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OPTIMAL INFLATION TAX AND STRUCTURAL REFORM

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This paper analyzes the optimal inflation tax in economies with structural imperfections in labor, commodity, and currency markets. The Friedman rule is a classic result in economics that claims that the optimal monetary policy is to set a zero nominal interest rate. This Ramsey equilibrium is robust in a wide range of environments without imperfections in input, output, or financial markets. In many developing countries, however, a large fraction of activity takes place in the "informal" sector. Roughly speaking, the informal sector is the untaxed and unregulated market sometimes referred to as the underground economy. We obtain three results. First, we show that when structural imperfections such as an informal sector exist, the optimal inflation tax is positive. Second, we show that structural imperfections introduce an important asymmetry in the welfare cost function. Third, we provide quantitative results.

Keywords: Inflation, Informal Sector, Ramsey Policy, Friedman Rule, Underground Economy, Currency Substitution, Incomplete Taxes

1. INTRODUCTION

The Friedman rule is a classic result in economics that claims that only monetary policies that generate a zero nominal interest rate will lead to optimal resource allocation [cf. Friedman (1969)]. In practice, this means that the economy should have either no inflation or deflation at a constant rate. On the basis of this result, some economists and policymakers have argued that price-level stability should be the main goal of a central bank. This "Ramsey equilibrium" result is robust in a wide range of dynamic, constrained, general equilibrium environments with distortionary taxes and commitment, but without imperfections in input, output, or financial markets.[1] For example, Chari and Kehoe (2002) show that the result holds in monetary models with a shopping time technology, cash-in-advance constraint, and money in the utility function. In all cases, markets are complete.

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In contrast, in many developing countries a large fraction of economic activity takes place in the "informal" sector (e.g., 20–50%). Roughly speaking, the informal sector is the untaxed and unregulated sector sometimes referred to as the underground economy. Two distinct reasons for this structural distortion are important in our analysis: First, activities in the informal sector are not observed by the authorities. Thus, agents may devote some fraction of their time to informal activities in order to evade taxes. This will occur when it is difficult for the government to measure economic activity or enforce compliance [cf. Stone and Paredes (1996)]. Second, activities in the informal sector may be less efficient than those in the formal sector. In this case, substitution between the two sectors can lead to welfare loss. Optimal tax policy balances the distortions that arise from direct labor taxation of the formal sector (only) and indirect taxation of both sectors via inflation.

We develop a model where taxation is incomplete due to an informal sector, and address the following questions: Does a Ramsey analysis of inflation as a tax support the Friedman rule when the government's ability to tax input, output, and financial markets is limited? Further, when such structural distortions occur, are they important quantitatively? To answer these questions, we study the problem of a government that wishes to finance optimally a given expenditure sequence. The economy has two types of markets distinguished solely by the government's ability to observe economic activity in them: All transactions in the formal sector are observable by the government. As a consequence, the government can monitor and tax the income from these transactions. In contrast, in the informal sector the government cannot observe or tax labor income, commodities, or financial transactions.

We follow the "Bewley approach" to incompleteness [cf. Ljungqvist and Sargent (2000)]: Markets are exogenously incomplete by assumption and we investigate the implications of varying the degree of incompleteness. In our setting, this means that the government is unable to tax input, output, and monetary transactions perfectly. In numerical experiments, we treat the size of the structural friction as exogenous and determine the quantitative impact of varying the distortion.[2] Our results are reminiscent of those of Sargent and Wallace (1981), who showed that exogenous fiscal policy is a key determinant of optimal monetary policy. In a similar spirit, we take structural conditions as given (i.e., limitations on the government's ability to tax) and characterize the link between exogenous structural conditions and monetary policy.

We obtain three main results. First, we prove that when structural imperfections exist the Friedman rule is not optimal. Second, we show that the optimal inflation tax can be positive, ranging from 0% to 22% for alternative calibrations. Third, we show how structural imperfections alter the welfare cost of inflation. A key finding is that structural imperfections "flatten out" the welfare cost function, leading to an important welfare asymmetry: The gain from reducing moderate inflation to the optimal (lower) level is small when structural imperfections are large. However, reducing inflation below the optimal level to an arbitrarily low level (e.g., zero) leads to large welfare losses. This result highlights the importance of structural reform for monetary policy.

2. MODEL

Consider a production economy with a single input (labor), a single output (consumption), and $t = 0, 1, \ldots,$ time periods. There is a formal sector and an informal sector in the input and output markets. The government

- can observe and tax transactions in the formal sector, and
- cannot observe and tax transactions in the informal sector.

For simplicity assume that commodity taxes are not available; thus, $\tau_c^F = \tau_c^I = 0$. Appendix A.2 shows that the analysis is robust to incomplete commodity taxes, $\tau_c^F > 0$ and $\tau_c^I = 0$.

2.1. Households

The economy has an infinitely lived representative agent with preferences

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t, h_t). \qquad \textbf{(1)}$$

Let $\beta \in (0, 1)$, c_t and h_t denote consumption and leisure in period t, respectively, and $u(\cdot)$ be a strictly concave, twice continuously differentiable function that satisfies the INADA conditions. The representative household is endowed with one unit of time in each period that can be used as leisure, h_t, to make transactions, s_t, or allocated to production in either the formal sector n_t^F, or in the informal sector n_t^I, with $1 = n_t^F + n_t^I + s_t + h_t$. By working in the informal sector, the agent can evade the taxes associated with formal job contracts. The labor tax in the formal sector is τ_n^F.

2.2. Transaction Technology

Let M_t denote fiat currency and P_t the price level. Thus $m_t = M_t/P_t$ is real money balances. As is standard, assume that real balances are costless to produce and useful for transaction purposes because they decrease the amount of time agents spend shopping. Denote the transaction technology by[3]

$$s_t \geq l\left(c_t, \frac{M_t}{P_t}\right) \equiv l(c_t, m_t). \qquad \textbf{(2)}$$

The transaction technology satisfies the following properties:

(a) $l(c, m) \geq 0$ and $l(0, m) = 0$,
(b) $l_c \geq 0$;

(c) $l_{cc} \geq 0, l_{mm} \geq 0, l_{cc}l_{mm} - l_{cm}^2 > 0$;[4]
(d) $l_m(c, m) = 0$ defines $m^* = m(c)$ such that $l_m < 0$ when $m < m^*$;
(e) $l(\cdot, \cdot)$ is homogeneous of degree k.

Property (d) implies that for a given level of consumption there is a point of "satiation" in real money balances. That is, for each level of consumption, there is a level of real balances such that an additional dollar does not decrease the amount of time agents spend shopping. According to Friedman (1969), "cash balances... are held to satiate, so that the real return from an extra dollar is zero." We assume the following.[5]

Assumption 1. $m^* = m(c) < \infty$.

Assumption 1 states that the satiation level of real money balances is finite. Otherwise, for each level of consumption, an extra dollar will decrease shopping time. From property (e), we can write $l(c, m) = L(m/c)c^k$, where $L' \leq 0$ and $L'' \geq 0$.

Appendix A.1 shows that the results we derive are qualitatively similar for other money demand specifications—cash and credit goods (CIA) and money-in-the-utility-function (MIUF).[6]

2.3. Household Problem

Let B_t denote government bonds with return i_t, and w_t^F and w_t^I denote the wage rates in each sector. The representative agent's one-period budget constraint is

$$c_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} \leq \frac{M_{t-1}}{P_t} + (1 + i_{t-1})\frac{B_{t-1}}{P_t} + \left(1 - \tau_n^F\right) w_t^F n_t^F + w_t^I \left(1 - n_t^F - s_t - h_t\right). \tag{3}$$

In present-value form, the budget constraint, with no-Ponzi game conditions for bonds and money, and with initial conditions, $B_{-1} = M_{-1} = 0$, is

$$\sum_{t=0}^{\infty} d_t c_t + \sum_{t=0}^{\infty} d_t I_t m_t \leq \sum_{t=0}^{\infty} d_t \left[\left(1 - \tau_n^F\right) w_t^F n_t^F + w_t^I \left(1 - n_t^F - s_t - h_t\right)\right], \tag{4}$$

where

$$m_t = \frac{M_t}{P_t}, \qquad 1 + r_t = (1 + i_t)\frac{P_t}{P_{t+1}}, \qquad I_t = \frac{i_t}{1 + i_t} \quad \text{and} \quad d_t = \frac{1}{\prod_{s=0}^{t-1}(1 + r_s)}.$$

The household's problem is to choose $\{c_t, h_t, m_t, b_t, s_t, n_t^F\}_{t=0}^{\infty}$ to maximize (1) subject to (2), (3), and the usual nonnegativity constraints, with $b_t = B_t/P_t$. The necessary conditions for an interior solution are (2), (3), and

$$\frac{u_h(t)}{u_c(t) - u_h(t)l_c(t)} = \left(1 - \tau_n^F\right) w_t^F, \tag{5}$$

$$w_t^I = \left(1 - \tau_n^F\right) w_t^F, \tag{6}$$

$$-l_m(t) = \frac{1}{\left(1-\tau_n^F\right) w_t^F} I_t, \tag{7}$$

$$\frac{u_c(t) - u_h(t) l_c(t)}{u_c(t+1) - u_h(t+1) l_c(t+1)} = \beta(1 + r_t). \tag{8}$$

Equation (7) implicitly defines a money demand function $\hat{m}_t(c_t, I_t, \tau_n^F, w_t^F)$. It is straightforward to show that the money demand function is increasing in consumption if and only if $l_{cm} < 0$.[7] As a consequence, we make the following assumption.

Assumption 2. $l_{cm} < 0$.

Assumption 2 implies a positive elasticity of this money demand function with respect to c (scale elasticity).[8] Estimates by Lucas (1990) and Mulligan and Sala-i-Martin (1997) suggest that this elasticity is close to 1.

The Friedman rule is equivalent to setting $I_t = 0$. The household chooses the bliss point in real balances, $\hat{m} = m^* = m(c)$, so that, given the consumption level, no more resources can be saved by increasing the amount of real money per unit of transaction.

2.4. Production Technology

There is a representative firm whose technology, $F(N) = N$, exhibits constant returns to scale. Assume that the single factor of production is labor services, where N is a CES aggregator of labor employed with formal contracts, N^F, and labor employed informally, N^I. Let

$$F(N^F, N^I) = [\lambda(N^F)^\rho + (1-\lambda)(N^I)^\rho]^{\frac{1}{\rho}}. \tag{9}$$

Parameter $0 < \lambda < 1$ measures the relative importance of formal and informal labor in production. It is also a key determinant of the marginal product of labor in each sector. Parameter $0 < \rho < 1$ affects the elasticity of substitution between the two types of employment. Easterly (1993) uses a similar formulation to investigate the impact of informal contracts on countries' growth rates. In the computational experiments, we use labor market statistics to calibrate production parameters λ and ρ. Our theoretical results do not depend on this formulation of the production function (e.g., a model with two production sectors gives the same results).

In every period the firm takes price as given and maximizes profit. The first-order conditions are $w_t^F = F_{n^F}(t)$ and $w_t^I = F_{n^I}(t)$. Using household equilibrium equation (6) and the firm's first-order conditions, it follows that

$$\frac{N^I}{N^F} = \left[\frac{1-\lambda}{\lambda} \frac{1}{1-\tau_n^F}\right]^{\frac{1}{1-\rho}}. \tag{10}$$

This ratio of labor inputs is essential in the computational experiments. Equation (10) shows that three parameters are crucial determinants of the size of the informal sector, measured by the ratio of labor inputs [i.e., the right-hand side of (10)].[9] We consider each parameter in some detail.

First, the tax τ_n^F on formal labor affects the labor/leisure choice via equation (5). However, this tax also stimulates labor market activity in the untaxed informal sector via equation (6). The innovation of our paper is to introduce the informal sector into the optimal inflation tax problem and show that it provides agents with an additional opportunity to evade taxes that can be important quantitatively. Specifically, in our model, agents can "substitute on two margins" in order to evade taxes. The standard margin is to substitute more (untaxed) leisure to evade the labor income tax via (5). The new margin is to substitute untaxed informal labor for taxed formal labor via (6).

Second, the new margin has potentially important welfare implications because there is an inherent difference in productivity in the formal and informal sectors. This difference is governed by parameter λ in (9). When $\lambda > 1/2$, the informal sector is less productive than the formal sector. Thus, the problem in the economy is not only that tax evasion occurs, but that when tax evasion occurs it stimulates greater activity in the less productive informal sector. This production distortion is costly in terms of welfare.

Third, $-1/(1-\rho)$ is the elasticity of substitution of informal labor supply with respect to the after-tax formal wage. Production parameter ρ determines how strongly N^I/N^F responds to changes in the tax. A higher substitutability between formal and informal labor implies that a small increase in the tax rate on formal income leads to a substantial increase in informal labor. Schneider and Enste (2000) claim that the most important cause of increases in informal activity is the rise of tax and social security burdens. They note that other variables that affect the level of informal activity are penalty rates and tax evasion detection probabilities, which are proxies for government institutions. Elasticity of substitution $-1/(1-\rho)$ reflects the strength of these institutions. Lemieux et al. (1994) document in a microlevel data set that taxes affect labor/leisure choices and stimulate labor market activity in the informal sector. We use their estimate to calibrate ρ in the computational experiments in Section 4.

2.5. Government

The government finances its expenditure stream $\{g_t\}_{t=0}^{\infty}$ by levying a tax on labor earnings in the formal sector at rate τ_n^F, by printing money, and by borrowing. The government's budget constraint is

$$g_t = \tau_n^F w_t^F n_t^F + \frac{B_t^g}{P_t} - (1+i_{t-1})\frac{B_{t-1}^g}{P_t} + \frac{M_t^g}{P_t} - \frac{M_{t-1}^g}{P_t}. \tag{11}$$

2.6. Competitive Equilibrium

A competitive equilibrium is a sequence of prices $\{w_t^F, w_t^I, P_t\}_{t=0}^{\infty}$, a sequence of government policies[10] $\{\tau_n^F, i_t, \pi_t\}_{t=0}^{\infty}$, a sequence of consumer choices $\{c_t, n_t^F, n_t^I, s_t, m_t, b_t\}_{t=0}^{\infty}$, and a sequence of firm choices $\{n_t^F, n_t^I\}_{t=0}^{\infty}$ such that

- given the sequence of prices and government policies, the consumer's allocation solves the representative household's problem;
- given the sequence of prices and government policies, the firm's allocation maximizes profit; and
- market clearing[11]

$$\begin{aligned} c_t + g_t &= F\left(n_t^F, n_t^I\right), \\ n_t^{s,j} &= n_t^{d,j} \quad \text{for} \quad j = F, I, \\ M_t &= M_t^g \quad \text{and} \quad B_t = B_t^g. \end{aligned} \tag{12}$$

3. RAMSEY PROBLEM

The Ramsey problem characterizes the optimal tax structure consistent with a competitive equilibrium for a given level of government spending. Assume that the government can finance its expenditure only by levying a distortionary tax on labor income, by issuing bonds or by printing money.[12] As a consequence, the second-best tax policy is found by choosing the allocation that maximizes the representative agent's welfare subject to the resource and implementability constraints. We will show that the appropriate Ramsey problem depends on whether the government faces restrictions on its set of policy instruments.

3.1. Ramsey Problem 1

Choose $\{c_t, h_t, m_t, n_t^F\}_{t=0}^{\infty}$ to maximize

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t, h_t)$$

subject to

$$c_t + g_t \leq F\left(n_t^F, n_t^I\right), \tag{13}$$

$$\sum_{t=0}^{\infty} \beta^t [u_c(t)c_t - u_h(t)(1 - h_t) + u_h(t)l(t)(1 - k)] = 0. \tag{14}$$

The first constraint is the standard resource constraint. The second constraint is the implementability constraint, which is a restriction that must be satisfied in order for an allocation from the centralized problem to be implemented as a competitive equilibrium. Implementability constraint (14) eliminates all prices and taxes in the representative agent's present-value budget constraint (4) to obtain quantities consistent with optimal firm and household behavior. Lucas and Stokey (1983) show that when markets are *complete*, (13) and (14) fully describe the set of competitive allocations that can be attained through feasible government policies. That is, Problem 1 is the appropriate Ramsey problem when the set of tax instruments is complete.

In contrast, when the government cannot monitor transactions in the informal sector, an additional restriction is required to implement market allocations. This requires a restatement of the Ramsey problem, which we provide below. To see this, let ψ be the Lagrange multiplier on implementability constraint (14). Now, maximize Problem 1 with respect to c_t and h_t.[13] To simplify notation, let $z_t = 1 - h_t - l(t)[1-k]$. It follows that

$$u_h(t)(1+\psi) - \psi u_{hh}(t)z_t = \{u_c(t) - u_h(t)l_c(t) + \psi[u_c(t) + u_{cc}(t)c_t - u_h(t)l_c(t)k] + \psi u_{hh}(t)z_t l_c(t)\}F_{n^I}(t). \quad \textbf{(15)}$$

If implementability constraint (14) does not bind ($\psi = 0$), then (15) is

$$\frac{u_h(t)}{u_c(t) - u_h(t)l_c(t)} = F_{n^I}(t). \quad \textbf{(16)}$$

This is the same condition as in the competitive equilibrium. This implies that when the implementability constraint binds ($\psi > 0$), the solution to Problem 1 is not the solution to the competitive equilibrium.

Since the Ramsey exercise characterizes tax patterns consistent with a competitive equilibrium, (16) must be imposed as an additional constraint. Intuitively, this constraint captures the fact that the planner's set of policy instruments is limited. Specifically, the government is restricted to setting the tax on informal labor equal to a fixed number ($\tau_n^I = 0$). In this case the government lacks an instrument to eliminate the wedge between the marginal rate of substitution between consumption and informal labor and the marginal rate of transformation [equation (16)]. See Chari and Kehoe (2000) for an explanation of this additional constraint in the presence of incomplete taxation. See also Correia (1996) and Jones et al. (1997) for examples of incomplete taxation and the optimality of a capital income tax in models with capital.

The main theoretical result follows directly from Problem 2, which is the appropriate Ramsey problem for an economy with an informal sector.

3.2. Ramsey Problem 2

Choose $\{c_t, h_t, m_t, n_t^F\}_{t=0}^{\infty}$ to maximize (1) subject to (13), (14), and (16).

PROPOSITION 1. *Assume that contracts cannot be monitored by the government in the informal sector and Assumptions* 1 *and* 2 *are satisfied. Then, the*

Friedman rule is not the optimal monetary policy when the transaction cost function is homogeneous of degree 1 *or greater.*[14]

Proof. By maximizing the Ramsey problem with respect to m_t, it follows that

$$-l_m(t)[\theta_t F_{n'}(t) - \psi u_h(t)(1-k) - \gamma_t F_{n'n'}(t)] = \frac{-\gamma_t u_h(t)^2 l_{cm}(t)}{[u_c(t) - u_h(t) l_c(t)]^2}, \quad \mathbf{(17)}$$

where θ_t, ψ, and γ_t are the Lagrange multipliers on resource constraint (13), implementability constraint (14), and implementability constraint (16). Given the assumptions on the utility and production functions, it is easy to verify that the term in brackets on the left-hand side is positive for any $k \geq 1$. Since (16) must hold in any competitive equilibrium, γ_t is always strictly positive. In addition, $l_{cm}(t)$ is negative by Assumption 2.[15] Thus, the right-hand side is positive, which implies that $-l_m(t) > 0$. This solution can be decentralized through a positive nominal interest rate [see equation (7)]. As a consequence, the Friedman rule is not optimal for any transaction technology with a finite "satiation level" that is homogeneous of degree 1 or greater.[16] ■

This result does not hold for homogeneous functions with a "satiation level" of real money balances that is not finite. In this case, the bliss point of real money balances is independent of the consumption level. Thus, Assumptions 1 and 2 are essential for proving Proposition 1. Assumption 1 imposes directly that, for each consumption level, there is a finite satiation level of real money balances. This standard "Friedman assumption" implies that after some finite level of money holdings m^*, the return from holding an extra dollar in terms of decreased shopping time is zero. Assumption 2 implies that money demand increases with consumption. It is not possible to ensure that Proposition 1 holds when the transaction cost technology is homogeneous of degree less than 1 because the sign of the term in brackets on the left-hand side of (17) cannot be determined.[17]

Proposition 1 stands in contrast with the finding by Correia and Teles (1996), who show that, in the absence of an informal sector, the Friedman rule is the optimal solution in monetary models with any type of homogeneous transaction cost function.[18] The intuition that motivates their result is that money is a free primary input (i.e., its production cost is negligible) and inflation is a unit tax. At the point of satiation in real balances the marginal benefit of an extra dollar is zero. Moreover, Correia and Teles (1996) show that under certain assumptions the marginal impact of an additional dollar of government revenue is also zero, at the satiation level. Thus, the Friedman rule is optimal in their model.

In our economy the government has two tax instruments: direct labor taxes, τ_n^F, and inflation. Because the government knows that agents will attempt to evade taxes on the two margins (i.e., by substituting untaxed leisure for labor and by substituting less efficient but untaxed informal labor for formal labor), the government must balance these distortions. In general, it does this by setting both $\tau_n^F > 0$ and a positive nominal interest rate. The positive nominal interest rate, which leads to a positive inflation tax, acts as a consumption tax. The inflation tax affects the traded consumption good in both sectors, regardless of the sector in which it is produced. Thus, the government tries to mitigate the distortion arising from labor tax evasion in the informal sector by imposing an inflation tax. The positive inflation tax expands the tax base, permitting a lower tax on formal labor, reducing both the distortion in the standard labor/leisure choice and the incentive to substitute into the less efficient informal market.

4. QUANTITATIVE EXPERIMENTS

In our economy, inflation serves as a proxy for the inability to tax labor and consumption directly in the informal sector. In an economy without distortions, Lucas (1994) argued that the cost of reducing moderate inflation of 4–5% to the rate prescribed by the Friedman rule could be substantial: 1–3% of GDP in the United States. These results were calculated under the assumption that the lost revenues would be replaced by revenues from a lump-sum tax.[19] Is this policy advice appropriate for economies with labor and goods market distortions of the type we have considered?

Table 1 shows the size of the informal economy relative to GNP for 17 countries. Clearly, the informal sector is significant in many countries. See Schneider and Enste (2000) and Friedman et al. (2000) for additional data, and for an

TABLE 1. Informal economy relative to GNP: Selected countries

Country	Informal economy (%) 1960	1995	Increase in informal economy (%)
Sweden	2	16	14
Denmark	4.5	17.5	13
Germany	2	13.2	11.2
United States	3.5	9	5.5
Switzerland	1	6.7	5.7
Netherlands		13.7	
Spain		22.4	
Italy		26	
Portugal		22.1	
Greece		29	
Japan		10.6	
Canada		14.8	
Brazil[a]		25–35	
Colombia[a]		25–35	
Mexico[a]		40–60	
Peru[a]		40–60	
Nigeria[a]		68–76	

[a] Average over 1990–1993.
Source: Schneider and Enste (2000, Tables 2, 3, and 7).

extensive discussion of the underground economy. We follow Schneider and Enste (2000, p. 79) and define the underground economy as "legal value-added creating activities which are not taxed or registered and where the largest part can be classified as 'black' or clandestine labor." This definition excludes unpaid household production, voluntary nonprofit services, and criminal activities.

Figure 1a shows a positive relationship between the inflation rate and the informal sector for a sample of 69 countries.[20] Figure 1b shows that the informal sector is negatively related to output per worker and Figure 1d shows a negative relationship between the informal sector and the efficiency of the judiciary.[21] These data are consistent with empirical observations documented by Campillo and Miron (1997), which show that institutional arrangements such as central bank independence or the exchange-rate mechanism are relatively unimportant determinants of why inflation differs across countries. They find that optimal tax considerations—greater expenditure needs and difficulty in collecting non-inflation taxes—are important determinants. Figures 1b and 1d also motivate our choice of production function (9), where $\lambda > 1/2$ corresponds to differences in

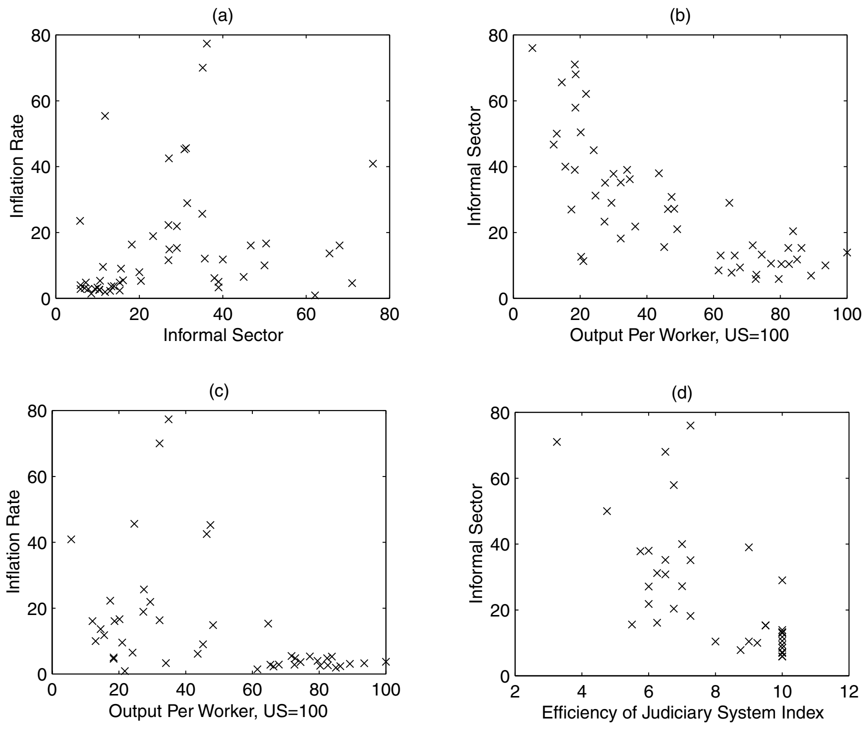

Figure 1. Inflation rate, informal sector, output per worker, and enforcement: Selected countries.

productivity between the two sectors and ρ reflects the strength of institutions such as the judiciary.

4.1. Commodity- and Labor-Market Distortions

The purpose of our quantitative analysis is to provide a numerical assessment of the welfare costs associated with informal labor- and commodity-market distortions. We calculate the welfare effects of alternative levels of inflation in the "distorted" and the "undistorted" economies. That is, we obtain a benchmark number for the welfare gains from reducing inflation in each economy. Implicitly, this provides a rough measure of the increase in aggregate welfare that would accrue from structural reform. The quantitative experiments require us to calibrate the theoretical model. We must determine functional forms for the (i) transaction technology, (ii) preferences, (iii) government policy, and (iv) production technology. We choose standard calibrations for (i), (ii), and (iii) using the U.S. economy as a baseline. The crucial difference in our paper is the specification and calibration of (iv).

Shopping-time transaction technology. We follow Mulligan and Sala-i-Martin (1997) and assume the parametric form

$$l(c, m) = cL(c/m) + \gamma c, \qquad \textbf{(18)}$$

where

$$L(z) = A\frac{(z - \bar{z})^2}{z}$$

is defined over $z \geq \bar{z}$. This functional form satisfies Assumptions 1 and 2. Mulligan and Sala-i-Martin (1997) note that it implies that the interest elasticity of money demand approaches zero as the nominal interest rate approaches zero in a way that conforms closely to empirical evidence. This transaction cost technology was calibrated such that, at a 4% inflation rate, shopping time as a fraction of GNP was 0.02 and the interest elasticity was 0.45.

Preferences. We use the standard CES utility function

$$u(c, h) = \frac{\sigma}{\sigma - 1}\left[c^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}}\right]. \qquad \textbf{(19)}$$

The parameters σ and α are chosen so that, one-third of the time endowment is spent in market activities. We also assume that households discount future utility at the rate of 2% per year, which implies that $\beta = 0.98$.

Government. Using the U.S. economy as a benchmark, we choose parameters so that the baseline inflation rate is $\pi = 4\%$, the tax rate on formal labor income is $\tau_n^F = 0.26$, and government purchases are about 20% of GNP.

Production technology. From (9) in Section 2,

$$F(N^F, N^I) = [\lambda(N^F)^\rho + (1-\lambda)(N^I)^\rho]^{\frac{1}{\rho}}.$$

Further, in Section 2 we derived (10),

$$\frac{N^I}{N^F} = \left(\frac{1-\lambda}{\lambda}\frac{1}{1-\tau_n^F}\right)^{\frac{1}{1-\rho}},$$

which shows that, in equilibrium, the size of the informal sector depends on three key parameters. The tax rate on formal labor τ_n^F is set to match U.S. data. The key production parameters that must be calibrated are ρ and λ. Lemieux et al. (1994) estimate $\rho = 0.71$ for Canada. Assuming that the United States and Canada have the same production technology, we let $\rho = 0.71$. Schneider and Enste (2000, Table 9) indicate that the size of the informal labor force as a percentage of the formal labor force for comparable OECD countries ranges from 6% to 12%. Therefore, we assign $N_I/N_F = 0.09$ for the United States. Then, (10) implies that $\lambda = 0.70$. When ρ is the same across countries, (10) indicates that λ is uniquely determined by the size of the informal sector (given τ_n^F).

To check robustness, we conduct experiments where ρ varies. The results are not sensitive to variations in the preference parameters. Finally, we do not examine extremely high inflation rates because any money demand relation is unlikely to be stable in this case.

Results: The welfare cost function. We summarize the baseline economies in Table 2. Note that α and σ are standard calibrations of preference parameters in (19); γ, A, and $\bar{z}$ are standard calibrations of money demand parameters in (18) [cf. Mulligan and Sala-i-Martin (1997)]; π is a baseline inflation rate of 4% standard in the literature; τ_n^F is the labor income tax in the formal sector; and the "informality parameters" λ and ρ in (9) are tied down by the labor market data ($\rho = 0.71$ and N_I/N_F), discussed earlier.

Table 3 summarizes selected statistics for these three benchmark economies. Column 2 in Table 3 varies the left-hand side of (10). Columns 3, 4, and 5 correspond to the parameters in Mulligan and Sala-i-Martin (1997). Finally, columns 6 and 7 show that the optimal inflation and nominal interest rates are quite different from the Friedman policy recommendation. In an economy with a "small" informal sector, the optimal monetary policy, $i = 1.02\%$, is close to the Friedman rule, $i = 0\%$.

TABLE 2. Baseline economies

Economy	α	σ	γ	A	$\bar{z}$	λ	ρ	π (%)	τ_n^F
1	1	1.7	0.01	0.009	1	0.7	0.71	4	0.26
2	1	1.7	0.01	0.013	1	0.63	0.71	4	0.26
3	1	1.7	0.01	0.019	1	0.56	0.71	4	0.26

TABLE 3. Quantitative results

Economies	Informal sector (%)	Shopping time/GNP	Interest elasticity	g/GNP	Optimal Inflation rate (%)	Optimal Interest rate (%)
1	9.23	0.021	0.46	0.23	−0.01	1.02
2	30	0.023	0.46	0.2	2	4
3	55	0.025	0.46	0.16	16	18.37

However, in an economy with a large informal sector, the optimal inflation and interest rate are significantly higher, $\pi = 16\%$ and $i = 18.37\%$, respectively.

In this shopping-time model, transaction costs increase with inflation, which implies that inflation causes agents to devote productive time to activities that enable them to economize on cash balances.[22] However, in an economy with an informal sector, a lower inflation rate means a higher tax rate on formal labor income, which leads to a larger informal sector, as we discussed previously. The optimal inflation rate balances these two distortions and depends crucially on the structural parameters, ρ and λ.

As in Lucas (1994, 2000), welfare is computed as a percentage increase or decrease relative to the baseline level of consumption, under the parameters in Table 2. Given a baseline economy with inflation rate π_b, we find the value of consumption increment $w(\pi_j)$ that satisfies

$$u\{c(\pi_j)[1 + w(\pi_j)], h(\pi_j)\} - u\{c(\pi_b), h(\pi_b)\} = 0.$$

Then, $w(\pi_j)$ is the increment to baseline consumption that makes the agent indifferent between steady states with inflation rates π_j and $\pi_b = 4\%$. In each economy, we calculate the steady state for different values of the inflation rate π_j, holding government purchases fixed. The government's budget constraint is balanced by adjusting the labor tax rate, as in Braun (1994), Mulligan and Sala-i-Martin (1997), and Lucas (2000).

Figure 2 shows how the welfare cost of inflation changes with the inflation rate for these three baseline economies. By construction, at the baseline inflation rate of 4%, welfare is zero in all three economies. In economy 1, a representative agent would be willing to give up 0.005 units of baseline consumption to move from the baseline inflation rate of 4% to the optimal inflation rate of essentially 0. The welfare gain is then 0.5% of baseline consumption. Figure 2 has several notable features. In economy 1, welfare has the same shape as in Braun (1994) and Lucas (2000). As inflation falls from its optimal level, the welfare cost of inflation increases sharply. However, when inflation is above its optimal rate, the welfare cost increases only gradually.

The shape of the welfare function is very important for policy purposes, particularly for a country considering a low inflation target. Figure 2 shows that the welfare cost of implementing the Friedman rule (loosely a zero inflation rate) would be substantial for economies with serious structural problems. The optimal inflation

TABLE 4. Welfare gains

Economy	(ρ, λ)	Optimal inflation, π^* (%)	Welfare gain (%): Inflation change, 4% to π^*%	Welfare gain (%): Inflation change, 25% to 4%
1 (low)	0.71, 0.7	−0.5	0.5	1.7
1_ρ (low)	0.80, 0.68	−0.5	0.5	1.7
2 (mod.)	0.71, 0.63	2	0.1	1
2_ρ (mod.)	0.80, 0.61	6	0.05	0.7
3 (high)	0.71, 0.56	16	0.5	−0.5
3_ρ (high)	0.80, 0.546	22	1.2	−1

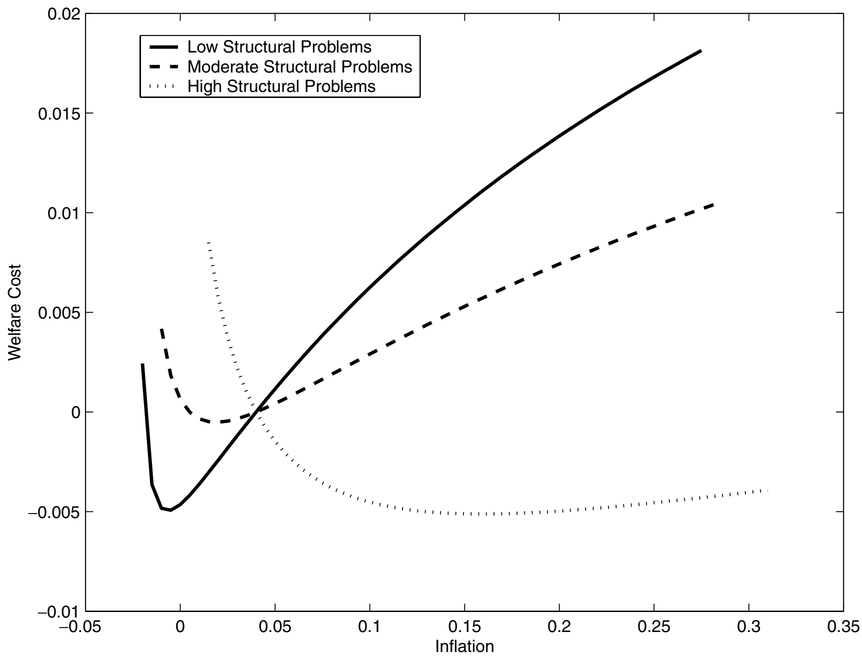

FIGURE 2. Welfare cost of inflation and structural problems.

rate is positive when structural problems are severe, and it is costly to drive inflation below its optimal level. Further, Figure 2 shows that severe structural distortions "flatten out" the welfare cost function, which implies that the cost of having inflation above the optimal level is small for economies with big structural problems.

Table 4 shows that the welfare gain of changing inflation from its baseline value of 4% to the optimal rate varies with the level of structural problems. For economies 1, 2, and 3 with the baseline parameters in Table 2, the welfare gain from changing inflation from the baseline of 4% to the optimal inflation rate is 0.5%, 0.1%,

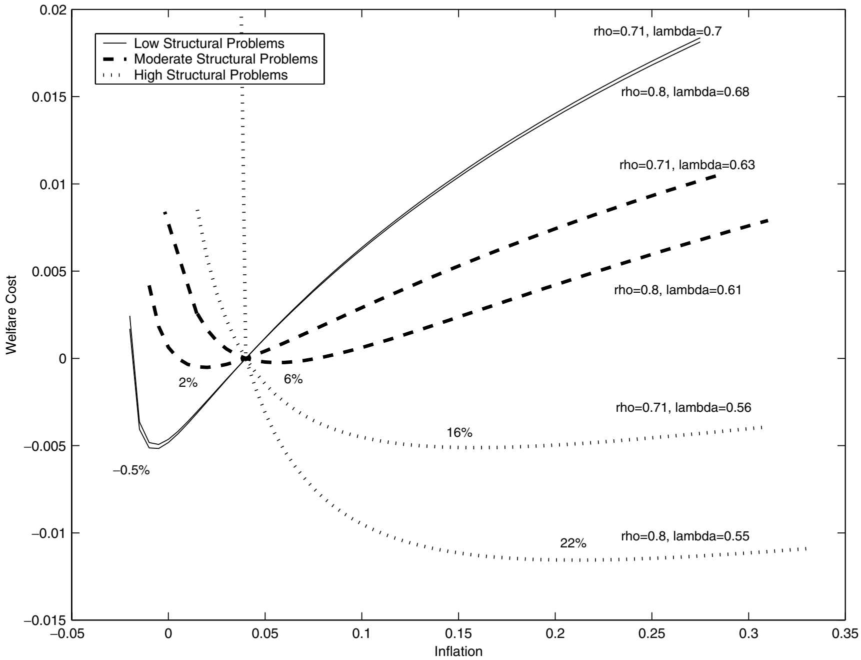

FIGURE 3. Informality parameter sensitivity.

and 0.5% of baseline consumption for economies with "low," "moderate," and "high" structural problems, respectively.[23] These welfare gains are not substantial. However, the last column of Table 4 shows that the welfare cost of adopting high-inflation policies depends on the level of structural problems. For instance, for economy 3 with a big informal sector, there is a welfare loss of −0.5% of baseline consumption associated with reducing inflation from 25% to a lower baseline level of 4%. In contrast, for economy 1 with low structural problems, the welfare gain of changing inflation from 25% to 4% is 1.7% of baseline consumption. This highlights the importance of structural reform.

Parameter sensitivity. The parameters used to calibrate preferences, government policy, and money demand in the baseline economy are standard. The new aspect of the analysis is production function (9), with key parameters ρ and λ. Rows (1_ρ), (2_ρ), and (3_ρ) of Table 4 report the results of experiments in which all other parameters remain the same but ρ is varied from 0.71 to 0.80. The theory in Section 2 established that ρ, τ_n^F, the size of the informal sector, and λ are related by (10). The model and the initial experiment assumed that ρ was the same across countries, and all parameters in the baseline economy were chosen to match the United States.[24] Because ρ measures the elasticity of substitution between formal and informal sectors, it is a proxy for the quality of institutions across countries.

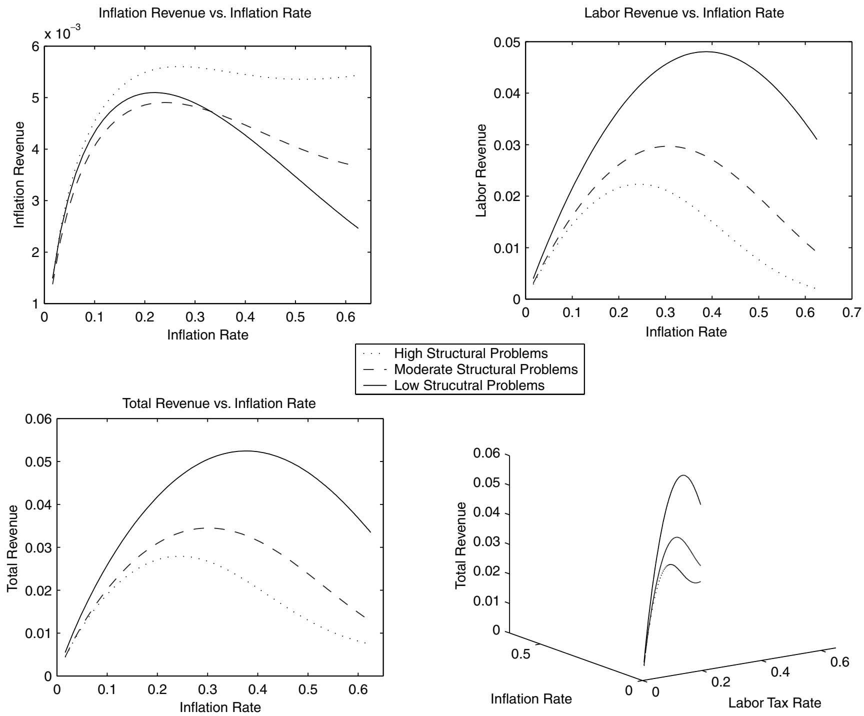

FIGURE 4. Government revenue and structural problems.

A higher ρ corresponds to weaker institutions. Through equation (10), when ρ changes, this implies the change in λ noted in column 3. The question we ask now is—what is the effect on the welfare cost function if ρ is higher? The data in Figure 1 suggest that countries with large informal sectors also have weaker institutions.

Figure 3 and Table 4 show three main effects when ρ increases. First, when structural problems are moderate, the optimal inflation rate increases from 2% in the baseline case when $\rho = 0.71$ to 6% when $\rho = 0.80$. When structural problems are high, the optimal inflation rate increases from 16% when $\rho = 0.71$ to 22% when $\rho = 0.8$. Second, Figures 2 and 3 clearly show that the welfare cost functions "flatten out" as structural problems increase. Third, the welfare gain from reducing inflation decreases as ρ increases.

Government revenue. Finally, Figure 4 indicates how structural problems affect government revenue. The first figure indicates that high structural problems cause the inflation revenue function to shift up somewhat and become flatter. However, the labor revenue function shifts down significantly. As a consequence, the total revenue function shifts down as structural problems increase. Figure 4 implies that seignorage is an important source of government revenue in economies with a large informal sector. This is consistent with Click (1998) who found that the share of seignorage over total government revenue has a negative relation to per capita GDP. When faced with an inefficient bureaucracy, high levels of corruption, and a weak judiciary system, firms hide their activities and, consequently, tax revenues fall relative to seignorage.

We summarize our findings for economies with limitations on the government's ability to tax labor and commodity markets: (i) The welfare cost of inflation increases sharply when inflation falls from its optimal level; (ii) the welfare cost associated with high inflation is small for economies with structural problems that impede uniform taxation, but substantial for economies without such structural problems; and (iii) structural distortions provide an additional margin on which agents can substitute in order to evade taxes. These quantitative exercises are, therefore, an alternative explanation for the persistence of high inflation in economies with structural problems of the type we describe. When an economy has a high level of structural problems, a quantitatively significant inflation policy can be constrained optimal.

4.2. Currency Market Distortions

From the preceding analysis, it is clear that, depending on the size of the informal sector, the optimal inflation policy requires not only analytical qualifications, but also raises important quantitative considerations. We now consider currency substitution (CS), a market distortion that is present in many developing countries. CS describes the replacement of domestic currency that loses value in the presence of inflation by a stronger and more reliable foreign money (frequently the U.S. dollar).[25] According to Giovannini and Turtelboom (1994), this phenomenon can be seen as the Gresham's law in reverse, since the "good" currency drives out the "bad."

In many developing countries, people hold U.S. dollars or another foreign money even when foreign currency is not legally acceptable.[26] In doing so, consumers insure their wealth against the instability of the domestic currency and they evade the inflation tax imposed by the home government.[27] CS is an informal arrangement in transaction activities that has the same "tax evasion" effect as informal labor contracts. Informal contracts decrease the tax base from which labor income revenue can be raised, and the presence of foreign currency decreases the monetary base from which domestic seigniorage can be raised by printing money. The higher the elasticity of substitution between domestic and foreign money, the more difficult it is for the government to finance deficits by an inflation tax.

In this section, we evaluate the effects of CS on our economic model. Assume now that agents can transact with both domestic currency, M, and foreign money, F. Modify the transaction technology as follows. Let

$$s_t \geq l\left(c_t, \frac{M_t}{P_t}, \frac{\xi_t F_t}{P_t}\right), \tag{20}$$

where ξ_t represents the exchange rate. Since prices in the model are flexible, purchasing power parity (PPP) holds; that is, $P_t = \xi_t P_t^*$, where P_t^* denotes the foreign currency price of the consumption good. Using the PPP equation in the transaction technology, it follows that $s_t \geq l(c_t, m_t, f_t)$, with $m_t = M_t/P_t$ and $f_t = F_t/P_t^*$.

Appendix A.3 shows that CS does not change Proposition 1. As a consequence, the Friedman rule is not the optimal monetary policy when labor markets are incomplete and both domestic and foreign money circulate as media of exchange.[28] Although, qualitatively, CS does not change the main results, it is important to verify how it affects the quantitative experiments.

We retain the previous specification of preference, production, and government policy. Let the shopping-time technology take the form

$$l(c, m, f) = cL\left(\frac{c}{m}, \frac{c}{f}\right) + \gamma c, \tag{21}$$

where $L(z_1, z_2) = A[(z_1 + \phi z_2) - \bar{z}]^2/(z_1 + \phi z_2)$. Under this specification, the demand for foreign money relative to domestic currency is given by

$$\frac{f_t}{m_t} = \left(\phi \frac{I_t}{I_t^*}\right)^{1/2}$$

with

$$I_t = \frac{i_t}{1 + i_t} \quad \text{and} \quad I_t^* = 1 - \frac{1}{1 + i_t} \frac{P_{t+1}}{P_t} \frac{P_t^*}{P_{t+1}^*} \tag{22}$$

If inflation in domestic currency increases, I_t^* decreases and agents replace domestic currency with foreign money. This is consistent with CS.

The parameter ϕ is the key determinant of the elasticity of substitution between domestic and foreign money.[29] We calibrate ϕ so that the demand for foreign currency relative to domestic is consistent with empirical evidence. According to the Joint Economic Committee (2000) report, countries are classified as "highly dollarized" if foreign currency deposits exceed 30% of a broad measure of the money supply. Let ϕ take the following values: 0, 0.01, 0.1, and 0.3. When $\phi = 0.3$ and home and foreign inflation rates are the same, 35.4% of total money holdings are in foreign currency. This is consistent with the Joint Economic Committee (2000) report.

Figures 5 and 6 show the effects of CS on the welfare cost of inflation for economies with low and high structural problems, respectively. In both economies there is a trade-off between the distortion introduced by informal labor contracts and by CS. As before, the welfare cost increases sharply when inflation falls from its optimal rate and it increases only gradually when inflation is above its optimal level. Figure 5 shows that, for an economy with low structural distortion, (i) the

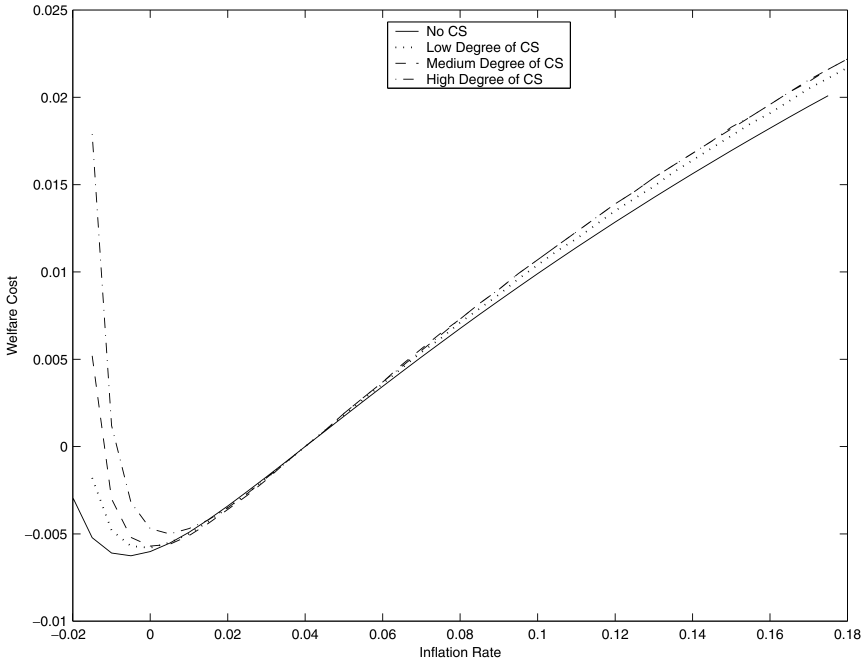

FIGURE 5. Welfare cost of inflation for different levels of CS for economies with low structural problems.

optimal inflation rate increases as the degree of CS increases, and (ii) the welfare gains of adopting the optimal policy decrease with CS. On the other hand, Figure 6 shows that, for an economy with high structural problems, (i) the optimal inflation rate decreases with CS, and (ii) the welfare gains of adopting the optimal inflation policy also decrease with CS.

CS leads the optimal inflation policy to be closer to the foreign inflation rate and it decreases the welfare gains from adopting the optimal inflation policy. The intuition is that agents will hold domestic currency if inflation in domestic currency is not sufficiently above the level in foreign currency. In this case the optimal policy is to set the interest rate closer to the foreign one. The welfare gain from reducing inflation decreases because agents have a new margin on which to substitute—they evade the domestic inflation tax by using foreign currency. This results in an upward shift in the welfare-cost-of-inflation curve. Although CS decreases the optimal inflation rate in economies with structural problems, these quantitative exercises reinforce the idea that the welfare benefits from reducing inflation from high levels, such as 25%, to the optimal level is very small. In addition, reducing inflation below the optimal level leads to significant welfare losses, as Figures 5 and 6 show.

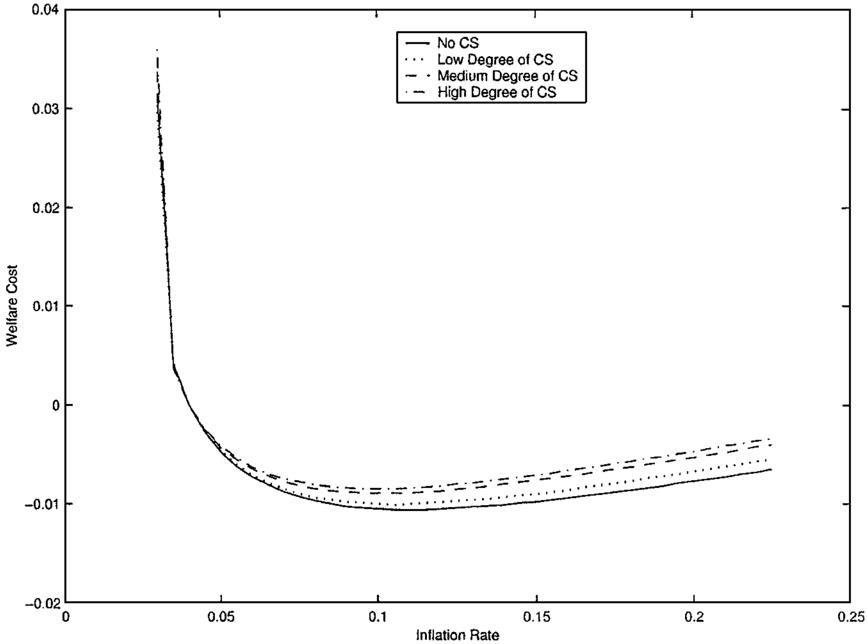

FIGURE 6. Welfare cost of inflation for different levels of CS for economies with high structural problems.

5. CONCLUDING REMARKS

Proposition 1 shows that the Friedman rule may not be optimal in an economy with structural problems in labor, goods, and financial markets. Since the inflation tax acts as a uniform consumption tax, it enables the government to reduce the distortion introduced in the economy by these imperfections. We also investigated whether the optimal inflation rate is quantitatively different from the Friedman policy. We found that it is, and calculated the welfare gains from adopting the optimal policy. In addition, we found that the welfare effect is highly asymmetric when structural problems are severe. The informal sector thus provides a public-finance-based explanation for positive inflation rates, especially in developing economies.

Our results indicate that if developing economies wish to adopt a low inflation policy they must also improve their institutional framework for enforcing formal contracts and collecting tax revenue. In this case the tax base and welfare will increase, and taxation via inflation will be unnecessary. Simply put, structural distortions introduce additional margins on which agents can substitute in order to evade taxes. The quantitative results indicate that these standard microeconomic tax evasion activities can have important implications for optimal monetary policy.

Finally, in many countries, inflation is both high and variable. In our model, government purchases are constant (about 20% of GDP). However, if government expenditure were stochastic, it would be interesting to determine conditions under which the optimal inflation rate varies. Lucas and Stokey (1983) consider the issue of stochastic government expenditures in the primal version of a Ramsey model with no physical capital but without an informal sector (i.e., complete markets). They derive a stochastic optimal government debt policy that responds to expenditure shocks and smooths tax distortions. Aiyagari et al. (2002) study the Ramsey allocation with incomplete markets (i.e., when only a risk-free asset exists). Even with this friction, taxes remain smooth.[30] We conjecture that when government expenditure is stochastic, the ability to insure is limited, and there is an informal sector, as is the case in many developing countries, the optimal inflation rate may vary. The precise interaction among incompleteness in the set of tax instruments (due to informal markets), limits on insurance, and their effect on inflation variability is a topic for future research.

NOTES

1. The "Ramsey problem" is to choose the optimal tax structure consistent with a competitive equilibrium when only distortionary taxes are available. Agents respond to tax distortions, and the government takes agents' responses into account when choosing its policy. When direct taxation is not possible, inflation can serve as an indirect tax on consumption [cf. Bryant and Wallace (1984), Villamil (1988), and Smith and Villamil (1998)].

2. Recent computational work in Bewley models has focused on the effect of incomplete loan markets (i.e., borrowing constraints) on agents' ability to self-insure when there are no external insurance opportunities [cf. Huggett (1993) and Akyol (2000)]. In contrast, our focus is on a classic public finance problem—incompleteness in the government's ability to tax agents.

3. Guidotti and Végh (1993) and Correia and Teles (1996) also use this specification.

4. This assumption ensures the existence of a unique interior optimum solution.

5. A finite "satiation level" of real balances is a standard assumption [cf. Friedman (1969), Phelps (1973), and Mulligan and Sala-i-Martin (1997)].

6. See Koresheva (2001) for an analysis of inflation with Schreft's (1992) cash-and-credit money demand specification.

7. A negative cross derivative of the transaction function means that the marginal cost of transaction due to an additional unit of consumption decreases with real money balances.

8. Using the implicit function theorem we can show that the scale elasticity and the interest rate elasticity are defined by $\epsilon_c = -(c/m)(l_{cm}/l_{mm})$ and $\epsilon_I = (I/m)\{1/[l_{mm}(1-\tau_n^F)w^F]\}$, respectively.

9. There are two important frictions in our model: tax evasion due to $\tau_n^F > 0$ and intrinsic differences in productivity in the two sectors, $\lambda > 1/2$. Equation (10) shows that these frictions disappear as $\tau_n^F \to 0$ and $\lambda \to 1/2$.

10. Where π_t is the rate of money creation.

11. Walras's law ensures that the government's budget constraint is satisfied.

12. Appendix A.2 shows that our results are robust to the introduction of a consumption tax as long as this tax is incomplete (i.e., the consumption tax can be levied on the formal sector but not on the informal sector).

13. For simplicity, assume that the utility function is strongly separable between c and h. This assumption is not essential, but it simplifies the analysis.

14. From this primal approach, it is not trivial to see that when $g_t = B_t^g = 0$, the Ramsey policy is to set $i_t = \tau_{nt}^F = 0$. See Lucas and Stokey (1983, examples 1 and 2). However, this can be easily seen from the dual problem where the planner chooses policies instead of allocations.

In this case, the present-value government budget constraint must be satisfied: $\sum_{t=0}^{\infty} q_t(\chi)g_t = \sum_{t=0}^{\infty} q_t(\chi)[\tau_{nt}^F F_{N^F}(t)N_t^F(\chi) + I_t m_t^g(\chi)]$, where χ is a vector of government policies and q_t is an Arrow–Debreu price. When $g_t = 0$, the Ramsey solution sets $i_t = \tau_{nt}^F = 0$.

15. If the transaction function is homogeneous of degree 1, $l_{cm} < 0$ follows from $l_{mm} > 0$.

16. When $l(\cdot)$ is a homogeneous function and $l_m(c,m) = 0$, this defines $m^* = \infty$. Then, $l_{cm}(c, m^*) = 0$ and the Friedman rule imply that the value of real balances is not finite.

17. By assumption, $l_{cm} < 0$. Thus, the right-hand side of (17) is positive. This implies that $l_m = 0$ cannot satisfy (17). Since a utility-maximizing agent will not hold additional cash beyond the "satiation level" of real balances, $l_m < 0$; thus, $-l_m > 0$. In this case, (17) will hold only if the term in brackets on the left is positive. This solution can be decentralized through a positive nominal interest rate.

18. In the absence of an informal sector, (17) is $-l_m(t)[\theta_t F_n^F(t) - \psi u_h(t)(1-k)] = 0$. This is the same equation found by Correia and Teles (1996). The term in brackets is different from zero, which implies that $-l_m(t) = 0$. This solution can be decentralized through a zero nominal interest rate. Thus the Friedman rule is the optimal monetary policy for any homogeneous transaction function in the absence of an informal sector. Notice that this result does not depend on the term l_{cm}. This is why it was necessary to characterize this term to establish Proposition 1.

19. Lucas (1994) claims that these results are robust to certain labor supply specifications.

20. Data for the size of the informal sector are from Friedman et al. (2000, Table 1). The inflation data are from the International Financial Statistics. Inflation was calculated by averaging the annual inflation rate in each country for the period from 1986 to 1995. We excluded countries with inflation rates above three digits.

21. Data for output per worker are from the Penn World Tables, and data for the efficiency of the judiciary system are from La Porta et al. (1998).

22. This result was also found by Cooley and Hansen (1989, 1991), Dotsey and Ireland (1996), and Lucas (2000). They show that inflation and employment in banking have been positively correlated over time in the United States and other countries.

23. In the last case, the optimal policy is inflation higher than the baseline value.

24. Lemieux et al. (1994) estimate $\rho = 0.71$ using Canadian labor market data. Canada, like the United States, has low structural distortion.

25. Porter and Judson (1996) estimate that 55–70% of U.S. dollars are held abroad, mainly in $100 bills.

26. According to the Joint Economic Committee (2000) staff report, in developing economies where foreign money is present, wages, taxes, and everyday expenses such as groceries and electric bills are paid for in domestic currency, but expensive items such as automobiles and houses are often paid for in foreign currency.

27. Agents pay a seigniorage tax on foreign currency of 4% in our experiments.

28. Végh (1989) shows in a similar model, but abstracting from the informal sector, that the Friedman rule is not the optimal monetary policy in the presence of CS.

29. Notice that $\phi = 0$ yields an economy without the CS phenomenon. Countries with $\phi = 0.01$ correspond to low CS, and those with $\phi = 0.3$ correspond to high CS.

30. Schmitt-Grohe and Uribe (2001) and Siu (2001) study optimal monetary policy with stochastic government expenditure and sticky prices. They find variability in taxes but very low variability in inflation.

31. Without an informal sector, the RHS is zero and thus $u_{c1}(t) = u_{c2}(t)$. This solution can be decentralized through a zero nominal interest rate [compare with equation (A.5)].

32. To simplify the algebra, let $u(c,m,h) = V[w(c,m)] + u(h)$. This will not drive the result. In fact, one can show under this assumption that the Friedman rule is optimal in the absence of an informal sector.

33. In Chari et al. (1996), $[u_m(t)]/[u_c(t)] = 0$.

34. Notice that in this case we do not need equation (18) to implement Ramsey allocations.

35. By maximizing this Ramsey problem with respect to h_t, we can show that

$$\theta_t F_n^I(t) = u_h(t) + \psi\{u_h(t) - u_{hh}(t)[1 - h_t - (1-k)l(t)]\}.$$

Substituting this result into (A.12) yields

$$-l_m(t)(u_h(t) + \psi\{ku_h(t) - u_{hh}(t)[1 - h_t - (1-k)l(t)]\}) = 0.$$

It is easy to verify that the term in braces is positive for any homogeneous transaction function of degree k. See Correia and Teles (1996).

36. The derivation uses the assumption that the consumption tax does not affect the transaction cost function. In practice, the amount of time spent shopping depends on the level of consumption expenditure (inclusive of consumption taxes), i.e., $l[(1+\tau_c)c_t, m]$ instead of $l(c, m)$. In this case, Guidotti and Végh (1993) show that even without an informal sector, the government should resort to inflationary finance.

37. The transaction technology satisfies the assumptions in Section 2, $l_{ff} \geq 0$ and $l_{mf} > 0$. In addition, $l_m = 0$ and $l_f = 0$ define $m^*(c, f)$ and $f^*(c, m)$ such that $l_m < 0$ and $l_f < 0$ for any $m < m^*$ and $f < f^*$, and m^* and f^* are finite numbers.

REFERENCES

Aiyagari, R., A. Marcet, J. Seppälä, & T. Sargent (2002) Optimal taxation without state contingent debt. *Journal of Political Economy*. Forthcoming.

Akyol, A. (2000) Optimal Monetary Policy in an Economy with Incomplete Markets and Idiosyncratic Risk. Manuscript, University of Iowa.

Braun, R.A. (1994) Another attempt to quantify the benefits of reducing inflation. *Federal Reserve Bank of Minneapolis, Quarterly Review* 18 (Fall), 17–25.

Bryant, J. & N. Wallace (1984) A price discrimination analysis of monetary policy. *Review of Economic Studies* 51, 279–288.

Campillo, M. & J. Miron (1997) Why does inflation differ across countries? In C. Romer & D. Romer (eds.), *Reducing Inflation: Motivation and Strategy*, chapter 9. Chicago: University of Chicago Press.

Chari, V., L.J. Christiano, & P. Kehoe (1996) Optimality of the Friedman rule in economies with distorting taxes. *Journal of Monetary Economics* 37, 203–223.

Chari, V. & P. Kehoe (2002) Optimal fiscal and monetary policy. In J. Taylor & M. Woodford (eds.), *Handbook of Macroeconomics*, Vol. 1c, Chapter 26. Amsterdam: Elsevier.

Click, R.W. (1998) Seigniorage in a cross-section of countries. *Journal of Money, Credit and Banking* 30, 154–171.

Cooley, T. & G. Hansen (1989) The inflation tax in a real business cycle model. *American Economic Review* 79, 733–748.

Cooley, T. & G. Hansen (1991) The welfare cost of moderate inflation. *Journal of Money, Credit and Banking* 23, 483–503.

Correia, I. (1996) Should capital income be taxed in the steady-state? *Journal of Public Economics* 60, 147–151.

Correia, I. & P. Teles (1996) Is the Friedman rule optimal when money is an intermediate good? *Journal of Monetary Economics* 38, 223–244.

Diamond, P.A. & J.A. Mirrlees (1971) Optimal taxation and public production. *American Economic Review* 61, 8–27.

Dotsey, M. & Ireland, P. (1996) The welfare cost of inflation in general equilibrium. *Journal of Monetary Economics* 37, 29–47.

Easterly, W. (1993) How much do distortions affect growth? *Journal of Monetary Economics* 32, 187–212.

Faig, M. (1988) Characterization of the optimal tax on money when it functions as a medium of exchange. *Journal of Monetary Economics* 22, 137–148.

Friedman, E., S. Johnson, D. Kaufmann, & P. Zoido-Lobaton (2000) Dodging the grabbing hand: The determinants of unofficial activity in 69 countries. *Journal of Public Economics* 76, 459–493.

Friedman, M. (1969) The optimum quantity of money. In M. Friedman (ed.), *The Quantity of Money and Other Essays*, pp. 1–50. Chicago: Aldine.

Giovannini, A. & B. Turtelboom (1994) Currency substitution. In Frederick van der Ploeg (ed.), *The Handbook of International Macroeconomics*. Oxford, UK: Basil Blackwell.

Guidotti, P. & C. Végh (1993) The optimal inflation tax when money reduces transaction costs. *Journal of Monetary Economics* 31, 189–205.

Huggett, M. (1993) The risk free rate in heterogeneous-agent incomplete insurance economies. *Journal of Economic Dynamics and Control* 17, 953–969.

Joint Economic Committee (2000) Basics of Dollarization. U.S. Congress, JEC, Office of the Chairman, Staff Report, January transcript available on-line at http://www.senate.gov/~jec.

Jones, L.E., R.E. Manuelli, & P.E. Rossi (1997) On the optimal taxation of capital income. *Journal of Economic Theory* 73, 93–117.

Kimbrough, K. (1986) The optimum quantity of money in the theory of public finance. *Journal of Monetary Economics* 18, 277–284.

Koresheva, T. (2001) Accounting for Inflation Rates in Developing Countries. Manuscript, University of Western Ontario.

La Porta, R., F. Lopes-de-Silanes, A. Shleifer, & R.W. Vishny (1998) Law and finance. *Journal of Political Economy* 106, 1113–1155.

Lemieux, T., B. Fortin, & P. Frechette (1994) The effect of taxes on labor supply in the underground economy. *American Economic Review* 84, 231–254.

Ljungqvist, L. & T.S. Sargent (2000) *Recursive Macroeconomic Theory*. Cambridege, MA: MIT Press.

Lucas, R.E., Jr. (1990) Supply-side economics: An analytical review. *Oxford Economic Papers* 42, 293–316.

Lucas, R.E., Jr. (1994) On the Welfare Cost of Inflation. CEPR Publication 394. Center for Economic Policy Research, Stanford University.

Lucas, R.E., Jr. (2000) Inflation and welfare. *Econometrica* 68, 247–274.

Lucas, R.E., Jr. & N. Stokey (1983) Optimal fiscal and monetary theory in world without capital. *Journal of Monetary Economics* 12, 55–93.

Mulligan, C.B. & X. Sala-i-Martin (1997) The optimum quantity of money: Theory and evidence. *Journal of Money, Credit and Banking* 29, 687–724.

Phelps, E. (1973) Inflation in the theory of public finance. *Swedish Journal of Economics* 75, 37–54.

Porter, R. & R. Judson (1996) The location of U.S. currency: how much is abroad? Federal Reserve Bulletin, v. 82, no. 10, October, pp. 883–903.

Sargent, T. & N. Wallace (1981) Some unpleasant monetarist arithmetic. *Federal Reserve Bank of Minneapolis Quarterly Review* 5(3), 1–17.

Schmitt-Grohe, S. & M. Uribe (2001) Optimal Fiscal and Monetary Policy Under Sticky Prices. Manuscript, Rutgers University.

Schneider, F. & D. Enste (2000) Shadow economies: Size, cause and consequences. *Journal of Economic Literature* 38, 77–114.

Schreft, S. (1992) Transaction costs and the use of cash and credit. *Economic Theory* 2: 283–296.

Siu, H. (2001) Optimal Fiscal and Monetary Policy with Sticky Prices. Manuscript, Northwestern University.

Smith, B. & A. Villamil (1998) Government borrowing using bonds with randomly determined returns: Welfare improving randomization in the context of deficit finance. *Journal of Monetary Economics* 41, 351–370.

Stone, A., B. Levy, & R. Paredes (1996) Public institutions and private transaction: A comparative analysis of the legal and regulatory environment for business transactions in Brazil and Chile. In L. Alston, T. Eggertsson, & D. North (eds.), *Empirical Studies in Institutional Change*. Cambridge, UK: Cambridge University Press.

Végh, C. (1989) The optimal inflation tax in the presence of currency substitution. *Journal of Monetary Economics* 24, 139–146.

Villamil, A. (1988) Price discriminating monetary policy: A non-uniform pricing approach. *Journal of Public Economics* 51, 378–386.

APPENDIX

Section A.1 considers two alternative models of money to show that our results are robust to the specification of money demand. Section A.2 shows that the results are robust to incomplete commodity taxation. Finally, Section A.3 shows that Proposition 1 is robust to currency substitution.

A.1. ALTERNATIVE SPECIFICATIONS OF MONEY DEMAND

A.1.1. Cash-in-Advance Constraint

Consider a production economy similar to the model in Section 2. In each period, there are two consumption goods: a cash good and a production good. The representative household is endowed with one unit of time that can be used as leisure, h_t, or allocated to production in the formal sector, n^F, or the informal sector, n^I. Preferences are

$$U = \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, h_t),$$

where c_{1t} and c_{2t} are the cash and the credit good, respectively. The one-period budget constraint is given by

$$c_{1t} + c_{2t} + \frac{M_t}{P_t} + \frac{B_t}{P_t} \leq \frac{M_{t-1}}{P_t} + (1 + i_{t-1})\frac{B_{t-1}}{P_t} + \left(1 - \tau_n^F\right) w_t^F n_t^F + w_t^I \left(1 - n_t^F - h_t\right).$$

In present-value form the budget constraint, with no-Ponzi game conditions for bonds and money, and with initial conditions $B_{-1} = M_{-1} = 0$, is

$$\sum_{t=0}^{\infty} d_t c_{1t} + \sum_{t=0}^{\infty} d_t c_{2t} + \sum_{t=0}^{\infty} d_t I_t m_t \leq \sum_{t=0}^{\infty} d_t [(1 - \tau_n^F) w_t^F n_t^F + w_t^I (1 - n_t^F - h_t)].$$

Assume that the purchase of cash goods must satisfy the following cash-in-advance (CIA) constraint:

$$c_{1t} \leq m_t.$$

The production function is as before and output can now be used for private consumption of either the cash good, c_{1t}, the credit good, c_{2t}, or for government consumption, g_t. The resource constraint is given by

$$c_{1t} + c_{2t} + g_t \leq F\left(N_t^F, N_t^I\right).$$

The household's equilibrium conditions are

$$\frac{u_h(t)}{u_{c2}(t)} = w_t^I, \tag{A.1}$$

$$w_t^I = \left(1 - \tau_n^F\right) w_t^F, \tag{A.2}$$

$$\frac{u_h(t)}{u_{c1}(t)} = \frac{\left(1 - \tau_n^F\right) w_t^F}{(1 + I_t)}, \tag{A.3}$$

$$\frac{u_{c2}(t)}{u_{c2}(t+1)} = \beta(1 + r_t), \tag{A.4}$$

$$\frac{u_{c1}(t)}{u_{c2}(t)} = (1 + I_t). \tag{A.5}$$

Notice that (A.5) implies that $u_{c1}(t) \geq u_{c2}(t)$.

Chari et al. (1996) show that, in the absence of an informal sector and under the assumption that the utility function is $u(c_1, c_2, h) = V[w(c_1, c_2), h]$, where w is a homothetic function, the optimal Friedman rule is the Ramsey equilibrium.

Now, consider the Ramsey equilibrium with informal labor contracts. The Ramsey problem is to choose allocations that maximize utility subject to the CIA constraint, the resource constraint, the implementability constraint,

$$\sum_{t=0}^{\infty} \beta^t [u_{c1}(t) c_{1t} + u_{c2}(t) c_{2t} - u_h(t)(1 - h_t)] = 0,$$

and the following market equilibrium conditions:

$$G(t) = \frac{u_h(t)}{u_{c2}(t)} = F_{n^I}(t), \tag{A.6}$$

$$u_{c1}(t) \geq u_{c2}(t). \tag{A.7}$$

PROPOSITION A.1. *Assume that contracts cannot be monitored by the government in the informal sector and the utility function is the same as in Chari et al.* (1996). *Then, the Friedman rule is not necessarily the optimal monetary policy.*

Proof. As in Chari et al. (1996), the utility function satisfies the property

$$\sum_{j=1}^{2} u_{cj,c1}(t) cj / u_{c1}(t) = \sum_{j=1}^{2} u_{cj,c2}(t) cj / u_{c2}(t). \tag{A.8}$$

Maximize the Ramsey problem with respect to c_1 and c_2 and use (A.8) to show that

$$\frac{\theta_t}{u_{c1}(t)} - \frac{\theta_t}{u_{c2}(t)} = \gamma \frac{G_{c2}}{u_{c2}} - \gamma \frac{G_{c1}}{u_{c1}},$$

where θ_t and γ_t are the Lagrange multipliers on the resource constraint and market equilibrium condition (A.6), respectively.[31] After rearranging this equation, it follows that

$$\frac{u_{c1}}{u_{c2}} = 1 - \frac{\gamma}{\theta} G_{c1} + \frac{u_{c1}}{u_{c2}} \frac{\gamma}{\theta} G_{c2}.$$

Now, noting that $u_{hci}/u_{ci} = V_{12}/V_i$ for $i = 1, 2$, we have that

$$\frac{u_{c1}}{u_{c2}} = 1 + \frac{\gamma}{\theta}\frac{u_{c2c1}u_h}{u_{c2}^2} - \frac{\gamma}{\theta}\frac{u_{c2c2}u_h}{u_{c2}^2}\frac{u_{c1}}{u_{c2}},$$

$$\frac{u_{c1}}{u_{c2}} = \frac{1 + \frac{\gamma}{\theta}\frac{u_{c2c1}u_h}{u_{c2}^2}}{1 + \frac{\gamma}{\theta}\frac{u_{c2c2}u_h}{u_{c2}^2}}. \tag{A.9}$$

The right-hand side of (A.9) can be greater than equal to, or less than 1. When it is less than or equal to 1, constraint (A.7) will bind and the Friedman rule is optimal. When it is greater than 1, the Friedman rule is not optimal [see equation (A.5)]. ■

A.1.2. Money in the Utility Function

Consider an economy with preferences

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, m_t, h_t),$$

where $m_t = M_t/P_t$ is real money balances. The resource constraint is given by

$$c_t + g_t \leq F\left(N_t^F, N_t^I\right).$$

In equilibrium,

$$\frac{u_h(t)}{u_c(t)} = F_{N^I}(t), \tag{A.10}$$

$$\frac{u_m(t)}{u_c(t)} = I_t. \tag{A.11}$$

The Ramsey allocations maximize preferences subject to the resource constraint, market equilibrium condition (A.10) and the following implementability constraint:

$$\sum_{t=0}^{\infty} \beta^t u[u_c(t)c_t + u_m(t)m(t) - u_h(t)(1 - h_t)] = 0.$$

As before, assume that $u(c, m, h) = V[w(c, m), h]$, where w is a homothetic function. Chari et al. (1996) show that, under this assumption and in the absence of an informal sector, the Friedman rule is the optimal monetary policy.[32] As in Chari et al. assume that $m \leq \bar{m}$.

PROPOSITION A.2. *Assume that contracts cannot be monitored by the government in the informal sector and that the utility function is the same as in Chari et al.* (1996). *Then, the Friedman rule is not the optimal monetary policy.*

Proof. Maximizing the Ramsey problem with respect to c and m and using the properties of the utility function, one can show that $[u_m(t)]/[u_c(t)] > 0$. This solution can be decentralized by a positive nominal interest rate.[33] See equation (A.11). ■

A.2. ROLE OF CONSUMPTION TAXES

We assumed at the outset that consumption taxes were not available. Assume now, in order to clarify our previous result, that the government can tax consumption at a uniform rate $\tau_c = \tau_c^F = \tau_c^I > 0$. For simplicity, assume that the transaction cost technology is not affected by the consumption tax. As in the previous analysis, the Ramsey problem is to choose $\{c_t, h_t, m_t, n_t^F\}_{t=0}^{\infty}$ to maximize the welfare of the representative agent (1), subject to the resource constraint (13), and the implementability condition (14).[34] When a uniform consumption tax is available, equations (13) and (14) describe completely the set of all competitive allocations that can be attained through government policies.

The solution of the consumption-tax Ramsey problem with respect to m_t is

$$-l_m(t)\left[\theta_t F_n^I(t) - \psi(1-k)u_h(t)\right] = 0. \qquad \textbf{(A.12)}$$

The term in brackets is positive for any degree of homogeneity of the transaction function,[35] which in turn implies that l_m must be zero. This Ramsey solution can be decentralized through a zero nominal interest rate, implying no inflation tax when a uniform consumption tax is possible. This result verifies the claim that when the government cannot levy a complete tax on consumption, because an informal sector is present, the inflation tax serves as an imperfect proxy. That is, inflation proxies for the government's inability to tax consumption in the informal sector.[36]

This consumption tax result indicates that, in principle, the government can replace the inflation tax by a uniform tax on consumption, τ_c. In practice, we believe that it is difficult for governments to monitor transactions of final goods in the informal sector, for the same reasons that the government cannot monitor labor contracts in the informal sector. In our economy, there is only one consumption good. Thus, we assumed at the outset that the government cannot tax consumption. Instead we could have considered an economy with two goods c^F and c^I, preferences $u(c)$ with $c = [\mu(c^F)^\rho + (1-\mu)(c^I)^\rho]^{\frac{1}{\rho}}$, and assumed that the government can levy tax $\tau_c^F > 0$ on the good in the formal sector, but not on the good in the informal sector. The important part of both the single- and two-consumption good specifications is that taxation of consumption is not possible in the informal sector, $\tau_c^I = 0$. Thus, the Diamond and Mirrlees (1971) conditions for production efficiency are not met. Effectively, money is an intermediate good in the model. As a consequence, it is optimal to tax it when an economy has an informal sector.

A.3. MODEL WITH CURRENCY SUBSTITUTION

We now augment the model in Section 2 to include currency substitution. Preferences, production technology, and the government sector are unchanged. Consumers may hold either domestic, M, or foreign, F, currency for transaction purposes. We consider the case of a small open economy. The transaction cost technology is given by (20). Since prices are flexible, PPP holds; that is, $P_t = \xi_t P_t^*$. Therefore, the transaction technology can be rewritten as[37] $s_t \geq l(c_t, m_t, f_t)$, where $m_t = \frac{M_t}{P_t}$ and $f_t = \frac{F_t}{P_t^*}$.

The representative household's one-period budget constraint is

$$c_t + \frac{M_t}{P_t} + \frac{\xi_t F_t}{P_t} + \frac{B_t}{P_t} \leq \frac{M_{t-1}}{P_t} + \frac{\xi_t F_{t-1}}{P_t} + (1+i_{t-1})\frac{B_{t-1}}{P_t} + \left(1-\tau_t^n\right) w_t^F n_t^F$$
$$+ w_t^I\left(1 - n_t^F - s_t - h_t\right).$$

The consumer's budget constraint in present-value form is

$$\sum_{t=0}^{\infty} d_t c_t + \sum_{t=0}^{\infty} d_t I_t m_t + \sum_{t=0}^{\infty} d_t I_t^* f_t \leq \sum_{t=0}^{\infty} d_t (1-\tau^n) w_t^F n_t^F + \sum_{t=0}^{\infty} d_t w_t^I \left(1 - h_t - s_t - n_t^F\right),$$

where

$$d_t = \frac{1}{\prod_{s=0}^{t-1}(1+r_s)}, \qquad (1+r_s) = (1+i_s)\frac{P_s}{P_{s+1}}, \qquad I_t = \frac{i_t}{1+i_t}, \quad \text{and}$$

$$I_t^* = 1 - \frac{1}{1+i_t}\frac{P_{t+1}}{P_t}\frac{P_t^*}{P_{t+1}^*}.$$

The household's conditions for an interior optimum are the same as in the case without foreign money plus the following condition:

$$-l_f(t) = \frac{1}{(1-\tau^n) w_t^F} I_t^*.$$

I_t^* depends on i_t, on the domestic inflation rate, and on the foreign inflation rate, which is given and cannot be controlled by the home government. The Ramsey problem has the same constraints as before, but the transaction technology now depends on foreign currency. Given that, it is straightforward to show that Proposition 1 remains unchanged.