level of assets in the SSBF corresponds to Aw in our model. We divide assets by entrepreneur net-worth, and compute firm return on assets using to (27) in section 11.1. Consistent with our previous empirical analysis, we consider only incorporated firms with assets over \$50,000. Entrepreneurs with zero or negative net-worth are omitted. The results are not sensitive to entrepreneur equity in the firm. If we include the owner’s equity in the firm, then

$$\begin{aligned} \text{realization} &= 1.4776 + 0.0042A + \text{error} \\ & \quad (0.0934) \quad (0.0119) \end{aligned}$$

Otherwise, if we exclude the owner’s equity in the firm then

$$\begin{aligned} \text{realization} &= 1.5175 - 0.0068A + \text{error} \\ & \quad (0.0926) \quad (0.0101) \end{aligned}$$

The numbers in parenthesis are the estimated standard deviation of the coefficient.

The absence of a positive correlation between assets and realizations indicates that differences in ability is not significant in the SSBF data. While we do not doubt the relevance of manager ability for firm success more broadly, we believe it does not matter in the SSBF for two reasons. First, most firms in the data set have operated for a number of years (in

³⁰The 1993 data set does not contain entrepreneur net-worth, thus we cannot use it. While the 1998 data set does not contain interest expenses, which should be added to profits to compute ROA, interest expenses are not large relative to profit. This missing interest data has little effect on the correlation coefficient.

1993 and 1998, 86% and 83% of firms operated for at least 5 years, respectively); firms operated by less able entrepreneurs are less likely to survive. Second, most firms with less able managers will not accumulate the required \$50,000 in assets we use as the minimum level for our analysis.

9 Concluding Remarks

This paper assesses the impact of differences in innate personal characteristics versus institutions on entrepreneurship – whether to operate a firm, its size, capital structure, terms of finance, owner consumption and net-worth. We examined SSBF data and constructed a dynamic, computable model to organize the facts. The returns generated by small firms are very risky, yet some entrepreneurs invest a substantial portion of personal net-worth in their business and even use personal funds to cover business losses when they are protected by limited liability if they declare bankruptcy. We show that intertemporal tradeoffs, the return distribution, and bankruptcy can explain this apparently puzzling willingness to bear risk for modest median returns.

The return distribution is important because the combination of asymmetric fat tails and bankruptcy means that the firm is insured against extreme loss, but may enjoy very high future upside gains. Most of the mass is centered around the middle of the distribution, which is attractive to individuals with standard degrees of risk aversion. Entrepreneurs trade off the value of absorbing a current loss against the expected discounted value of future gains from maintaining the firm. Thus, to answer the question posed by the paper — Entrepreneurs need not have significantly different personal characteristics such as willingness to bear risk or optimism to explain the facts in the SSBF data. However, institutions are very important (e.g., bankruptcy laws and access to credit) and interact with modest differences in personal characteristics. This suggests that policies which effect the institutional environment within which firms operate are very important.

10 Appendix A: Experiments

10.1 Experiment 1: Bankruptcy Exclusion Parameter T & Cost δ

Table 8 Benchmark Exogenous Variables: $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$

T	10	11	12	13	14	15	16	20
μ	1.62	1.55	1.49	1.51	1.52	1.52	1.51	1.50
σ	0.90	0.83	0.75	0.74	0.76	0.76	0.76	0.78
b %	20.6	21.5	22.0	19.8	18.4	17.7	17.3	15.4
fit	0.046	0.042	0.037	0.034	0.034	0.034	0.035	0.036
median A %	46.9	48.1	49.2	47.0	45.3	44.3	43.8	41.3
default %	4.7	4.4	4.2	3.8	3.5	3.3	3.1	2.5
cons. %	3.7	3.6	3.6	3.6	3.6	3.6	3.6	3.5
neg Eq. %	10.2	10.6	10.8	10.5	10.8	11.1	11.6	11.1

Table 9 Comparative statics for T : Fix $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$

T	6	7	8	9	10	11	12	13	14	15	16	20
fit	0.095	0.085	0.076	0.066	0.053	0.042	0.054	0.065	0.073	0.079	0.084	0.107
med A %	56.2	54.4	52.7	51.1	49.6	48.1	46.7	45.4	44.3	43.3	42.4	38.9
default %	6.1	5.6	5.3	5.0	4.7	4.4	4.2	4.0	3.8	3.6	3.5	2.9
cons. %	3.7	3.7	3.7	3.7	3.7	3.6	3.6	3.6	3.6	3.6	3.6	3.6
neg Eq. %	8.4	8.8	9.3	9.7	10.0	10.6	11.1	11.9	13.4	15.1	17.0	21.0

Table 10 Welfare Effect as T Varies: % increase or decrease of net-worth compared to benchmark

risk aversion ρ	0.9	1.2	1.5	1.8	2.1	2.5	3.0	3.5	4.0
$T = 6$	36.9	11.2	7.7	6.1	5.0	3.9	3.1	2.6	2.2
$T = 7$	27.5	8.1	5.6	4.4	3.6	2.8	2.2	1.9	1.6
$T = 8$	19.8	5.5	3.9	3.0	2.4	1.9	1.5	1.3	1.1
$T = 9$	13.5	3.2	2.4	1.8	1.5	1.2	0.9	0.7	0.6
$T = 10$	6.3	1.3	1.1	0.8	0.7	0.5	0.4	0.3	0.3
$T = 11$	—	—	—	—	—	—	—	—	—
$T = 12$	-3.6	-0.7	-0.9	-0.7	-0.5	-0.4	-0.3	-0.3	-0.2
$T = 13$	-4.4	-3.2	-1.5	-1.3	-1.0	-0.8	-0.6	-0.5	-0.4
$T = 14$	-7.6	-4.4	-2.1	-1.8	-1.4	-1.0	-0.8	-0.6	-0.5
$T = 15$	-10.5	-5.5	-3.2	-2.1	-1.7	-1.3	-1.0	-0.8	-0.6
$T = 16$	-12.7	-6.5	-3.8	-2.4	-2.0	-1.5	-1.1	-0.9	-0.7
$T = 20$	-20.4	-9.4	-5.7	-4.0	-2.8	-1.8	-1.5	-1.2	-1.0

Table 11 Interest Rate as T Varies

risk aversion ρ	0.9	1.2	1.5	1.8	2.1	2.5	3.0	3.5	4.0
$T = 6$	18.0	15.3	14.2	14.0	14.3	14.4	14.3	14.1	14.1
$T = 7$	17.7	14.9	13.7	13.5	13.8	13.8	13.7	13.6	13.5
$T = 8$	17.3	14.5	13.3	13.0	13.3	13.3	13.2	13.1	13.0
$T = 9$	17.0	14.1	12.9	12.5	12.8	12.9	12.7	12.6	12.5
$T = 10$	16.6	13.7	12.4	12.1	12.3	12.4	12.2	12.0	11.9
$T = 11$	16.3	13.3	12.0	11.6	11.9	11.9	11.6	11.5	11.4
$T = 12$	16.0	12.9	11.7	11.2	11.4	11.4	11.2	11.0	10.9
$T = 13$	15.6	12.6	11.3	10.8	11.0	10.9	10.7	10.6	10.4
$T = 14$	15.3	12.3	10.9	10.5	10.6	10.5	10.3	10.2	10.1
$T = 15$	15.0	12.0	10.7	10.1	10.2	10.2	10.0	9.8	9.7
$T = 16$	14.7	11.8	10.4	9.8	9.8	9.8	9.6	9.5	9.4
$T = 20$	13.6	10.7	9.3	8.7	8.5	8.8	8.6	8.5	8.4

Table 12 Higher Cost δ : $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.30$

T	10	11	12	13	14	15	16	20
μ	1.79	1.67	1.55	1.50	1.52	1.52	1.51	1.50
σ	1.08	0.95	0.81	0.74	0.76	0.76	0.76	0.78
b %	14.9	16.9	19.8	20.1	18.4	17.6	17.2	15.4
fit	0.052	0.046	0.040	0.035	0.034	0.034	0.035	0.036
median A %	39.8	42.6	46.3	47.3	45.3	44.3	43.6	41.3
default %	4.0	4.0	4.0	3.8	3.5	3.2	3.1	2.5
cons. %	3.8	3.7	3.7	3.6	3.6	3.6	3.6	3.5
neg Eq. %	8.7	9.2	10.2	10.5	10.7	11.0	11.4	11.1

Table 13 Comparative Statics for δ : Fix $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$

δ	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.80	1.00
fit	0.042	0.042	0.046	0.050	0.054	0.057	0.060	0.063	0.065
median A %	48.3	48.1	48.0	47.9	47.8	47.8	47.7	47.6	47.5
default %	4.5	4.4	4.4	4.4	4.3	4.3	4.3	4.2	4.2
cons. %	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6
neg Eq. %	10.8	10.6	10.3	10.2	10.1	10.1	10.0	9.9	9.7

Table 14 Welfare Effect as δ Varies: % increase or decrease of net-worth compared to benchmark

risk aversion ρ	0.9	1.2	1.5	1.8	2.1	2.5	3.0	3.5	4.0
$\delta = 0.00$	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.0	0.0
$\delta = 0.10$	—	—	—	—	—	—	—	—	—
$\delta = 0.20$	0.0	0.0	0.0	0.0	-0.1	-0.1	0.0	0.0	0.0
$\delta = 0.30$	0.0	0.0	0.0	0.0	-0.2	-0.1	-0.1	-0.1	0.0
$\delta = 0.40$	0.0	0.0	0.0	0.0	-0.2	-0.2	-0.1	-0.1	-0.1
$\delta = 0.50$	0.0	0.0	0.0	-0.1	-0.3	-0.2	-0.2	-0.1	-0.1
$\delta = 0.60$	0.0	0.0	0.0	-0.1	-0.4	-0.3	-0.2	-0.1	-0.1
$\delta = 0.80$	0.0	0.0	0.0	-0.2	-0.5	-0.3	-0.2	-0.2	-0.1
$\delta = 1.00$	0.0	0.0	0.0	-0.2	-0.5	-0.4	-0.3	-0.2	-0.1

10.2 Experiment 2: μ and b

Table 15 Comparative Statics for μ : $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$

μ	1.15	1.25	1.35	1.45	1.55	1.65	1.75	1.85
fit	0.224	0.146	0.109	0.074	0.042	0.080	0.117	0.153
median A %	74.3	65.4	58.3	52.7	48.1	44.4	41.2	38.6
default %	4.2	4.2	4.3	4.4	4.4	4.5	4.6	4.7
cons. %	2.8	3.0	3.2	3.5	3.6	3.8	4.0	4.1
neg Eq. %	8.4	8.9	9.5	10.0	10.6	11.1	11.7	12.3

Table 16 Comparative Statics for b : $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$

b	0.10	0.15	0.20	0.21	0.25	0.30	0.35	0.40	0.50
fit	0.145	0.071	0.048	0.042	0.067	0.094	0.113	0.126	0.145
median A %	46.9	47.1	47.8	48.1	49.0	50.5	51.8	52.4	52.5
default %	3.0	3.6	4.3	4.4	4.8	5.2	5.4	5.6	5.9
cons. %	3.6	3.6	3.6	3.6	3.7	3.7	3.7	3.6	3.6
neg Eq. %	5.7	7.7	10.0	10.6	11.7	12.9	13.6	14.2	15.0

Table 17 Welfare Effect as b Varies: % increase or decrease in net-worth compared to benchmark
 Note: More risk averse agents are unaffected because the credit constraint does not bind for them

risk aversion ρ	0.9	1.2	1.5	1.8	2.1	2.5	3.0	3.5	4.0
$b = 0.100$	-13.1	-8.5	-6.2	-4.9	-3.7	-2.2	-0.9	-0.2	-0.1
$b = 0.150$	-6.1	-4.8	-3.2	-2.0	-1.4	-0.4	0.0	0.0	0.0
$b = 0.200$	-1.8	-0.2	-0.6	-0.4	-0.1	0.0	0.0	0.0	0.0
$b = 0.215$	—	—	—	—	—	—	—	—	—
$b = 0.250$	8.2	1.7	1.3	0.6	0.0	0.0	0.0	0.0	0.0
$b = 0.300$	14.8	4.2	2.4	0.7	0.0	0.0	0.0	0.0	0.0
$b = 0.350$	20.9	6.0	2.7	0.7	0.0	0.0	0.0	0.0	0.0
$b = 0.400$	26.6	7.2	2.7	0.7	0.0	0.0	0.0	0.0	0.0
$b = 0.500$	35.0	7.5	2.7	0.7	0.0	0.0	0.0	0.0	0.0

10.3 Experiment 3: Slight optimism is consistent with the data

Table 18 5% Optimism: $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$, optimism=5%

T	10	11	12	13	14	15	16	20
μ	1.69	1.65	1.61	1.58	1.55	1.52	1.50	1.48
σ	0.75	0.71	0.68	0.66	0.64	0.63	0.62	0.62
b %	26.4	26.2	26.3	26.7	27.0	27.3	27.2	24.4
fit	0.032	0.030	0.029	0.028	0.028	0.028	0.028	0.029
median A %	55.1	54.9	54.8	54.7	54.7	54.7	54.5	51.5
default %	4.7	4.4	4.1	3.9	3.7	3.6	3.4	2.7
cons. %	4.4	4.4	4.3	4.3	4.2	4.2	4.1	4.0
neg Eq. %	12.6	13.4	14.5	15.9	17.1	17.7	17.8	16.2

Table 19 10% Optimism: $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$, optimism=10%

T	10	11	12	13	14	15	16	20
μ	1.92	1.89	1.83	1.79	1.76	1.73	1.70	1.61
σ	0.83	0.81	0.77	0.74	0.72	0.70	0.69	0.63
b %	26.6	26.2	27.0	27.2	27.3	27.3	27.3	27.4
fit	0.030	0.030	0.029	0.029	0.029	0.029	0.029	0.028
median A %	54.9	54.1	54.8	54.8	54.8	54.8	54.8	54.7
default %	4.4	4.0	3.8	3.6	3.4	3.3	3.1	2.7
cons. %	5.2	5.1	5.1	5.0	5.0	4.9	4.9	4.7
neg Eq. %	15.8	16.7	17.5	17.8	17.8	17.8	17.7	17.6

Appendix B: Proofs

Proof of Proposition 1. First, substitute $V_S(w) = w^{1-\rho}v_S$ and $V_B(w) = w^{1-\rho}v_B$ into the right-hand side of the objective of problem 1 and in constraint 2. Thus, we get

$$V_S(w) = \max_{c,A,\epsilon,\bar{v}} u(c) + \beta \left[\int_{\mathfrak{B}} ((1+r)(w - \epsilon A - c))^{1-\rho} v_B dF(x) + \int_{\mathfrak{B}^c} (A(x - \bar{v}) + (1+r)(w - \epsilon A - c))^{1-\rho} v_S dF(x) \right];$$

Subject to:

$$\int_{\mathfrak{B}} (1 - \delta)x dF(x) + \int_{\mathfrak{B}^c} \bar{v} dF(x) \geq (1 - \epsilon)1 + r_B \quad (16)$$

$$x \in \mathfrak{B} \iff v_B \left((1+r)(w - \epsilon A - c) \right)^{1-\rho} > v_S \left(A(x - \bar{v}) + (1+r)(w - \epsilon A - c) \right)^{1-\rho} \quad (17)$$

$$(1 - \epsilon)A \leq bw \quad (18)$$

$$c, A \geq 0, 0 \leq \epsilon \leq 1. \quad (19)$$

Let $\lambda > 0$ and let current wealth be w . We must prove that $V_S(\lambda w) = \lambda^{1-\rho}w$.

Suppose that the entrepreneur chooses a level of consumption λc , increases the firm's assets to λA , and keeps ϵ fixed. When wealth is λw then

$$\lambda^{1-\rho}v_B \left((1+r)(w - \epsilon A - c) \right)^{1-\rho} = v_B \left((1+r)(\lambda w - \epsilon \lambda A - \lambda c) \right)^{1-\rho}, \text{ and}$$

$$\lambda^{1-\rho}v_S \left(A(x - \bar{v}) + (1+r)(w - \epsilon A - c) \right)^{1-\rho} = v_S \left(\lambda A(x - \bar{v}) + (1+r)(\lambda w - \epsilon \lambda A - \lambda c) \right)^{1-\rho}$$

This and (17) imply that bankruptcy set \mathfrak{B} remains unchanged. Thus, (16), (18) and (19) are satisfied. Next, note that the right-hand side of the objective changes by the factor $\lambda^{1-\rho}$. Because $V_S(\lambda w)$ is the maximum utility of the entrepreneur given wealth λw , it follows that

$$V_S(\lambda w) \geq \lambda^{1-\rho}V_S(w), \quad (20)$$

for all $\lambda > 0$. Thus,

$$V_S(w) = V_S\left(\frac{1}{\lambda}\lambda w\right) \geq \frac{1}{\lambda^{1-\rho}}V_S(\lambda w),$$

which implies that (20) holds with equality. Substituting $w = 1$ and $\lambda = w$ in (20) immediately implies that $V_S(w) = w^{1-\rho}v_S$. The proof that $V_B(w) = w^{1-\rho}v_B$ is similar. ■

Lemma 1 *Constraint 1 of Problem 1 binds.*

Proof of Lemma 1. Immediate: Suppose by way of contradiction that constraint (1) is slack. Then \bar{v} can be lowered thereby increasing $w'_s(x)$, which increases the objective of problem 1.³¹ ■

Lemma 2 *Suppose that \mathfrak{B} is non-empty. Let*

$$x^* = \bar{v} - \left[1 - \left(\frac{v_B}{v_S} \right)^{\frac{1}{1-\rho}} \right] \frac{(1+r)(1-\epsilon A - c)}{A} \quad (21)$$

Then $\mathfrak{B} = \{x | \underline{x} \leq x < x^\}$. Conversely, if $x^* > \underline{x}$, then bankruptcy set \mathfrak{B} is non-empty.³²*

Proof of Lemma 2. If the entrepreneur chooses to default, the entrepreneur's utility is

$$u^B(x) = \left[\eta Ax + (1+r)(1-\epsilon A - c) \right]^{1-\rho} v_B. \quad (22)$$

Otherwise, if the entrepreneur does not default, then the utility is

$$u^S(x) = \left[A(x - \bar{v}) + (1+r)(1-\epsilon A - c) \right]^{1-\rho} v_S. \quad (23)$$

Note that $x \in \mathfrak{B}$ if $u^B(x) > u^S(x)$ and $x \notin \mathfrak{B}$ if $u^S(x) \geq u^B(x)$.

Suppose that $u^S(x) \geq u^B(x)$. We show that $u^S(x') > u^B(x')$ for all $x' > x$. Note that

$$\frac{d(u^S(x) - u^B(x))}{dx} = \frac{(1-\rho)(1-\eta)Av_S}{\left[\eta Ax + (1+r)(1-\epsilon A - c) \right]^\rho v_B} > 0$$

Thus, $u^S(x) - u^B(x) \geq 0$ implies that $u^S(x') > u^B(x')$ for all $x' > x$. Similarly, $u^B(x) > u^S(x)$ implies $u^B(x') > u^S(x')$ for all $x' < x$. Let x^* solve $u^B(x^*) = u^S(x^*)$. Then the bankruptcy set is given by $\mathfrak{B} = \{x | \underline{x} \leq x < x^*\}$. (22) and (23) imply

$$\left[\eta Ax^* + (1+r)(1-\epsilon A - c) \right] \left(\frac{v_B}{1-\rho} \right)^{1-\rho} = \left[A(x^* - \bar{v}) + (1+r)(1-\epsilon A - c) \right] \left(\frac{v_S}{1-\rho} \right)^{1-\rho},$$

which implies (21).

³¹The direct effect is to increase the entrepreneur's payoff by decreasing required payments to the bank and the indirect effect is to lower the bankruptcy probability.

³²At realization x^* , the entrepreneur is indifferent between default and continuing to operate the firm. Thus, (2) must hold with equality. Solving (2) for x^* implies (21).

Now suppose that x^* is given by (21) and $x^* > \underline{x}$. Then by construction, $u^S(x^*) = u^B(x^*)$. Further, the monotonicity result established above implies $u^B(x) > u^S(x)$ for all $x < x^*$ and $u^S(x) \leq u^B(x)$ for all $x \geq x^*$. Thus, the bankruptcy set is given by $\mathfrak{B} = \{x | \underline{x} \leq x < x^*\}$. ■

Proof of Proposition 2. Let $\Gamma(v_S)$ be the maximum entrepreneur utility in Problem 3. We must prove there exists v_S^* such that $\Gamma(v_S^*) = v_S^*$. First let $\rho > 1$. Suppose that $v_S = 0$. Then $v_B < 0$. As a consequence, $\Gamma(0) < 0$. Now let \hat{v}_S be the entrepreneur's expected utility from autarky.

$$\hat{v}_S = \max_{c_0, c_1, \dots} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

Subject to:

$$\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} \leq w \quad \text{and} \quad c_0, c_1, \dots \geq 0,$$

Note that if $v_S = \hat{v}_S$ and we choose $A = 0$ in problem 3 then we get the autarky utility \hat{v}_S . Thus, optimization implies that $\Gamma(\hat{v}_S) \geq \hat{v}_S$. Since Γ is continuous, the intermediate value theorem implies that there exists a fixed point v_S^* .

For $\rho \leq 1$ we re-normalize $u_\rho(x) = (x^{1-\rho} - 1)/(1 - \rho)$. Then $\lim_{\rho \rightarrow 1} u_\rho(x) = \ln(x)$. Suppose that $v_S = 0$ and that $u(x) = \ln(x)$. We show that $\Gamma(v_S) < 0$.

Let $w_0 = 1 - \epsilon A$ be the amount of net-worth not invested in the firm. Because the continuation payoff from non-default is zero we get

$$\Gamma(0) = \max_{c_0, c_1, \dots, c_T} \sum_{t=0}^T \beta^t \ln(c_t) \tag{24}$$

Subject to:

$$\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} \leq w_0$$

Furthermore, it is sufficient to prove that the objective of (24) is negative for $w_0 = 1$, because the objective is increasing in w_0 .

The first order conditions immediately reveal that

$$c_t = (1+r)^t \beta^t c_0, \quad c_0 = \frac{1-\beta}{1-\beta^{T+1}}. \tag{25}$$

Substituting (25) into the objective of (24) yields

$$\sum_{t=0}^T \beta^t \ln((1+r)^t \beta^t) + \sum_{t=0}^T \beta^t \ln(c_0). \quad (26)$$

If $\beta(1+r) = 1$ then (26) is strictly less than 0. Thus, there exists $\bar{r}(\beta)$ with $(1+\bar{r}(\beta))\beta > 1$ such that $\Gamma(0) < 0$ for all $r \leq \bar{r}(\beta)$. By continuity there exists $\underline{\rho} < 1$ such that $\Gamma(0) < 0$ for $\rho \geq \underline{\rho}$. Finally, $\Gamma(\hat{v}_S) \geq \hat{v}_S$ for the autarky level of utility \hat{v}_S . Thus, continuity of Γ implies the existence of a fixed point v_S^* . ■

Appendix C: Match Criterion

We compare criterion (15) to the alternative square distance criterion.

Table 20 Supremum Norm: $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$, optimism=0.0%

T	10	11	12	13	14	15	16	20
μ	1.62	1.55	1.49	1.51	1.52	1.52	1.51	1.50
σ	0.90	0.83	0.75	0.74	0.76	0.76	0.76	0.78
b %	20.6	21.5	22.0	19.8	18.4	17.7	17.3	15.4
fit	0.046	0.042	0.037	0.034	0.034	0.034	0.035	0.036
median A %	46.9	48.1	49.2	47.0	45.3	44.3	43.8	41.3
default %	4.7	4.4	4.2	3.8	3.5	3.3	3.1	2.5
cons. %	3.7	3.6	3.6	3.6	3.6	3.6	3.6	3.5
neg Eq. %	10.2	10.6	10.8	10.5	10.8	11.1	11.6	11.1

Table 21 Square Norm: $r_B = 1.2\%$, $r = 4.5\%$, $\beta = 0.97$, $\delta = 0.10$, optimism=0.0%

T	10	11	12	13	14	15	16	20
μ	1.53	1.49	1.47	1.46	1.44	1.41	1.42	1.41
σ	0.74	0.70	0.70	0.69	0.67	0.65	0.67	0.69
b %	21.4	21.8	20.9	20.3	20.3	20.7	19.3	17.3
fit	0.020	0.019	0.019	0.019	0.019	0.019	0.019	0.020
median A %	50.2	50.8	49.6	49.0	48.9	49.3	47.4	44.7
default %	4.7	4.4	4.0	3.8	3.6	3.4	3.2	2.6
cons. %	3.7	3.7	3.6	3.6	3.6	3.5	3.5	3.5
neg Eq. %	9.8	10.2	10.1	10.3	10.9	11.7	11.8	11.5

11 Appendix D

11.1 Construction of the Distribution of Firm Returns

We use the 1993 SSBF to compute the return on assets (ROA) because it includes interest payments. We exclude unincorporated firms because the SSBF data do not account for the entrepreneur’s wage from running the firm. The firm’s nominal after-tax ROA is:³³

$$x = \frac{\text{Profit after taxes} + \text{Interest Paid}}{\text{Assets}} + 1. \quad (27)$$

Interest paid is added to after tax profit because the ROA must include payments to both debt and equity holders.³⁴ The nominal rate is adjusted by 3% for inflation (BLS CPI 1993). ROA is computed instead of return on equity because many firms had negative equity (about 16% in the 1993 SSBF and 21% in 1998). Many of these firms stay in business because owners use personal funds to “bail out the firm.” Computing a ROA and modeling owners’ allocations of equity and debt accounts for this.³⁵

11.2 Numerical Procedure

Given model parameters, compute solutions to problem 3 as follows. For fixed v_S , use the first order conditions to solve for the optimum. (9) is always slack, since $c + \epsilon A = 1$ would imply zero future consumption. We need only verify if (10) and (or) (11) bind by checking for positive Lagrange multipliers in the first order conditions. Inserting the solution of the first order conditions into the objective yields $\Gamma(v_S)$. To find a fixed point, compute slope $\Gamma'(v_S)$ by the Envelope Theorem or compute the difference of Γ between v_S and a point v'_S , giving solution ϵ , A , c , \bar{v} . Section 5 explains how to go from these point estimates to cdfs. Compute $\underline{\rho}$ from the first order condition using the fact that $v_S \rightarrow \infty$ as $\rho \downarrow \underline{\rho}$.³⁶

³³The 1993 SSBF has 4637 observations representing 4,994,157 firms with 50.1% incorporated; the 1998 SSBF has 3554, representing 5,282,786 firms with 42.0% incorporated. Section 2 reports the S&P500 ROA in 1993, computed from Compustat’s Research Insight 7.6 database (440 companies with complete data):

$$x = \frac{IBCOM + XINT + DVP}{AT} + 1$$

IBCOM is income before extraordinary items, XINT is interest expense, DVP is preferred stock and dividends and AT is total assets. The median, mean, standard deviation, skewness and kurtosis are computed and confidence intervals are calculated using 5000 bootstrap iterations.

³⁴We use after tax returns as this is relevant for an entrepreneur to decide how much net-equity to invest.

³⁵Computing ROE is misleading for firms near distress. For firms with low but positive equity, small profit gives a high percentage return. Also, many loans are collateralized; book value of equity understates owner contribution (the “correct” value of equity).

³⁶Choose a large value for v_S , solve for the remaining parameters including ρ , which approximates $\underline{\rho}$. In other words, rather than solving the fixed point problem for v_S , solve it for ρ .

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