

Problem Set 1

- (1) Consider the exponential family submodels,

$$p_\epsilon^k(z) = c(\epsilon, p^k) \exp(\epsilon D(p^k, z)) p^k(z),$$

where $c(\epsilon, p^k)$ is just the normalizing constant at ϵ for the semiparametric model, p^k , at the k th iteration. And $D(p^k, z)$ is the efficient influence function, EIF, for estimation of the parameter $\psi(p)$ in the semiparametric model, \mathcal{P} .

- (a) Given a random sample, $\{z_i, \dots, z_n\}$, describe how to find the MLE for ϵ first, assuming that you had an explicit form for c , and then if you did not.
(b) Show that evaluating the log likelihood score equation,

$$\frac{d}{d\epsilon} \sum_{i=1}^n \log p_\epsilon^k(z_i) |_{\epsilon=0} = \sum_{i=1}^n D(p^k, z_i).$$

- (c) Conclude that iterating the MLE solutions for ϵ , eventually producing a p^k for which $\hat{\epsilon} \approx 0$ yields an efficient estimator of the parameter of interest $\psi(p^k)$.
(2) Reconsider question one for the other Diaz and Rosenblum subfamily

$$p_\epsilon^k(z) = c(\epsilon, p^k) (1 + \exp(-2\epsilon D(p^k, z)))^{-1} p^k(z),$$

and comment on the advantages if any of this formulation.

- (3) Using the code provided in the lecture for the Diaz and Rosenblum estimator of the standard missing data problem, design a *small* simulation experiment to compare its performance with various naive estimators like the mean of the observed y_i 's, and the Horvitz-Thompson estimator. Or, preferably, as an alternative design a small simulation experiment for the Diaz QTET estimator intended to compare approaches. The third installment of the Lecture Notes contains some initial steps in this direction.