Contract Model Applications: Bankruptcy

Anne P. Villamil *

Institute for Advanced Studies
Vienna, Austria
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Outline of Part 4: Contract Model Application to Bankruptcy

1. Consumer Bankruptcy: Livshits, MacGee and Tertilt
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   (b) Bankruptcy institutions:
      i. US: Fresh start
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*Department of Economics, University of Illinois, 1206 S. 6th Street, Champaign, IL 61820 USA, avillami@uiuc.edu

1 Introduction

Institutions affect the ability of firms to raise finance. For example, Beim and Calomiris (2001) note that financial markets depend on the legal system in four general ways. First, legal systems define property rights. Second, they specify the types of contracts that are permissible and procedures to enforce contracts. Third, they permit a firm to establish itself as a legal entity, with different firm types having different rights and responsibilities (e.g., corporations have limited liability, but the debt of an unincorporated firm is the owner's personal liability). Fourth, legal systems define and restrict the financial system by specifying laws to buy and sell securities, pledge collateral, and resolve firm insolvencies.

Despite recent progress in empirically documenting the links between institutions and finance, an integrated view of how institutions and enforcement affect finance has not yet been developed. Contract models are well suited for understanding this issue. There are at least two branches in the contract literature, but neither branch models the contract enforcement process:[1]

- *Optimal contract models* focus on deriving the form of the contract, but have not yet been used for quantitative policy analysis due to data problems (cf., Freixas and Rochet [2] and the references therein).

- *Dynamic contracting models* have been used for quantitative analysis, but the models usually focus on optimal social insurance when there are incentive or commitment problems (cf., Ljungqvist and Sargent [7], chapter 15, and the references therein).[2]

We will study computable contracting models of consumer and firm bankruptcy that can be used to analyze policy, e.g., the effect of particular legal reforms.

2 Consumer Bankruptcy

Consider the model of consumer bankruptcy by Livshits, MacGee and Tertilt [6].[3] They provide an interesting quantitative assessment of consumer bankruptcy rules

[1] With the exception of Krasa and Villamil [4] and Krasa, Sharma and Villamil [3], contract models assume either perfect ex post enforcement or self-enforcing contracts.

[2] These models usually study a social planner who wishes to design an efficient contract to supply insurance, subject to incentive constraints.

[3] See also Chatterjee, Corbae, Nakajima and Rios-Rull [1] for a related model of unsecured consumer bankruptcy with an infinitely lived agent and an exogenous income process.

in life cycle model of insurance with incomplete markets calibrated to U.S. and German data. Households face idiosyncratic uncertainty about net asset holdings and labor income. The paper quantitatively analyzes two different bankruptcy institutions for unsecured consumer credit:

- U.S. Bankruptcy Code, Chapter 7: Debtors discharge debt via bankruptcy and all claims on future income are extinguished. When bankruptcy occurs under Chapter 7, the debtor gives up all non-exempt property owned at the time the bankruptcy petition is filed. If the court grants a discharge, the debtor is not liable for any other pre-bankruptcy debts and no claims can be made against future earnings. Chapter 7 simultaneously liquidates all non-exempt assets for the benefit of creditors and protects the insolvent debtor.[4]

- Continental Europe: Consumer bankruptcy restructures consumer debt payments and limits the amount of wage income that can be seized. Prior to 1999 consumers in Germany were liable for any debt until the end of life.[5]

Livshits, MacGee and Tertilt [6] argue that bankruptcy has two effects:

1. Credit Market Distortion: Bankruptcy weakens agents' ability to commit to repay debt in the future. This limits their ability to borrow to smooth consumption across *time.*

2. Insurance: When markets are incomplete, bankruptcy increases the ability to smooth across *states* because it introduces contingencies into non-contingent debt contracts. The easier it is to discharge debt, the greater the insurance against bad shocks – divorce, job loss, medical problems.

Taking contracts and markets as given, Livshits, MacGee and Tertilt argue that any evaluation of bankruptcy must consider the quantitative tradeoff between the negative effect of the credit market distortion and the positive effect from insuring

[4]In the U.S. there are five types of bankruptcy, Chapters 7, 9, 11, 12 and 13. Chapter 7 is the most common type of bankruptcy for individuals; Chapter 13 is the second. Chapter 13 requires debtors to repay creditors under a court approved plan, and is used when a debtor is better off repaying but needs more time than creditors will allow. For example, if a debtor misses mortgage payments and faces foreclosure due to a temporary job loss, Chapter 13 allows the debtor up to 3 years to repay. Chapter 11 is designed for corporations seeking to reorganize debts while continuing to operate, Chapter 12 is the analog for family farms and Chapter 9 is for government bodies. Businesses can file Chapter 7 or 11, but not 13.

[5]The current law allows for discharge of debt after a 7-year court approved payment period.

against bad shocks. They consider a heterogeneous agents life cycle model with *unsecured* consumer credit where each period agents[6]

- make labor-leisure and consumption-savings decisions
- decide whether to file for bankruptcy, given the bankruptcy rule (i.e., the percent of wages that can be seized and whether debt is discharged)

They consider three types of default costs. First, future exclusion from credit markets (i.e., the inability to borrow and save in a default period). Second, a transaction cost on consumption during the default period. Third, the income seized when bankruptcy is filed.

2.1 The Model

The economy consists of overlapping generations of ex ante identical households that live for J periods. The households face idiosyncratic uncertainty, but there is no aggregate uncertainty. Markets are exogenously incomplete: The only asset is a person specific one-period non-contingent bond.

Consumer/Producer: Utility in period j is:
$u(c_j, 1 - h_j)$, where
c_j is consumption
h_j is work effort
β_i is the discount factor of a household of age i.

Output is determined by productivity, work effort and the labor endowment:
$y_j = z_j \bar{e}_j h_j$, where
$\bar{e}_j$ is the household's endowment of efficiency units of labor
z_j is the random household productivity parameter at age j

There are two sources of uncertainty:

1. Income shock z_j: The household productivity parameter follows a Markov chain with an age independent transition matrix $\Pi(z'|z)$. Initial productivity at age 0, z_0, is drawn from a stationary distribution.
2. Expense shock κ: Idiosyncratic expense shock $\kappa \geq 0$, $\kappa \in K$ where K is a finite set of all possible expense shocks. The probability of each shock is $\pi(\kappa)$. These shocks are iid and independent of the income shock.

[6] Key model features are that consumers cannot commit to future repayment or fully insure.

Financial Markets: The borrowing and lending market is perfectly competitive. Financial intermediaries accept deposits from savers and make one-period, non-contingent loans with face value d. Debt obligation d is the amount to be repaid next period, where $d > 0$ is borrowing and $d < 0$ is saving. Although loans are non-contingent, bankruptcy introduces a partial contingency.

Intermediaries observe the borrower's total level of borrowing d, current productivity shock z, and age.

$q^b(d, z, j)$ is the price of the loan issued by the bank to the household

τ is the transaction cost of making a loan that is proportional to loan size

Each period, the intermediary maximizes expected profit. Because the market is competitive, intermediaries earn zero expected profit on loans. Thus, the expected value of repayments must equal the cost of the loan to the intermediary. Further, cross subsidization of interest rates across different borrowers will not occur and the interest paid to savers satisfies $q^s = \frac{1}{1+r}$.

Bankruptcy: A bankruptcy rule has two elements

1. The law of motion for household debt takes one of two forms

   (a) Fresh Start: Full discharge of all debts.

   (b) No Fresh Start: Remaining debt, after seizure of income, is rolled over at a specified interest rate $\bar{r}$ with bond price $\bar{q} = \frac{1}{1+\bar{r}}$. Debt is not discharged, but is rescheduled (payments are postponed and the interest rate may be reduced). All household assets can be seized and the wage garnishment rule is linear:

   $$\Gamma = [\max\{y - \bar{y}, 0\}]g$$

   Γ is the total amount of wages garnished and transferred to creditors, $\bar{y}$ is an earning exemption that cannot be seized, and $g \in [0, 1]$ is the marginal rate of garnishment. The garnishment technology is costless.

2. The garnishment rule specifies the amount of household assets and earnings that can be seized by creditors.

During the bankruptcy period, the debtor faces two further punishments:

- Transaction cost λ is proportional to consumption expenditures during the default period.

- A household that defaults cannot save or borrow during the default period.

Timing within a Period: At the beginning of the period, each household observes shocks z (productivity) and κ (expense). If the household receives an expense shock, debt is increased by κ (or savings decreased).

The household decides to:

- File for bankruptcy or not, $I = 0, 1$.
- How much to work, $h \in [0, 1]$.
- How much to consume, $c \geq 0$.
- Next period's debt, d', taking bond prices as given.

Work occurs and wages are directly deposited into a "bank account." If the household is bankrupt, the amount garnished Γ is deducted and the household keeps the rest. I is the household's default decision.

- Bankrupt households cannot save in the period they declare bankruptcy and consume all assets net of garnishment Γ and transaction cost λ. The new debt level depends on the bankruptcy rule.
- Non-bankrupt households choose net assets for next period and current consumption.

Household Problem. Fresh Start

$$V(d, z, \kappa, j) = \max_{c,h,d',I} u(c, 1-h) + \beta_j EV(d', z', \kappa', j+1)$$

subject to:

$$c + d + \kappa = \bar{e}_j zh + q^b(d', z, j)d', \text{ if } I = 0$$

$$c = (1-\lambda)[\bar{e}_j zh - \Gamma] \text{ and } d' = 0, \text{ if } I = 1$$

$$c \geq 0, h \in [0, 1], V(\cdot, \cdot, \cdot, J+1) = 0$$

Household Problem. No Fresh Start

$$V(d, z, \kappa, j) = \max_{c,h,d',I} u(c, 1-h) + \beta_j EV(d', z', \kappa', j+1)$$

subject to:

$$c + d + \kappa = \bar{e}_j zh + q^b(d', z, j)d', \text{ if } I = 0$$

$$c = (1 - \lambda)[\bar{e}_j zh - \Gamma], \text{ if } I = 1$$

$$d' = \max\{[d + \kappa - \Gamma], 0\}/\bar{q}, \text{ if } I = 1$$

$$c \geq 0, h \in [0, 1], V(\cdot, \cdot, \cdot, J + 1) = 0$$

Intermediaries. There is no aggregate risk, so the price of a bond is determined by the default probability and the risk free bond price.

$\theta(d', z, j)$ is the probability that a household declares bankruptcy tomorrow. The household is of age j, with current productivity shock z, and total debt d'.

$\bar{q}^b = \frac{1}{1+r^s+\tau}$ is the price of a bond with zero default probability.

$q^b(d', z, j) = (1 - \theta(d', z, j))\bar{q}^b$ is the zero profit condition without garnishment and full discharge of debt.

The bond price under FS with wage garnishment is:

$$q^{FS}(d', z, j) = (1 - \theta(d', z, j))\bar{q}^b + \theta(d', z, j)E[\frac{\Gamma}{d' + \kappa'}|I = 1]\bar{q}^b$$

The bond price under NFS with wage garnishment is:

$$q^{NFS}(d', z, j) = (1-\theta(d', z, j))\bar{q}^b+\theta(d', z, j)E[\frac{\Gamma + q(d'', z', j + 1)d''}{d' + \kappa'}|I = 1]\bar{q}^b$$

where $d'' = \frac{\max\{d'+\kappa'-\Gamma,0\}}{\bar{q}}$.

Given a bankruptcy rule $(\bar{y}, g)$ and a risk free bond price q^s, a *competitive equilibrium* with FS or NFS is a value function $V(\cdot)$, a set of policy functions $h(\cdot)$, $d(\cdot)$, $I(\cdot)$, a default probability $\theta(d', z, j)$, and a pricing function $q^b(\cdot)$ such that

1. $V(\cdot)$ satisfies the FS or NFS functional equation and $h(\cdot)$, $d(\cdot)$, $I(\cdot)$ are the associated policy functions.

2. The default probabilities are give by

$$\theta(d', z, j) = \sum_K \sum_Z I(d', z', \kappa', j + 1)\pi(z'|z)\pi(\kappa')$$

3. Bond prices $q^b(\cdot)$ are determined by the FS or NFS zero profit condition.

2.2 Calibration

Livshits, MacGee and Tertilt [6] calibrate the model for the US and Germany.

1. Preferences: Households live for 10 periods and the length of a period is 5 years. Life begins at age 20. The utility function is

$$\sum_{j=1}^{10}\Big(\prod_{i=1}^{j-1}\beta_i\Big)\Big(\big[c_j^{\chi}(1-h_j)^{1-\chi}\big]^{1-\sigma}-1\Big)/(1-\sigma)$$

   The parameters are standard (e.g., $\sigma = 2$) where $1/\sigma$ is the intertemporal elasticity of substitution. χ is consumption's share of the composite commodity – the household spends 40% of time working. The discount factor is 0.96. The savings interest rate is 4%, and the transaction cost is 5% (based on data from credit card companies).

2. Income Process: They use the PSID for the US and the GSOEP for Germany. Productivity shocks have 5 possible values, and productivity is (total labor income)/(total hours worked). The transition probability is the probability of moving between earning quintiles over a 5 year period. See the paper. They also calibrate the life cycle endowment of the efficiency units of labor (see the paper).

3. Expense Uncertainty: Expense shocks take on three values, and there are two probabilities. The shocks are medical expense, divorce, and the cost of an unwanted child. See the paper for calibration.

4. Bankruptcy Rules: The parameters are λ, g, $\bar{y}$, and $\bar{r}$. See the paper.

2.3 Results

Livshits, MacGee and Tertilt [6] obtain three important results.

1. The welfare implications of different bankruptcy rules are sensitive to the type and size of a consumer's uncertainty.

2. The different bankruptcy laws in the U.S. and Germany are consistent with their different stochastic processes for income and expenses (e.g., presumably due to different institutions in the two countries).

3. The bankruptcy rules have a very small effect on labor decisions.

3 Firm Bankruptcy

The previous paper considered an unsecured lending model of default for personal bankruptcy. However, Budria, Diaz-Gimenez,Quadrini and Rios-Rull [9] indicate that "bankruptcy occurs often in households who fail in their business projects." The 1998 SCF data indicate the share of income from businesses for non-bankrupt households is 10.3%, but for bankrupt households the share is - 0.7%. Thus, it is important to model firm distress if we wish to understand household bankruptcy. Krasa and Villamil propose the following model of small firm finance with secured lending.

3.1 The Model

Consider an economy with a risk neutral firm and investor. The firm owns a constant returns to scale technology. Each unit of input yields a random output $y \in Y = \{\underline{y}, ..., \bar{y}\}$. The firm has initial wealth $\omega = \omega_l + \omega_c$, where ω_l are liquid assets (such as transaction accounts, CDs, stocks, bonds, mutual funds), and illiquid assets (such as equity in vehicles and residences).[7] The firm can invest liquid assets directly in the project, but illiquid assets can be pledged as collateral to obtain a loan from an investor. Assume that the supply of funds is perfectly elastic. Thus, the lender will supply funds as long as he/she receives at least market return $1 + r_m$. Only the firm privately observes output realization y unless enforcement occurs.

Agents arrange finance by writing a contract, $\{F, \epsilon, r, \sigma_d\}$, where

- F is the total amount of funds invested in the project;
- ϵ is the percent of private equity the firm provides and $1-\epsilon$ is the percent of debt the investor provides;
- r is the interest rate on the loan with principal and interest $\bar{v}(\epsilon, r) = (1 - \epsilon)(1 + r)$, per unit of investment; and
- $\sigma_d(y)$ is the probability that the firm defaults in state y.

Default means that the firm does not repay principal and interest $\bar{v}(\epsilon, r)$.

[7]These categories correspond to variables in the SSBF data that are used to calibrate the model. The data are described below.

When the firm defaults, the investor can request costly judicial enforcement. The investor pays cost $c > 0$ and the court determines the value of the firm's total assets per unit of funds invested,

$$a(y) = y + (1 + r_m)\frac{\omega - \epsilon F}{F}$$

Term y is the total value of the firm's investment project and term $\omega - \epsilon F$ is the amount of owner funds invested at market rate $1 + r_m$. F is the total amount of funds invested in the firm, and these funds consist of two parts:

- ϵF is private equity since the firm is not publicly traded.
- $(1 - \epsilon)F$ is debt, measured by assets minus equity.

Assume that the court cannot seize some percentage η of the total assets, e.g., due to exemptions set by the states. The court distributes the remaining assets to the investor, less cost c. Net firm assets that the court transfers to the investor are

$$(1 - \eta)a(y) - c$$

Enforcement is described by two key parameters:

1. Enforcement cost, c, which is a deadweight loss to the contracting parties, per unit of funds, F. This cost includes the cost of receiving and interpreting accounting statements, legal fees, court fees, and corruption (e.g., bribes).
2. Debtor protection, η, which is the percentage of firm assets that the court may not seize. This parameter is determined by factors such as the exemptions (set by states in the U.S.), inflation, and the length of bankruptcy proceedings. The higher these factors are, the higher η, and the weaker creditor protection (or the stronger debtor protection).

In computational experiments these parameters are varied in order to quantitatively determine the effect of changes in enforcement institutions on firm finance. Parameter η is a key variable for unincorporated firms because the firm's debts are the owner's personal liability. Hence, η affects the firm's incentive to default.[8]

[8] Berkowitz and White (2001, Table 1) document wide variation in bankruptcy exemptions by state. For example, in 1993 seven U.S. states (AR, FL, IA, KS, MN, OK, TX) permitted individuals who file for bankruptcy to shelter an unlimited amount of wealth as equity in a principle residence (this is known as the homestead exemption). In contrast, twenty-seven U.S. states capped the maximum possible amount of equity that can be sheltered in a principal homestead at $20,000 or less (in MD this amount was zero). In most states the homestead exemption is the largest means by which debtors can shelter wealth. The two exceptions are Kentucky (a horse of unlimited value can be sheltered) and Hawaii (a car of unlimited value can be sheltered).

In Krasa and Villamil show that contract $\{F, \epsilon, r, \sigma_d\}$ is derived from a contract problem that is a reduced form of a game with incomplete information and limited commitment. As in Krasa, Sharma and Villamil [3], this contract accommodates both *inability to pay* and *willful default*. The firm is unable to pay if assets are less than the amount owed, $\bar{v}$, and unwilling to pay if the firm defaults even though total assets are sufficient to pay $\bar{v}$.

Definition 1. Consider the payment from the firm to the investor per unit of funds invested, $\bar{v} = (1-\epsilon)(1+r)$.

1. If $a(y) < \bar{v}(\epsilon, r)$, default occurs because the firm is unable to pay.
2. If $a(y) \geq \bar{v}(\epsilon, r)$, then if default occurs it is willful.[9]

3.2 The Investment Problem and Equilibrium Contract

Recall that $\sigma_d(x)$ is the probability that the firm defaults. Let $\beta(y)$ denote the ex-ante probability of realization y. If the firm defaults, then the investor will update $\beta(y)$ to $\beta(y|d) = \frac{\sigma_d(y)\beta(y)}{\sum_{x\in Y}\sigma_d(y)\beta(y)}$. Therefore, the contract problem is:

Problem 1. At $t = 0$, choose $\{F, \epsilon, r, \sigma_d\}$ to maximize

$$E[u_E(y)] = \sum_{y\in Y}\eta a(y)\sigma_d(y)\beta(y) + \sum_{y\in Y}\big(a(y)-\bar{v}(\epsilon,r)\big)\big(1-\sigma_d(y)\big)\beta(y) + (\epsilon_0-\epsilon)(1+r_m)$$

Subject to:

$$\frac{1}{1-\epsilon}\sum_{y\in Y}\Big[\big((1-\eta)a(y)-c\big)\sigma_d(y) + \bar{v}(\epsilon,r)\big(1-\sigma_d(y)\big)\Big]\beta(y) \geq 1+r_m \quad (1)$$

$$E[u_E(y)] \geq \omega(1+r_m) \quad (2)$$

$$F\epsilon \leq \omega_l \quad (3)$$

$$\sigma_d(y) = \begin{cases} 1 & \text{if } \bar{v}(\epsilon,r) > (1-\eta)a(y) \\ 0 & \text{if } \bar{v}(\epsilon,r) < (1-\eta)a(y) \\ \alpha\in[0,1] & \text{if } \bar{v}(\epsilon,r) = (1-\eta)a(y) \end{cases} \quad (4)$$

[9] This is "willful default" because total assets $a(y)$ exceed the amount owed, $\bar{v}$, but the bankruptcy code protects the debtor from judgments against the exempt portion of assets.

If $\sigma_d(y) > 0$ for an $y \in Y$ (i.e., bankruptcy occurs) then:

$$\sum_{y \in Y}(1-\eta)a(y)\beta(y|d) - c \geq (1-\eta)a(\underline{y}) \tag{5}$$

$$\bar{v}(\epsilon, r) \leq (1-\eta)\sum_{y \in Y} a(y)\beta(y|d) - c \tag{6}$$

The objective is the firm's ex-ante expected payoff in default and non-default states, respectively, plus the opportunity cost of funds not invested in the firm. Given realization y, the firm defaults with probability $\sigma_d(y)$. When default occurs and the investor requests enforcement, the court seizes $1-\eta$ percent of assets $a(y)$, and the firm therefore retains $\eta a(y)$. Otherwise, with probability $1-\sigma_d(y)$ the firm repays the debt and retains the residual $a(y) - \bar{v}(\epsilon, r)$. ϵ_0 is the total amount of the firm's initial wealth at $t = 0$, thus $\epsilon_0 - \epsilon$ is the amount the firm invests in the outside market opportunity with return $1 + r_m$.

The following constraints may bind, depending on parameter values. (1) is the investor's ex-ante expected return.[10] We assume that the investor's supply of funds is perfectly elastic, thus as long as the expected return is at least $1 + r_m$ the firm will receive the required funds. (2) is the firm's voluntary participation constraint. The expected payoff from operating the project must be at least as great as the amount that the firm's owner can obtain from investing all initial wealth in the market investment opportunity. (3) specifies that the firm owner's personal funds invested in the project cannot exceed liquid assets. The owner can invest more finds by borrowing from the investor and using illiquid assets as collateral. (4) requires the firm's default decision to maximize expected payoffs. If $\bar{v}$ exceeds the expected payment from bankruptcy, $(1-\eta)a(y)$, the firm chooses to default (i.e., not pay $\bar{v}$). If the inequality is reversed, the firm chooses to pay $\bar{v}$ with probability 1. If the payments are the same, the firm may randomize with probability α.

Finally, (5) and (6) ensure that firm and investor expectations about judicial enforcement are rational. That is, if ex-ante the investor is expected to request enforcement when the firm does not make a payment, then it should be optimal for the investor to request enforcement ex-post. For a given loan size, Krasa, Sharma and Villamil [3] show that constraints (5) and (6) do not bind for the values of c that apply to the U.S. economy.

[10] The investor's payoff is $F\big((1-\eta)a(y) - c\big)$ if bankruptcy occurs, and $F\bar{v}$, otherwise. This expected payment is divided by the total amount of funds supplied by the investor, $(1-\epsilon)F$.

3.3 The SSBF Data and Calibration

Krasa and Villamil use data from the National Survey of Small Business Finance (SSBF) to calibrate the model. The SSBF is a survey administered by The Board of Governors of the Federal Reserve System and the U.S. Small Business Administration. Surveys were conducted in (1987), (1993), (1998) and (2001), with the (2001) results scheduled for release in January 2003. Each survey is a cross sectional sample of about 4000 non-farm, nonfinancial, non-real estate U.S. businesses, using the SBA classification for small firms as those with less than 500 full-time equivalent employees. The firms represent about 5 million businesses.[11] Each survey contains information on the characteristics of small firms and the primary owner (e.g., firm owner and age, industry, type of business organization), the firm's income statement and balance sheet, details on the use and source of financial services, and the firm's most recent borrowing experience (including trade credit and capital injections such as equity). The data set is publicly available at: *http://www.federalreserve.gov/*. There is also an internal version that contains additional data.[12]

The model is designed to perform policy experiments. For example, to show how changes in legal institutions (c, η) affect components of the optimal contract (r, σ_d, ϵ). The general methodology for this type of analysis is outlined in Kydland and Prescott [5]. In order to conduct quantitative analysis, the following parameters must be calibrated. (i) *Preferences:* This is a standard exercise in calibration.[13] (ii) *Endowments:* This has been a significant problem for quantitative analysis of financial contracting models because it requires detailed balance sheet information. The SSBF contains the required information. (iii) *Technology/distributions*: This has also been a significant problem because it requires measurement of firm return distributions per unit of funds invested. (iv) *Legal parameters:* The two legal parameters are the percent of assets that are exempt from seizure by the bankruptcy court and the bankruptcy cost.

3.4 Computational Procedures

We now explain the procedure to compute solutions to the calibrated model. When the last two constraints of Problem 1 do not bind, Krasa and Villamil show that

[11] Full time equivalents are full-time employees plus one half of part-time employees.

[12] See Woken [10] for an excellent discussion of the data set.

[13] However, risk aversion significantly complicates the derivation of the optimal contract in the theoretical analysis.

the problem can be written in a form that is readily amenable to computation. The objective is again the firm's expected payoff. The only remaining constraint is a version of the investor's finance constraint. All other constraints are substituted into the problem. Denote by $x^\star$ the lowest non-default state; ϵ is the percent of funds that the firm supplies in the form of private equity.

Problem 2. At $t = 0$, choose ϵ and $y^\star \in \{y_1, \ldots, y_n\}$ to maximize

$$E[u_E(y)] = \frac{\omega_l}{\epsilon}\Big[E[Y] - c\beta(y < y^\star) - (1 + r_m)\Big] + \omega(1 + r_m)$$

Subject to

$$\epsilon = K\left[1 + r_m + c\beta(y < y^\star) - (1 - \eta)\left(\sum_{y<y^\star} y\beta(y) + y^\star\beta(y) \geq y^\star)\right)\right]$$

where $K = \Big[(1 + r_m)\Big(1 + (1 - \eta)\frac{\omega_c}{\omega_l}\Big)\Big]^{-1}$.

In order to compute solutions numerically, proceed as follows. For each choice of $y^\star$, the constraint provides the value of ϵ that can be used to compute the firm's expected payoff (the objective of Problem 2), given the particular choice of $y^\star$. By iterating over all possible choices for $y^\star \in \{y_1, \ldots, y_n\}$, determine the optimal $y^\star$. The constraint then implies the optimal percentage of private equity, ϵ. Krasa and Villamil prove that the remaining variables can be determined as follows:

1. *Amount of funds*: Because it is optimal for the firm to commit all liquid assets to a project, $\epsilon F = \omega_l$. We can compute F from ϵ and ω_l.

2. *Firm collateral*: This is determined from the definition of assets $a(y) = y + (1 + r_m)\frac{(\omega - \epsilon F)}{F}$.

3. *Implied interest rate*: Because $(1 - \eta)a(y^\star) = \bar{v}(\epsilon, r) = (1 - \epsilon)(1 + r)$ for the optimal contract, r can be computed directly.

4. *Willful default states:* Willful default occurs in all project states y where $\bar{v}(\epsilon, r) \leq y < y^\star$.

3.5 Results

The model shows that:

1. The default rate is increasing in η (debtor protection) and c, given ϵ_0.
2. The default rate is decreasing in initial equity level ϵ_0, given c and η.
3. Equilibrium credit rationing can occur for plausible parameter values.[14]

[14] See Park [8] for an application of the model to Korea.

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